

13. (a) The free-body diagram for the crate is shown below.  $\vec{T}$  is the tension force of the rope on the crate,  $\vec{F}_N$  is the normal force of the floor on the crate,  $m\vec{g}$  is the force of gravity, and  $\vec{f}$  is the force of friction. We take the  $+x$  direction to be horizontal to the right and the  $+y$  direction to be up. We assume the crate is motionless. The equations for the  $x$  and the  $y$  components of the force according to Newton's second law are:

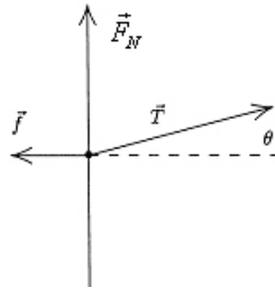
$$\begin{aligned} T \cos \theta - f &= 0 \\ T \sin \theta + F_N - mg &= 0 \end{aligned}$$

where  $\theta = 15^\circ$  is the angle between the rope and the horizontal. The first equation gives  $f = T \cos \theta$  and the second gives  $F_N = mg - T \sin \theta$ . If the crate is to remain at rest,  $f$  must be less than  $\mu_s F_N$ , or  $T \cos \theta < \mu_s (mg - T \sin \theta)$ . When the tension force is sufficient to just start the crate moving, we must have

$$T \cos \theta = \mu_s (mg - T \sin \theta).$$

We solve for the tension:

$$\begin{aligned} T &= \frac{\mu_s mg}{\cos \theta + \mu_s \sin \theta} \\ &= \frac{(0.50)(68)(9.8)}{\cos 15^\circ + 0.50 \sin 15^\circ} \\ &= 304 \approx 3.0 \times 10^2 \text{ N.} \end{aligned}$$



(b) The second law equations for the moving crate are

$$\begin{aligned} T \cos \theta - f &= ma \\ F_N + T \sin \theta - mg &= 0. \end{aligned}$$

Now  $f = \mu_k F_N$ , and the second equation gives  $F_N = mg - T \sin \theta$ , which yields  $f = \mu_k (mg - T \sin \theta)$ . This expression is substituted for  $f$  in the first equation to obtain

$$T \cos \theta - \mu_k (mg - T \sin \theta) = ma,$$

so the acceleration is

$$a = \frac{T(\cos \theta + \mu_k \sin \theta)}{m} - \mu_k g.$$

Numerically, it is given by

$$a = \frac{(304 \text{ N})(\cos 15^\circ + 0.35 \sin 15^\circ)}{68 \text{ kg}} - (0.35)(9.8 \text{ m/s}^2) = 1.3 \text{ m/s}^2.$$