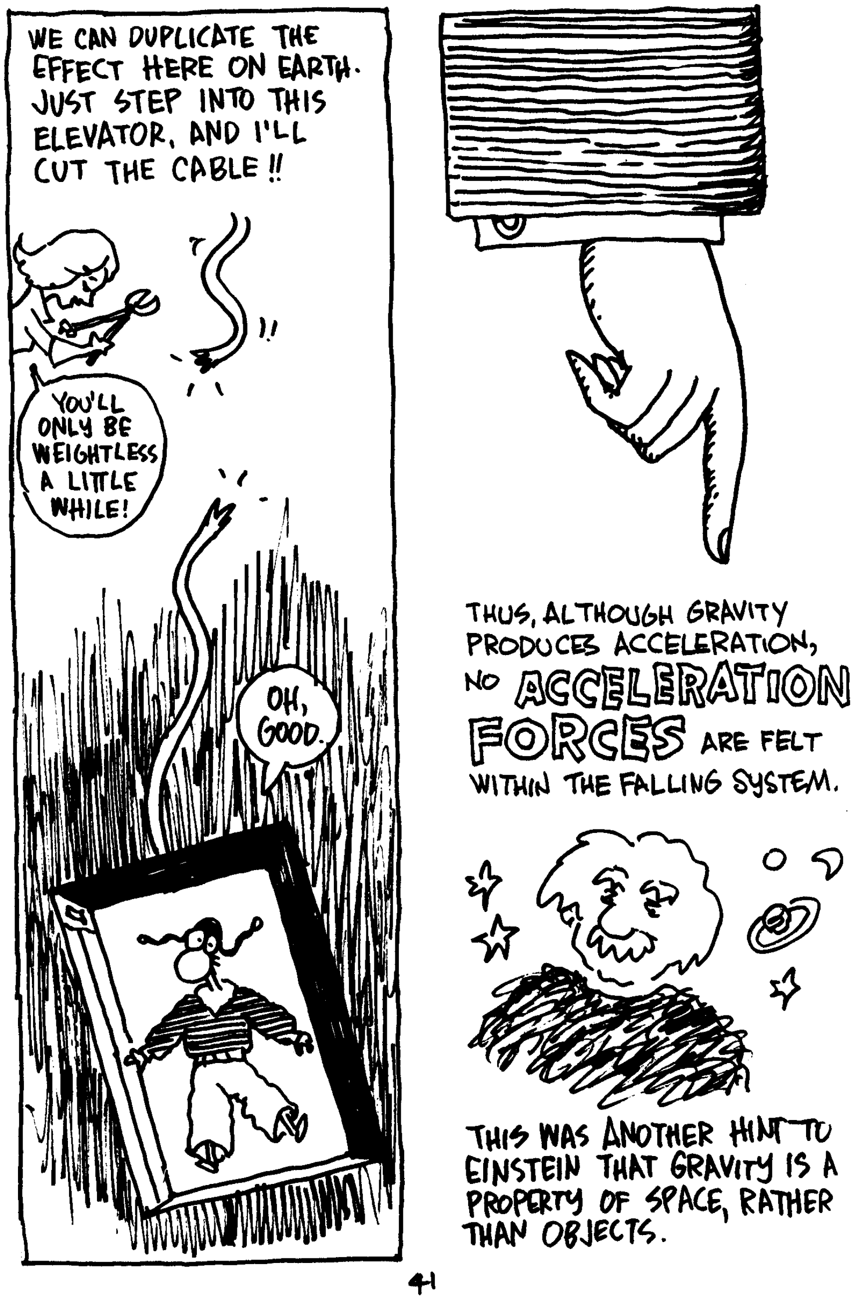
Lecture 9
Chapter 13
Gravitation

Gravitation

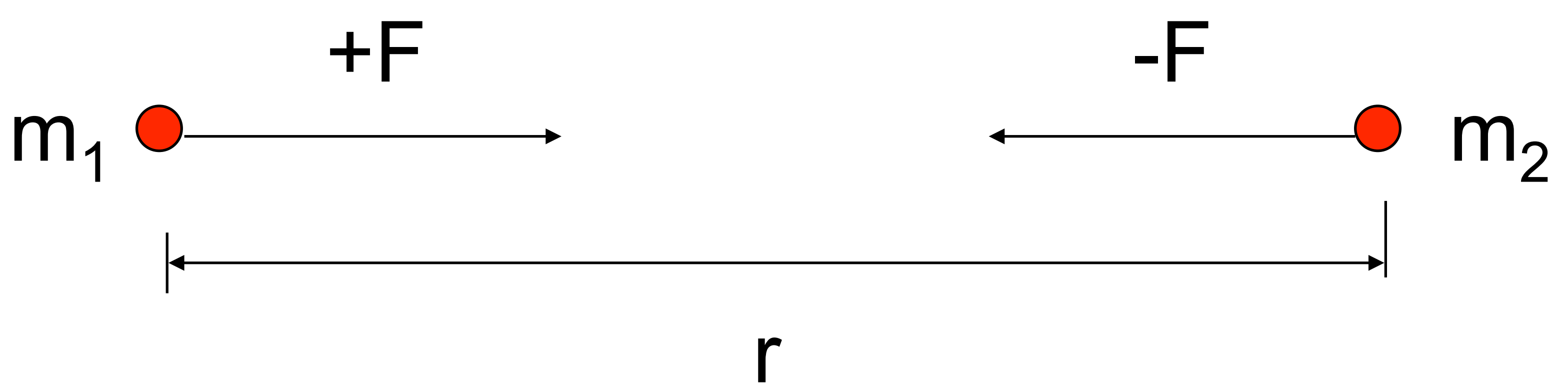


UNIVERSAL GRAVITATION

For any two masses in the universe:

$$F = \frac{Gm_1 m_2}{r^2}$$

G = a constant evaluated
by Henry Cavendish



Two people pass in a hall. Find the gravitational force between them.

$$m_1 = m_2 = 70kg$$

$$r = 1m$$

$$F = \frac{Gm_1m_2}{r^2}$$

$$F = \frac{(6.7 \times 10^{-11})(70)(70)}{1^2} = 3.3 \times 10^{-7} N$$

1 millionth of an ounce

NEWTON: G DOES NOT CHANGE WITH MATTER

For masses near the earth $mg = \frac{GMm}{r^2}$

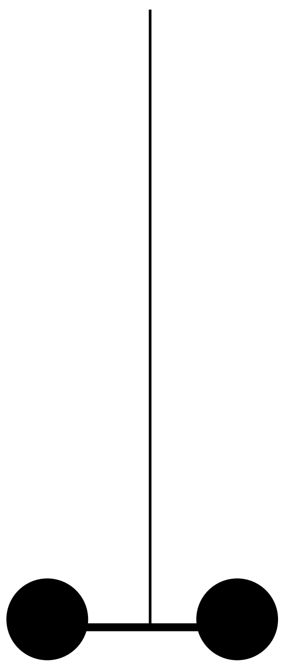
Therefore, $G = g\left[\frac{r^2}{M}\right]$

Newton built pendula of different materials, and measured g at a fixed location, finding it to remain constant.

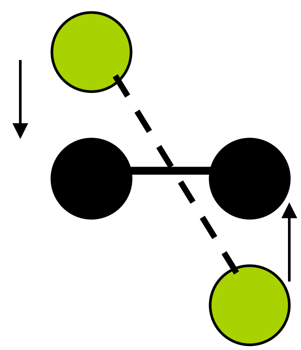
Therefore he concluded that G is independent of the kind of matter. All that counts is mass.

CAVENDISH: MEASURING G

Torsion Pendulum



Side View



Top View

Modern value:

$$G = 6.674 \times 10^{-11} Nm^2 / kg^2$$

Definition of Weight

- The weight of an object on the earth is the gravitational force the earth exerts on the object.

$$W = \frac{mGM_E}{R_E^{\ 2}} = mg$$

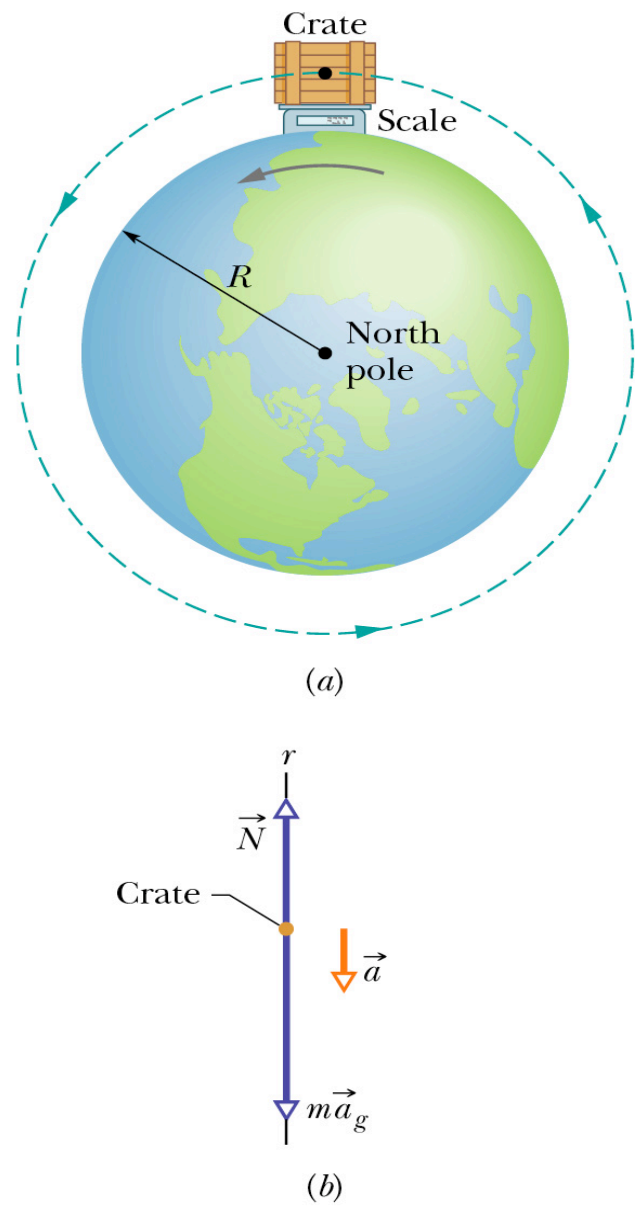
$$R_E = 6400 km$$

$$W = (70 kg)(9.8) = 668 N$$

- How much less would he weigh at the equator due to Earth's rotation?

$$N - mg = -ma_c$$

$$N = m(g - a_c)$$



Amount you weigh less at the equator

$$N - mg = -ma_c$$

$$N = m(g - a_c) = mg - ma_c$$

$$N = 668N - 70a_c$$

$$a_c = \frac{v^2}{R_E}$$

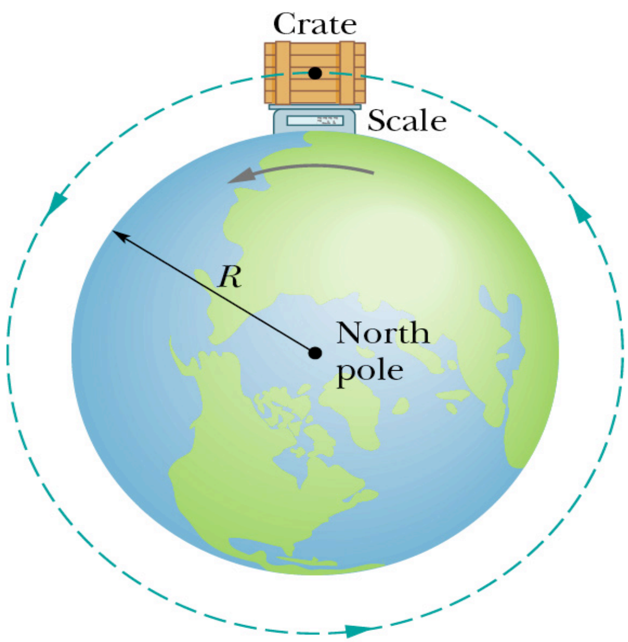
$$v = \frac{2\pi R_E}{T} = \frac{6.28(6400km)}{24(3600)} = 465m/s$$

$$a_c = \frac{(465)^2}{6400000} = 0.034m/s^2$$

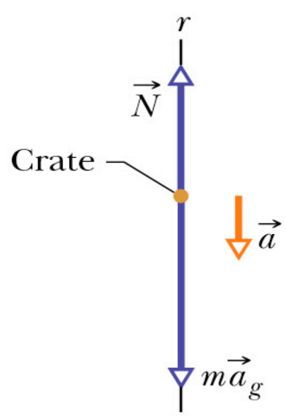
At the equator, the amount you weigh less is:

$$70a_c = (70)(0.034) = 0.21N = 0.04lb = 0.64oz.$$

$$1N = 0.22lb$$



(a)



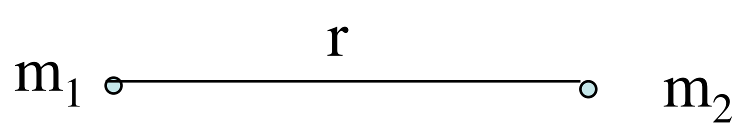
(b)

Variation of g near Earth's Surface

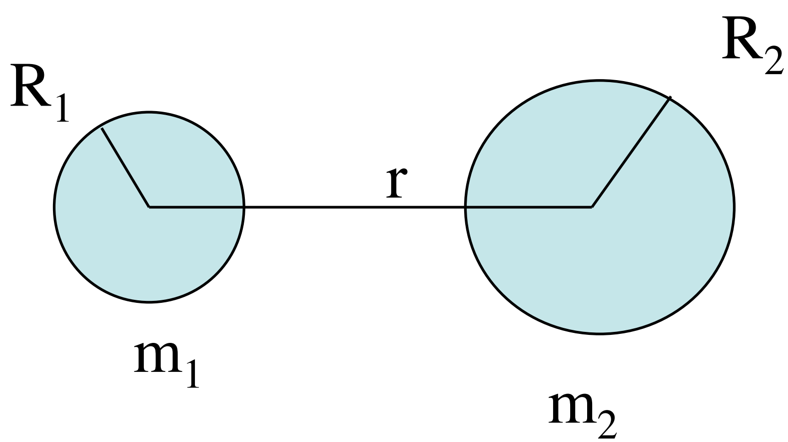
Location	$g(m/s^2)$	Altitude (km)
Charlottesville	9.80	0
Latitude 0^0 (sea level)	9.78	0
Latitude 90^0(sea level)	9.83	0
Mt Everest	9.80	8.8
Space shuttle orbit	8.70	400
Communications satellite	0.225	35,700

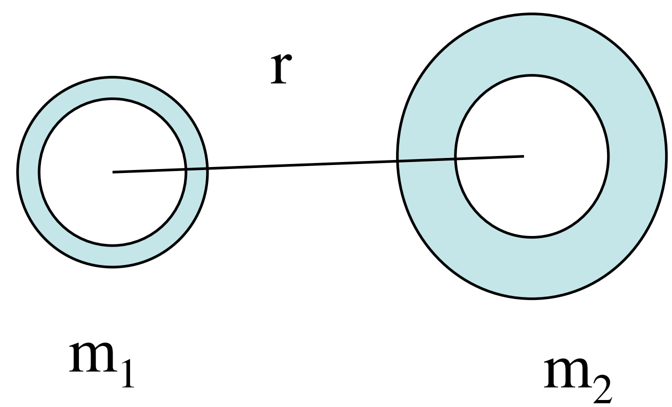
Some properties of Newton's Gravitational Inverse Square Force Law

$$F = \frac{Gm_1 m_2}{r^2}$$



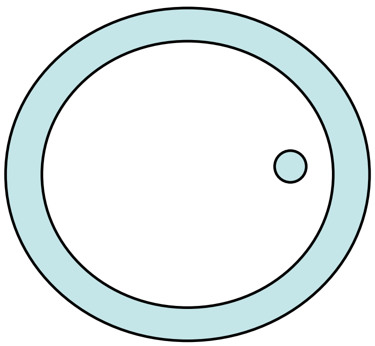
1. The force between two solid spherical masses or two shells of different radii is the same as the difference between two point masses separated by their centers





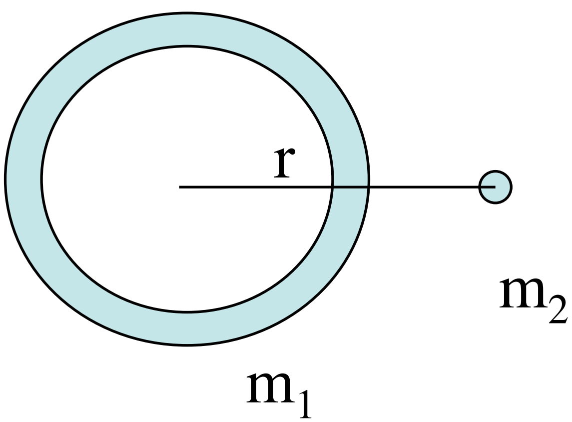
Newton's Shell Theorem

A uniform shell of matter exerts no net gravitational force on a particle located inside it



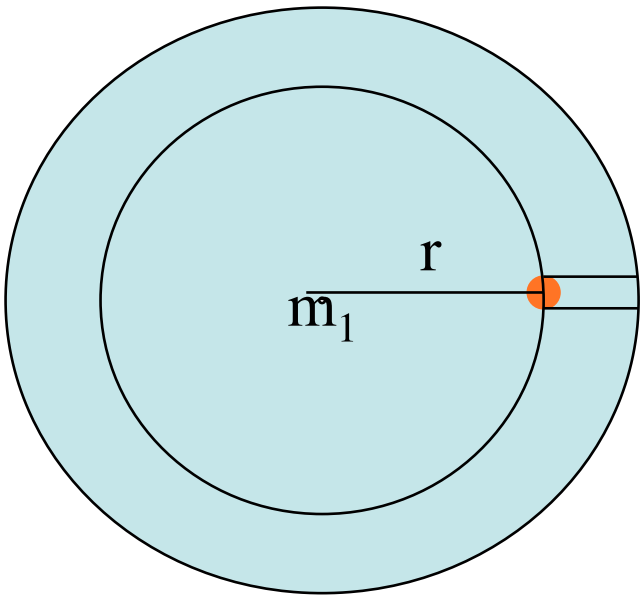
$F = 0$

A uniform spherical shell of matter attracts a particle that is outside the shell as if all the shell's mass were concentrated at its center.



$$F = \frac{Gm_1m_2}{r^2}$$

Newton's Shell Theorem



m_1 — Mass inside inner circle

m_2 — Red ball inside hole

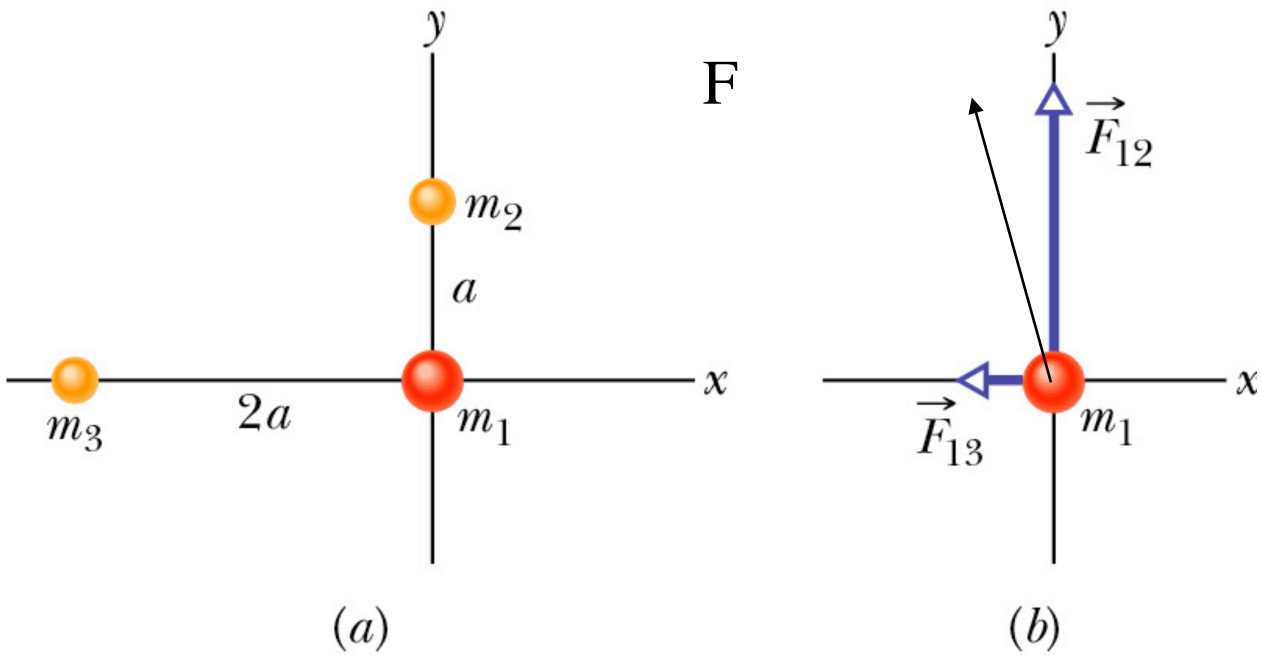
$$F = \frac{Gm_1m_2}{r^2}$$

How is mass defined?

1. You can measure the force acting on it and measure the acceleration and take the ratio. $m_i = F/a$
   This mass is called the inertial mass.

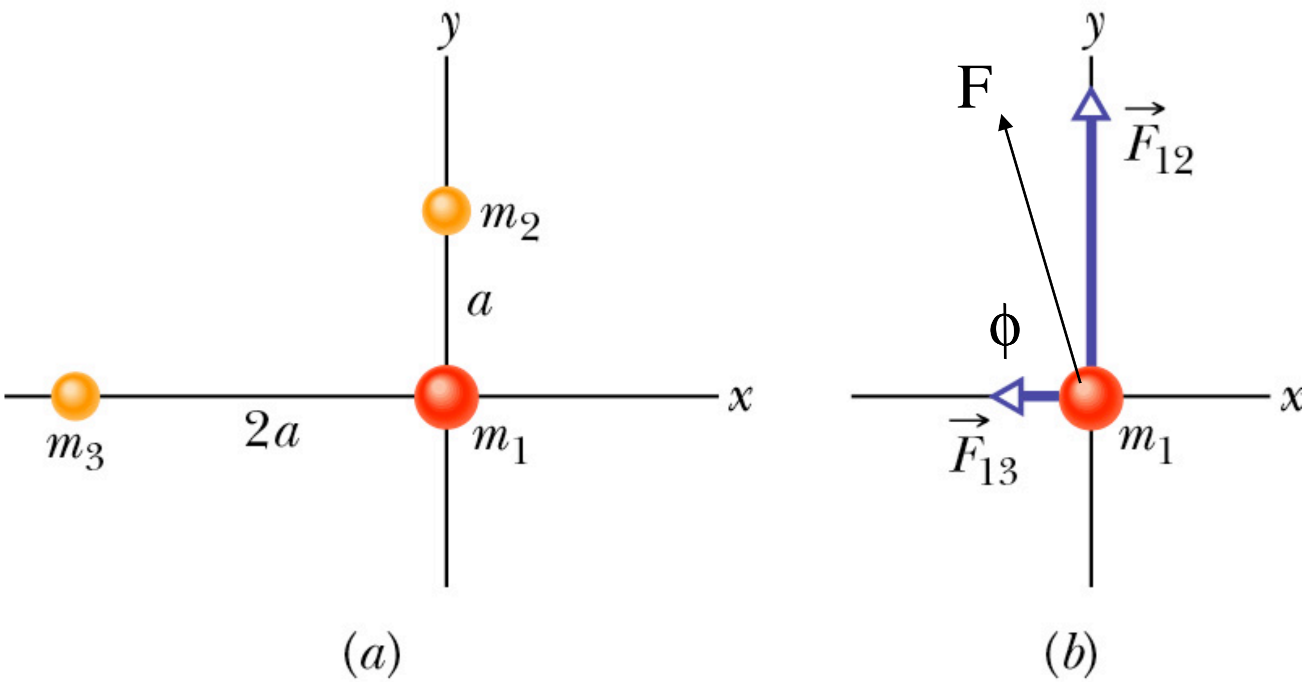
2. You can weigh it and divide by g. This is called the gravitational mass. For all practical purposes they are equivalent,

Principle of Superposition



The force F on mass m_1 is the vector sum of the forces on it due to m_2 and m_3

Principle of Superposition



$$\vec{F} = \vec{F}_{12} + \vec{F}_{13}$$

$$\left|\vec{F}_{12}\right| = m_1 m_2 G / a^2$$

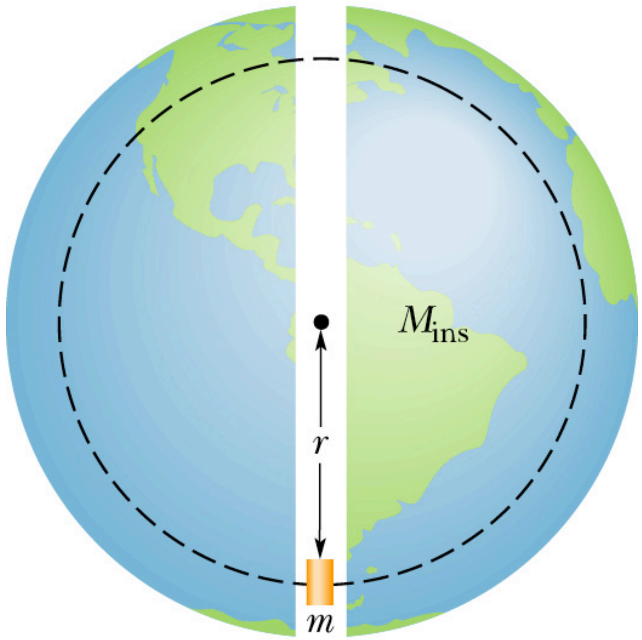
$$\left|\vec{F}_{13}\right| = m_1 m_3 G / 4a^2$$

$$\left|\vec{F}\right| = \left(\sqrt{m^2{}_2 + \tfrac{1}{16} m^2{}_3}\right) m_1 G / a^2$$

$$\phi = \tan^{-1}\frac{F_{12}}{F_{13}} = \tan^{-1}\frac{4m_2}{m_3}$$

If $m_2 = m_3$, then ϕ=76.0 deg

Gravitational force on a particle inside the Earth



$$F = \frac{GmM_{ins}}{r^2}$$

$$M_{ins} = \rho V_{ins} = \rho \frac{4\pi r^3}{3}$$

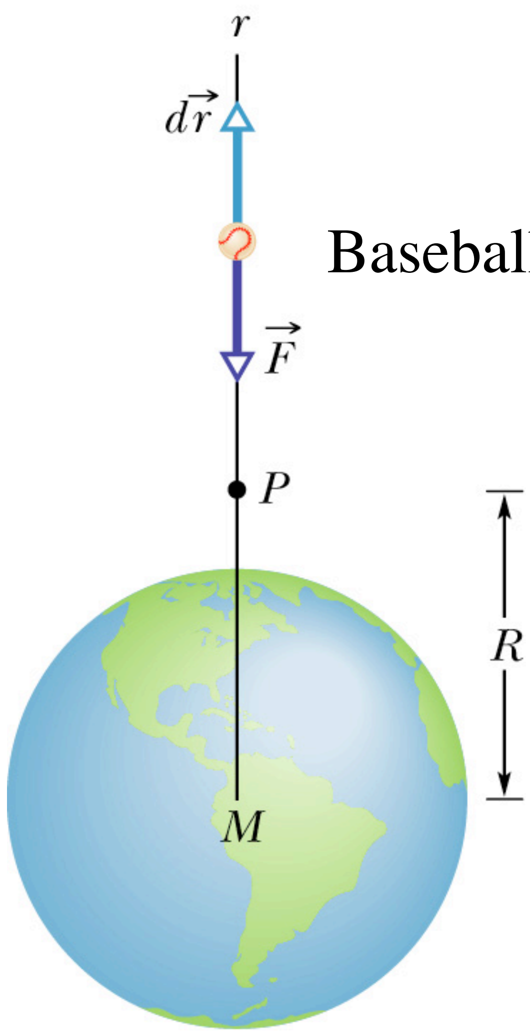
$$F = \frac{Gm_s}{r^2} \rho \frac{4\pi r^3}{3}$$

$$\vec{F} = \frac{4\pi G m \rho}{3} \vec{r}$$

$$\vec{F} = -k\vec{r}$$

Doesn't this force remind you of a mass on a spring?
What is the resultant motion look like?

We want to find the potential energy for the gravitational force acting on a particle far outside the Earth



Baseball

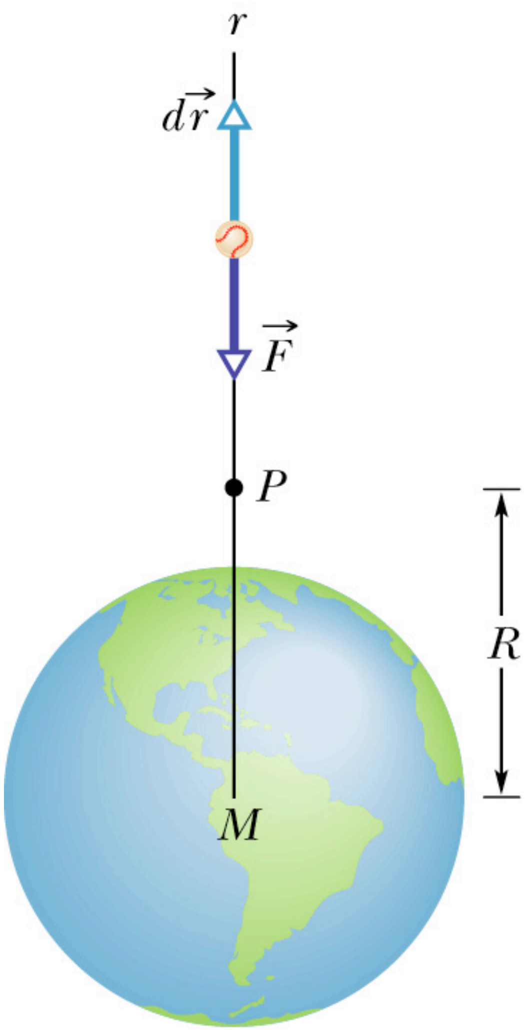
Let's find the work done on a mass moving in a gravitational field.

Consider a super steroid user hitting a baseball directly away from earth and the work done by gravity in slowing it down.

Neglect friction due to the air.

$$W = \text{Force} \times \text{distance}$$

Gravitational Potential Energy



$$F = \frac{Gm_1m_2}{r^2}$$

$$F(r) = -\Delta U = -\frac{dU}{dr}$$

Note the minus sign
Angle =180 deg

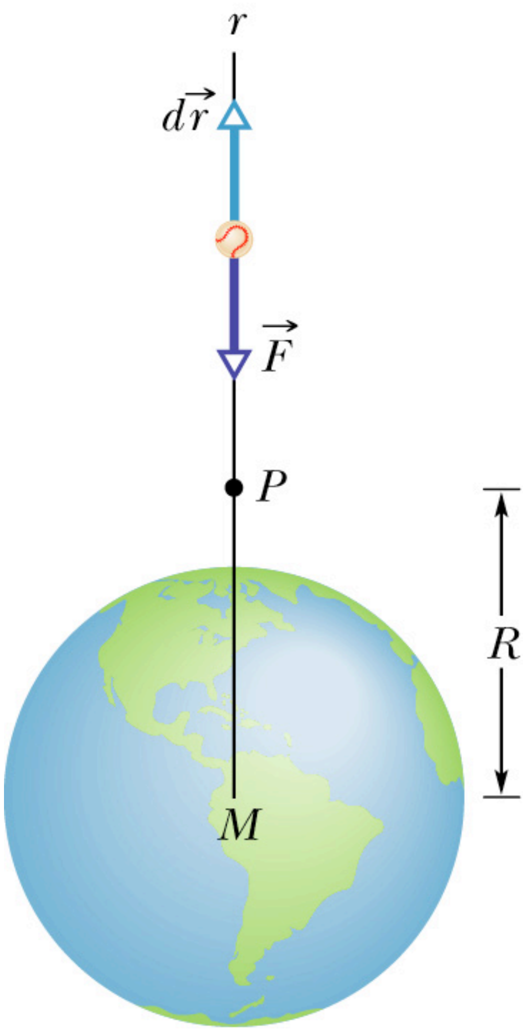
$$W = \int_R^\infty \vec{F}.d\vec{r}$$

$$W = -\int_R^\infty GMm\frac{1}{r^2}\,dr = -GMm\int_R^\infty \frac{1}{r^2}\,dr$$

$$W = +GMm\frac{1}{r}\bigg|_R^\infty = GMm\left(\frac{1}{\infty} - \frac{1}{R}\right)$$

$$W = -\frac{GMm}{R}$$

Gravitational Potential Energy



$$W = +GMm\frac{1}{r}\Big|_R^{\infty} = GMm\left(\frac{1}{\infty} - \frac{1}{R}\right) = -\frac{GMm}{R}$$

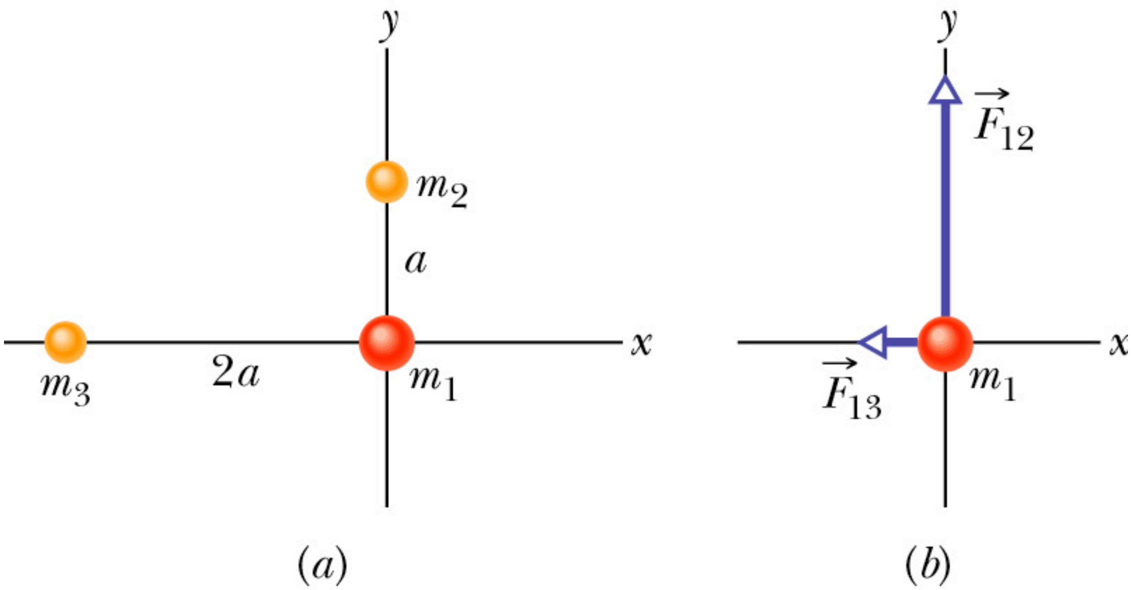
$$W = -(U_\infty - U(R))$$

$$U_\infty - U = \text{-W}$$

$$\text{U}_\infty = 0$$

$$U = W = -\frac{GmM}{R}$$

Gravitational Potential energy of a system



$$U = -\frac{GmM}{r}$$

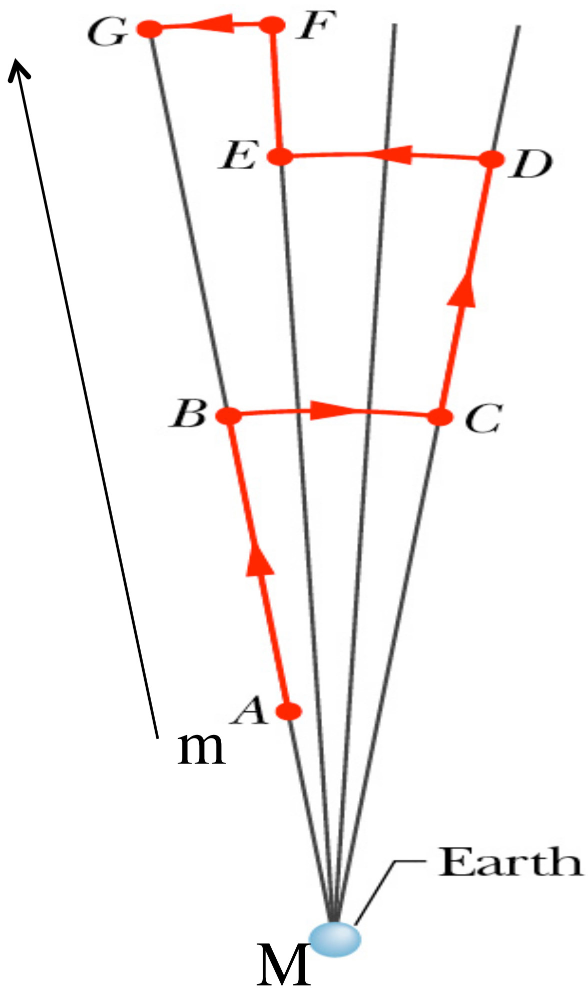
$$U_{12} = -\frac{Gm_1 m_2}{a}$$

Use principle of superposition again

$$U = U_{12} + U_{13} + U_{23}$$

$$U = -\left(\frac{Gm_1 m_2}{a} + \frac{Gm_1 m_3}{2a} + \frac{Gm_2 m_3}{\sqrt{5}a}\right)$$

Path Independence - Conservative Force



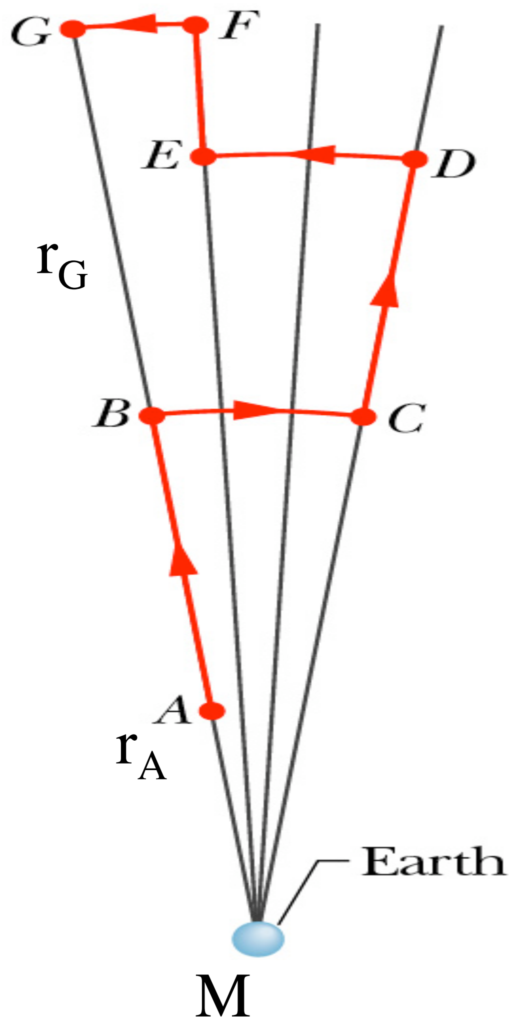
$$F = -\frac{dU}{dr}$$

$$U = -\frac{GmM}{r}$$

$$F = -\frac{GmM}{r^2}$$

The minus sign means the
force points inward toward big M

Difference in potential energy between two points only depends on end points.



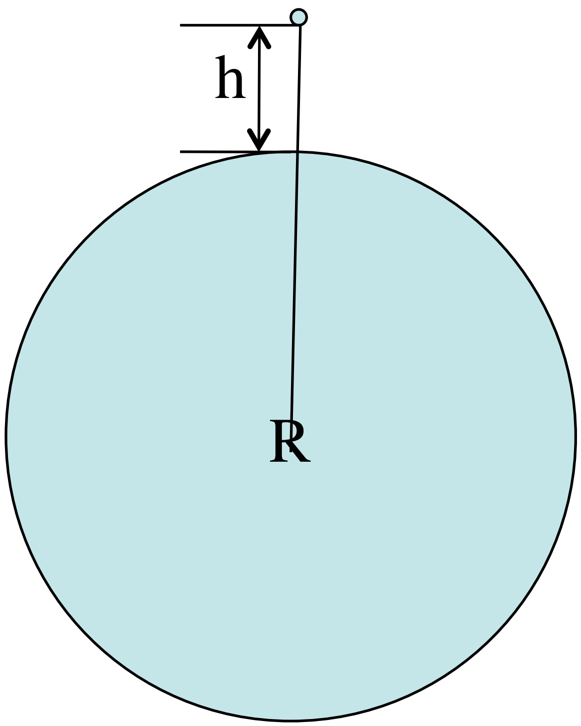
The difference in potential energy in a mass m moving from A to G is

$$U = -\frac{GmM}{r}$$

$$U_G - U_A = -\frac{GmM}{r_G} - (-\frac{GmM}{r_A})$$

$$U_G - U_A = -GmM(\frac{1}{r_G} - \frac{1}{r_A})$$

How is our old definition U=mgh related to our new definition of potential energy



$$\Delta U = U(h+R) - U(R)$$

$$\Delta U = -\frac{mMG}{h+R} + \frac{mMG}{R}$$

$$\Delta U = mMG(-\frac{1}{h+R} + \frac{1}{R}) = mMG(\frac{-R+h+R}{R(R+h)})$$

$$\Delta U = \frac{mMG}{R^2}(\frac{h}{(1+\frac{h}{R})})$$

Now neglect $\frac{h}{R}$ $\quad h \ll R$

$$\Delta U \cong \frac{mMG}{R^2} h = mgh$$

$$g = \frac{MG}{R^2}$$

So if we measure U relative to the surface we get the same result

Some consequences of a $1/r^2$ potential
Escape Speed

There is a certain minimum initial speed that when you fire a projectile upward it will never return.

It has total energy $E = K + U$

At the surface of the Earth it has $E = \frac{1}{2}mv^2 + -(\frac{GMm}{r})$

When it just reaches infinity it has 0 kinetic energy and 0 potential energy so its total energy is zero. Since energy is conserved it must also have 0 at the Earth's surface.

$$0 = \frac{1}{2}mv^2 + -(\frac{GMm}{R})$$

Solve for v

$$v = \sqrt{\frac{2GM}{R}}$$

Some escape speeds

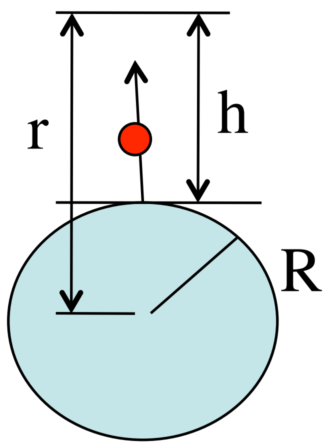
Moon 2.38 km/s = 5,331 mi/hr

Earth 11.2 km/s = 25,088 mi/hr

Sun 618 km/s = 1,384,320 mi/hr

Problem 39 Ed 6

- A projectile is fired vertically from the surface of the earth with a speed of 10 km/s. Neglecting air drag, how far will it go?



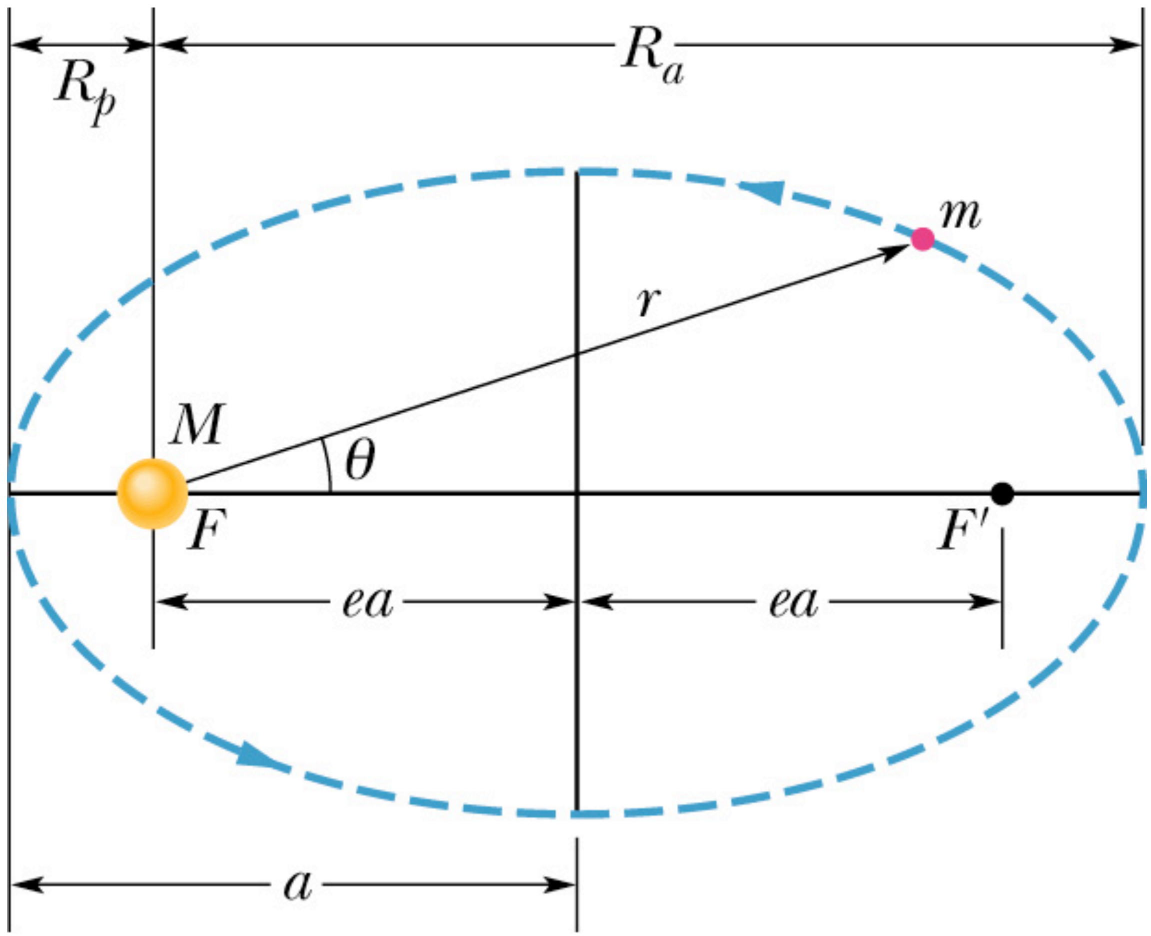
Kepler's Laws

1. Law of Orbits: All planets move in elliptical orbits with the sun at one focus

2. The Law of Areas: A line that connects a planet to the sun sweeps out equal areas in the plane of the planets orbit in equal time intervals. dA/dt is constant

3. The Law of Periods: The square of the period of any planet is proportional to the cube of the semimajor axis of its orbit

1. All planets move in elliptical orbits with the sun at one focus



$$E = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

$$\frac{1}{r} = c(1 + e\cos\theta)$$

$$e = \sqrt{1 + \frac{2EL^2}{G^2m^3M^2}}$$

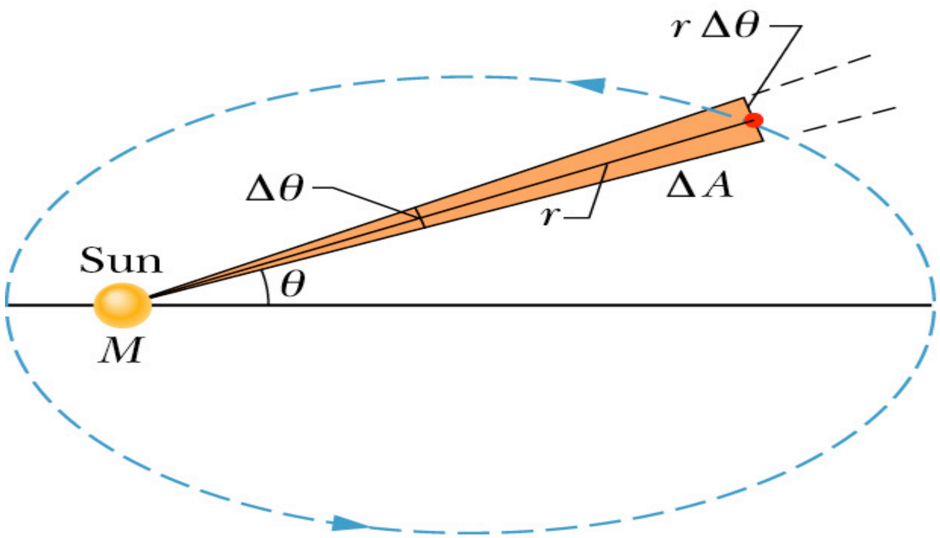
$e > 1$ $E > 0$ hyperbola

$e = 1$ $E = 0$ parabola

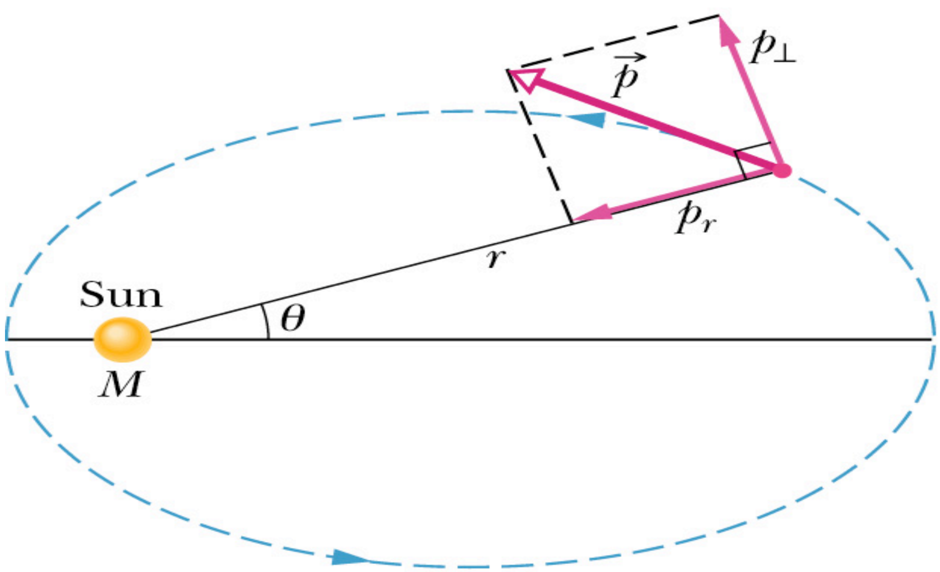
$e < 1$ $E < 0$ ellipse

$e = 0$ $E < 0$ circle

2. Law of Areas



(a)



(b)

$$\Delta A = \frac{1}{2}(r\Delta\theta)(r) = \frac{1}{2}r^2\Delta\theta$$

$$\frac{dA}{dt} = \tfrac{1}{2}r^2\frac{d\theta}{dt}$$

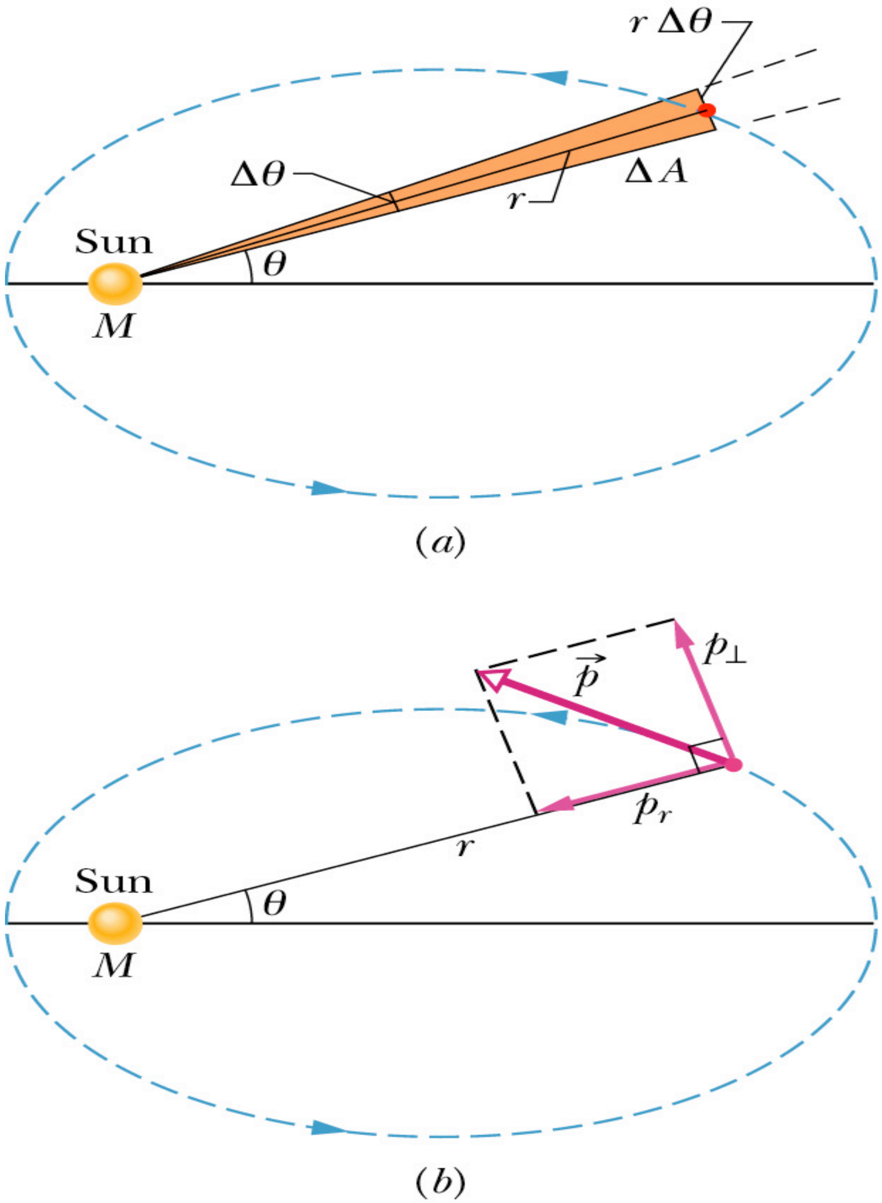
w

$$\frac{dA}{dt} = \tfrac{1}{2}r^2\frac{d\theta}{dt} = \tfrac{1}{2}r^2\omega$$

$$L = rp_\perp = rmv_\perp = mr^2\omega$$

L is a conserved quantity, since the torque is r x F = 0

2. Law of Areas



$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{1}{2} r^2 \omega$$

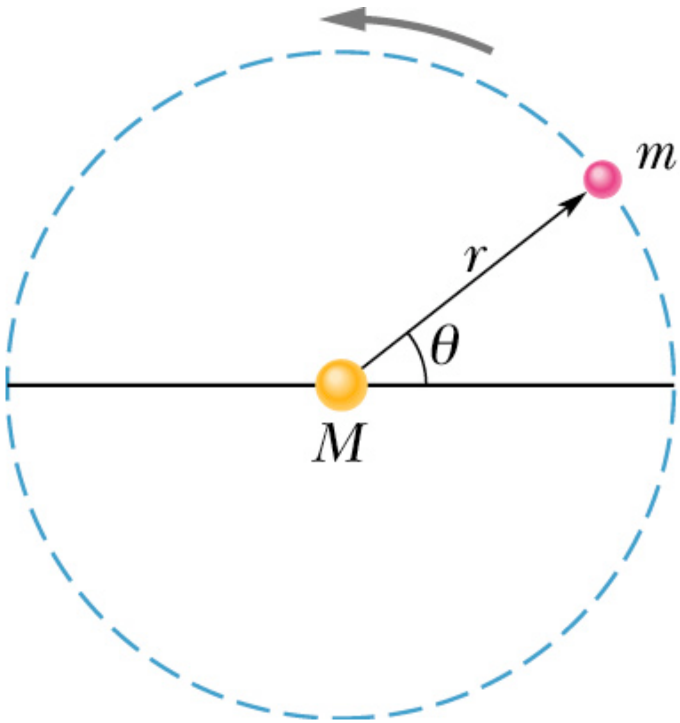
$$L = rp_\perp = rmv_\perp = mr^2\omega$$

$$r^2\omega = \frac{L}{m} = \text{constant}$$

$$\frac{dA}{dt} = \frac{L}{2m} = \text{constant}$$

Since L is a constant, dA/dt = constant. As the earth moves around the sun, it sweeps out equal areas in equal times

Law of Periods



Consider a circular orbit more like the Earth

Gravitational force is balanced by force due to centripetal acceleration

$$\frac{GmM}{r^2} = mv^2/r$$

$$= m\omega^2 r = m(2\pi/T)^2 r$$

$$\frac{GmM}{r^2} = m(2\pi/T)^2 r$$

$$\frac{T^2}{a^3} = 3\times10^{-34}\, y^2/m^3$$

$$T^2 = \frac{4\pi^2}{GM} r^3$$

True for any central force and elliptical orbit. r=a

See Table13-3 page 344 use Elmo

Energies for orbiting satellites or planets (True in general for inverse square law)

For a satellite in a circular orbit we again write

$$\frac{GmM}{r^2} = mv^2/r$$

$$K = \tfrac{1}{2}mv^2 = GmM/2r$$

$$K = -U/2$$

The total energy is E = K + U $\quad = \dfrac{GMm}{2r} - \dfrac{GMm}{r}$

$$= -GMm/2r$$

Note that the total energy is the negative of the kinetic energy.

A negative total energy means the system is bound.

For an elliptical orbit E = - GMm/2a where a is the semi major axis

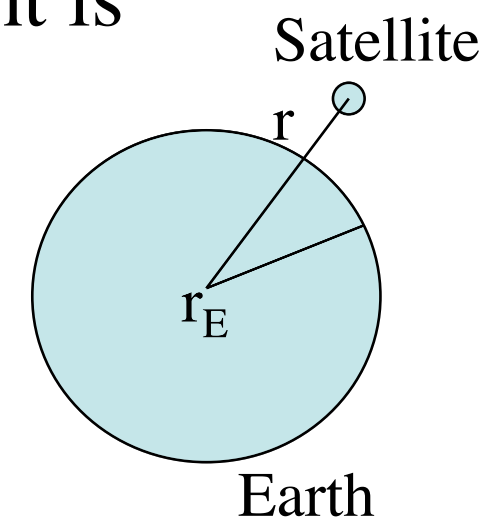
Websites

http://galileoandeinstein.physics.virginia.edu/more_stuff/Applets/home.html

http://galileoandeinstein.physics.virginia.edu/more_stuff/flashlets/home.htm

What is the weight of Satellite in orbit?

Suppose we have a geosynchronous communication satellite in orbit a distance 42,000 km from the center of the earth. If it weighs 1000 N on earth, how much does it weigh at that distance? The weight is



$$F = \frac{G m_S M_E}{r^2}$$

$$\frac{F_r}{F_E} = \frac{{r_E}^2}{r^2}$$

$$F_r = F_E \frac{{r_E}^2}{r^2} \qquad (1000N)\left[\frac{6400}{42000}\right]^2 = 23N$$

In the previous problem the distance to the geosynchronous TV Satellite was given as 42,000km. How do you get that number?

$$\frac{GmM_E}{r^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{GM_E}{r}}$$

$$v = \sqrt{\frac{GM_E}{r}} = \frac{2\pi r}{T}$$

$$T = \frac{2\pi r^{3/2}}{\sqrt{GM_E}}$$

$$r = \left[\frac{GM_E T^2}{4\pi^2}\right]^{1/3}$$

Geosynchronous satellites have the same period as the earth

$$T = 24hr = 8.6\times 10^4 s$$

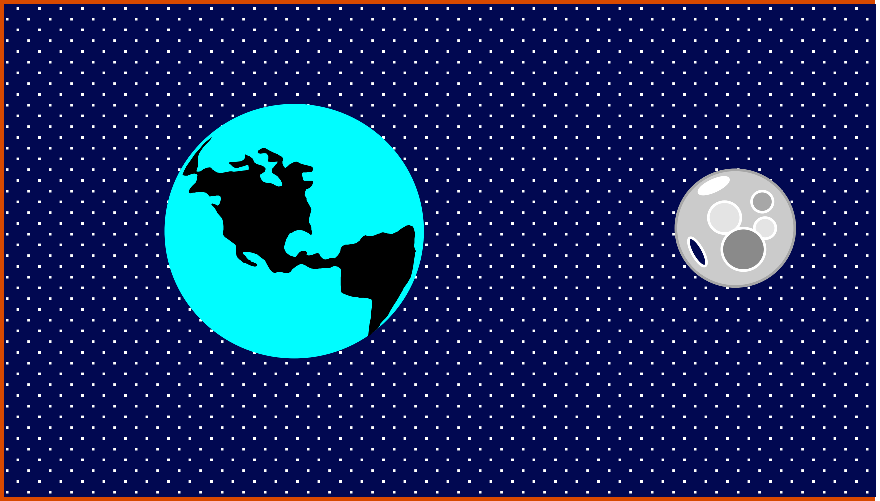
$$r = \left[\frac{GM_E T^2}{4\pi^2}\right]^{\frac{1}{3}} = \left[\frac{6.67\times 10^{-11}\times 5.98\times 10^{24}\times 74.0\times 10^8}{4\pi^2}\right]^{\frac{1}{3}}$$

$$= 42,000km = 26,000mi$$

ConcepTest 12.1a Earth and Moon I

Which is stronger, Earth's pull on the Moon, or the Moon's pull on Earth?

1) **the Earth pulls harder on the Moon**
2) **the Moon pulls harder on the Earth**
3) **they pull on each other equally**
4) **there is no force between the Earth and the Moon**
5) **it depends upon where the Moon is in its orbit at that time**



ConcepTest 12.1b Earth and Moon II

If the distance to the Moon were doubled, then the force of attraction between Earth and the Moon would be:

1) **one quarter**
2) **one half**
3) **the same**
4) **two times**
5) **four times**

ConcepTest 12.5 In the Space Shuttle

Astronauts in the space shuttle float because:

1) They are so far from Earth that Earth's gravity doesn't act any more.

2) Gravity's force pulling them inward is cancelled by the centripetal force pushing them outward.

3) While gravity is trying to pull them inward, they are trying to continue on a straight-line path.

4) Their weight is reduced in space so the force of gravity is much weaker.

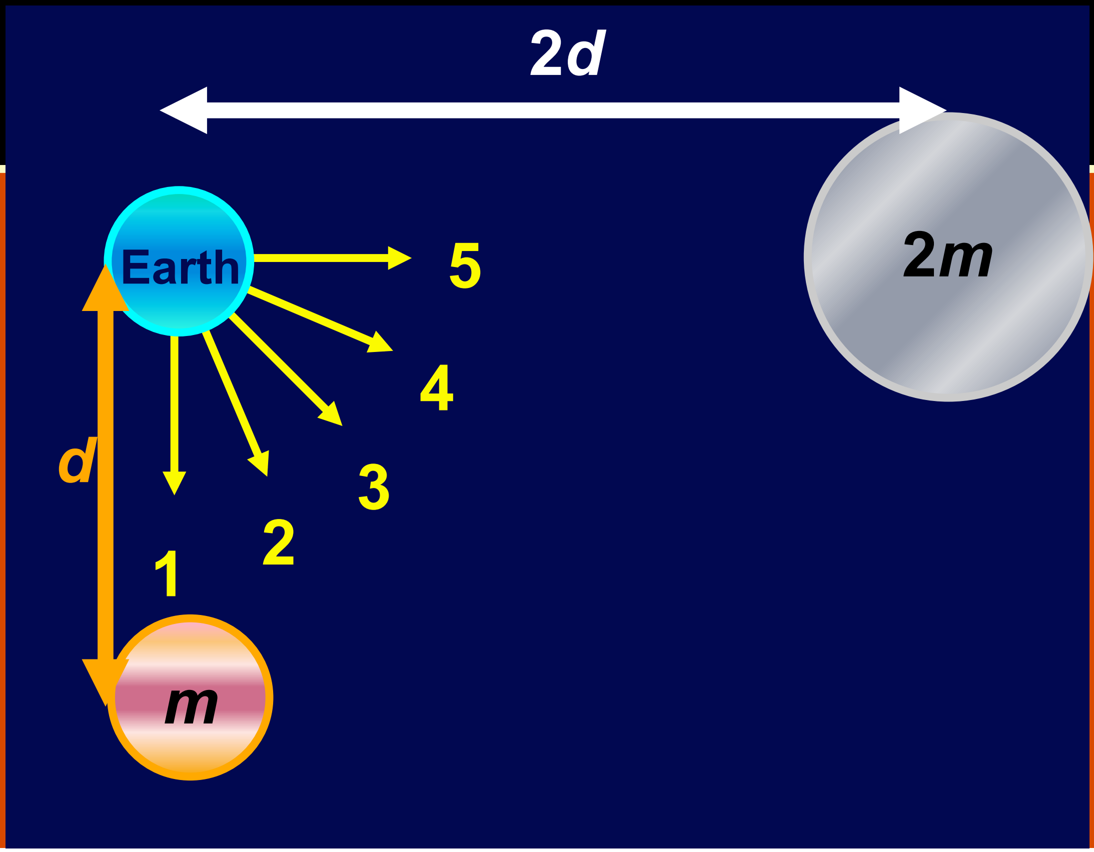
ConcepTest 12.6 Guess my Weight

If you weigh yourself at the equator of Earth, would you get a bigger, smaller or similar value than if you weigh yourself at one of the poles?

1) bigger value
2) smaller value
3) same value

ConcepTest 12.7 Force Vectors

A planet of mass *m* is a distance *d* from Earth. Another planet of mass 2*m* is a distance 2*d* from Earth. Which force vector best represents the direction of the total gravitation force on Earth?

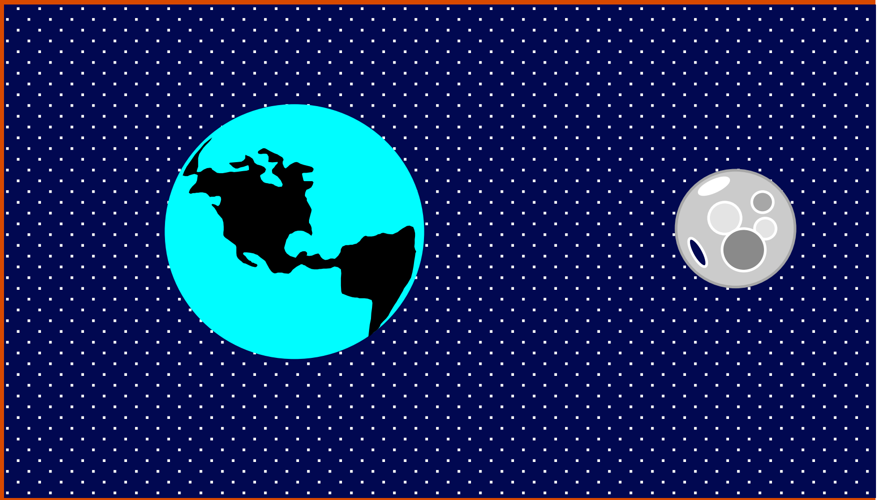


ConcepTest 12.1a Earth and Moon I

Which is stronger, Earth's pull on the Moon, or the Moon's pull on Earth?

1) the Earth pulls harder on the Moon
2) the Moon pulls harder on the Earth
3) they pull on each other equally
4) there is no force between the Earth and the Moon
5) it depends upon where the Moon is in its orbit at that time

By Newton's 3rd Law, the forces are equal and opposite.



ConcepTest 12.1b Earth and Moon II

If the distance to the Moon were doubled, then the force of attraction between Earth and the Moon would be:

1) one quarter
2) one half
3) the same
4) two times
5) four times

The gravitational force depends inversely on the distance squared. So if you increase the distance by a factor of 2, the force will decrease by a factor of 4.

$$F = G\frac{Mm}{R^2}$$

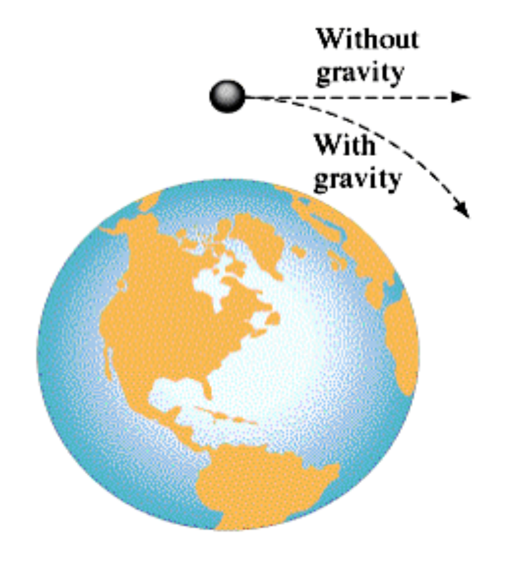
Follow-up: What distance would increase the force by a factor of 2?

ConcepTest 12.5 In the Space Shuttle

Astronauts in the space shuttle float because:

1) They are so far from Earth that Earth's gravity doesn't act any more.
2) Gravity's force pulling them inward is cancelled by the centripetal force pushing them outward.
3) While gravity is trying to pull them inward, they are trying to continue on a straight-line path.
4) Their weight is reduced in space so the force of gravity is much weaker.

Astronauts in the space shuttle float because they are in "free fall" around Earth, just like a satellite or the Moon. Again, it is gravity that provides the centripetal force that keeps them in circular motion.



Follow-up: How weak is the value of *g* at an altitude of 300 km?

ConcepTest 12.6 Guess my Weight

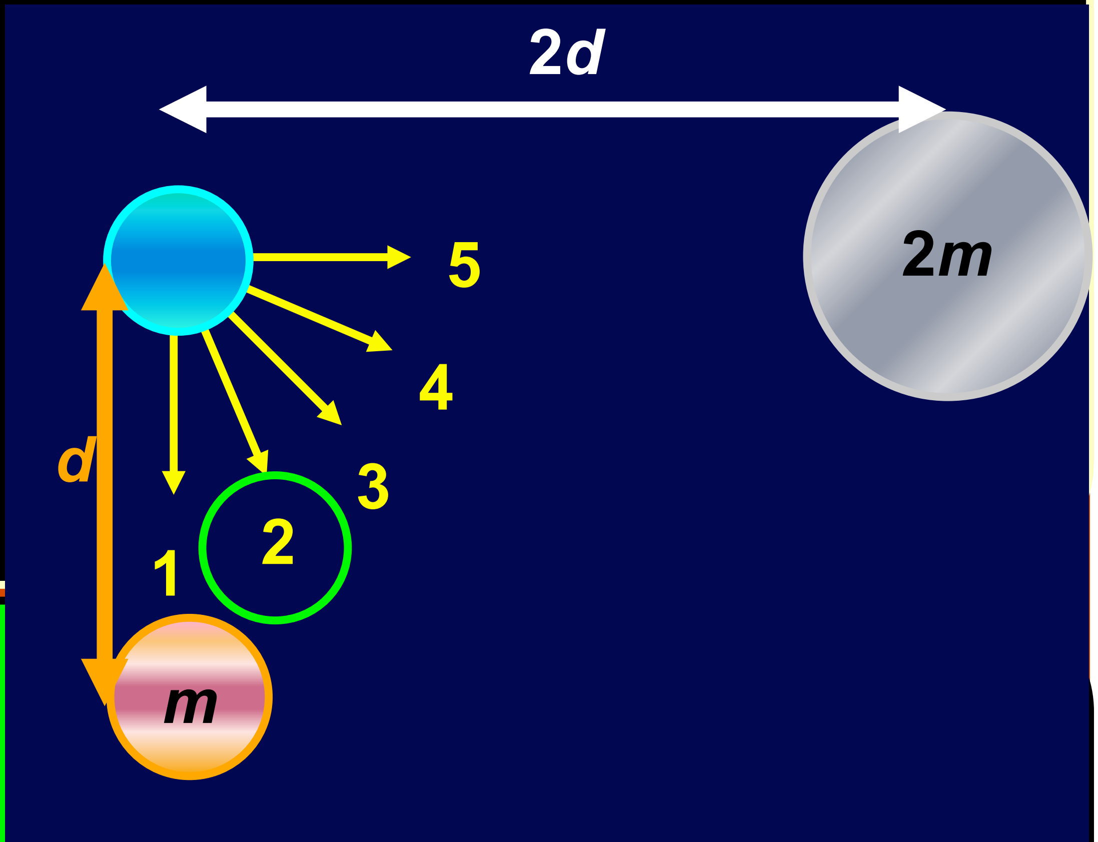
If you weigh yourself at the equator of Earth, would you get a bigger, smaller or similar value than if you weigh yourself at one of the poles?

1) bigger value
2) smaller value
3) same value

The weight that a scale reads is the **normal force** exerted by the floor (or the scale). At the equator, **you are in circular motion**, so there must be a **net inward force** toward Earth's center. This means that the **normal force must be slightly less than *mg***. So the scale would register something less than your actual weight.

ConcepTest 12.7 Force Vectors

A planet of mass m is a distance d from Earth. Another planet of mass $2m$ is a distance $2d$ from Earth. Which force vector best represents the direction of the total gravitation force on Earth?



The force of gravity on the Earth due to m is greater than the force due to $2m$, which means that the force component pointing down in the figure is greater than the component pointing to the right.

$$F_{2m} = GM_E(2m) / (2d)^2 = 1/2\ GMm / d^2$$

$$F_m = GM_E\, m / d^2 = GMm / d^2$$