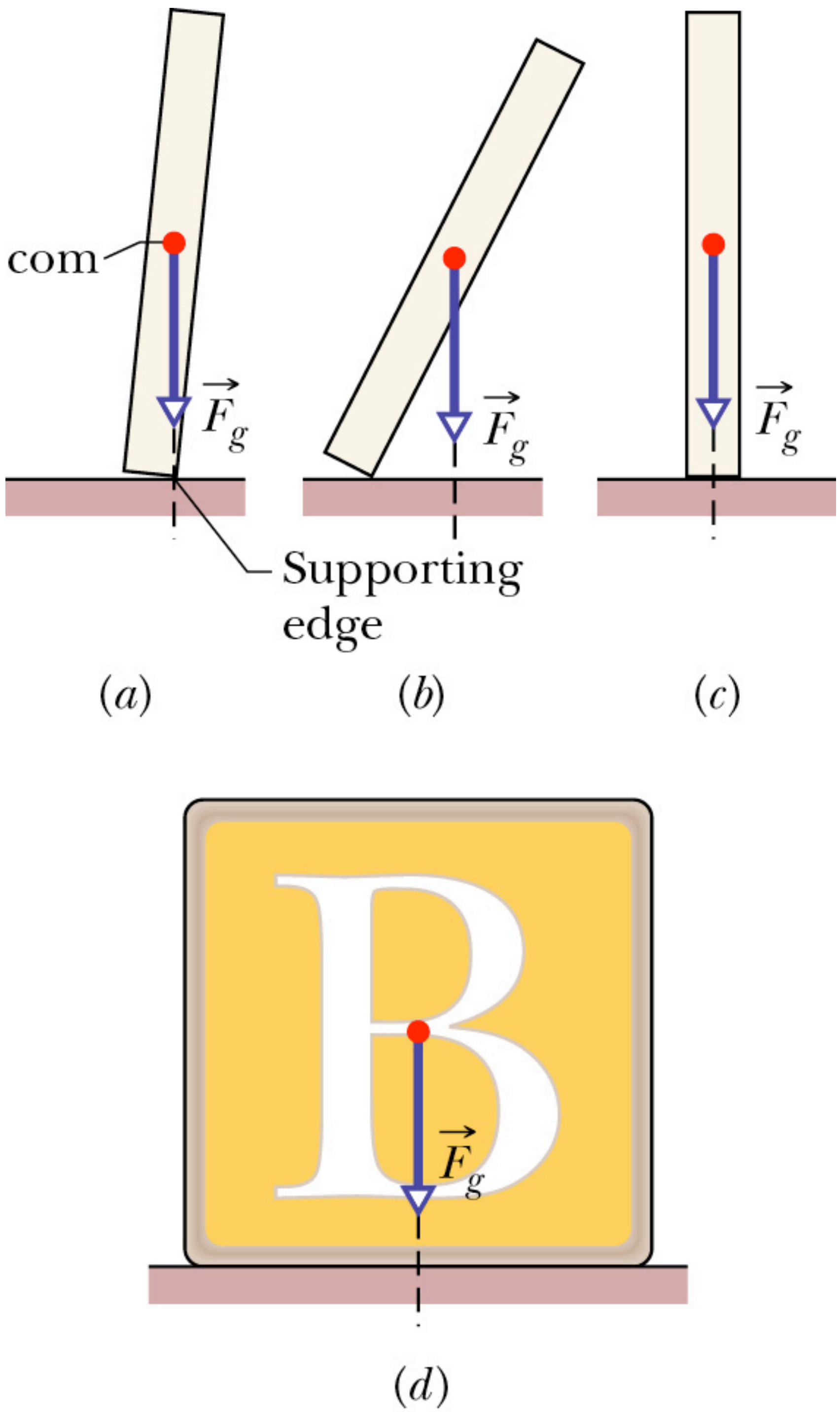


Lecture 8
Equilibrium and Elasticity

July 19 EQUILIBRIUM AND ELASTICITY CHAPTER 12
Give a sharp blow one end of a stick on the table. Find center of percussion.
Baseball bat center of percussion
Equilibrium demos?
Pile of tiles
Ladder with weight.
Stretching and compressing metals.

FLUIDS IN MOTION CHAPTER 14
Bathroom scale atmosphere crusher
Pascal Vases
Venturi tube
Crush the soda can
Magdeburg hemispheres
Archimedes principle weigh cylinder in water
Heat up water and watch floating object sink as density increases.
Cartesian Diver
Weigh thumb in water
Buoyancy of air
Weight of Air
Surface tension and adhesive forces
Blow a few bubbles on overhead projector . Note as bubble heats up, pressure increases inside bubble, and the radius of the bubble decreases.
Air flow around objects

Equilibrium

- Equilibrium P_{com} = constant, L_{com} = constant
- Static Equilibrium   constant =0
- Stable static equilibrium
- Unstable static equilibrium

Balance of net Forces and net Torques

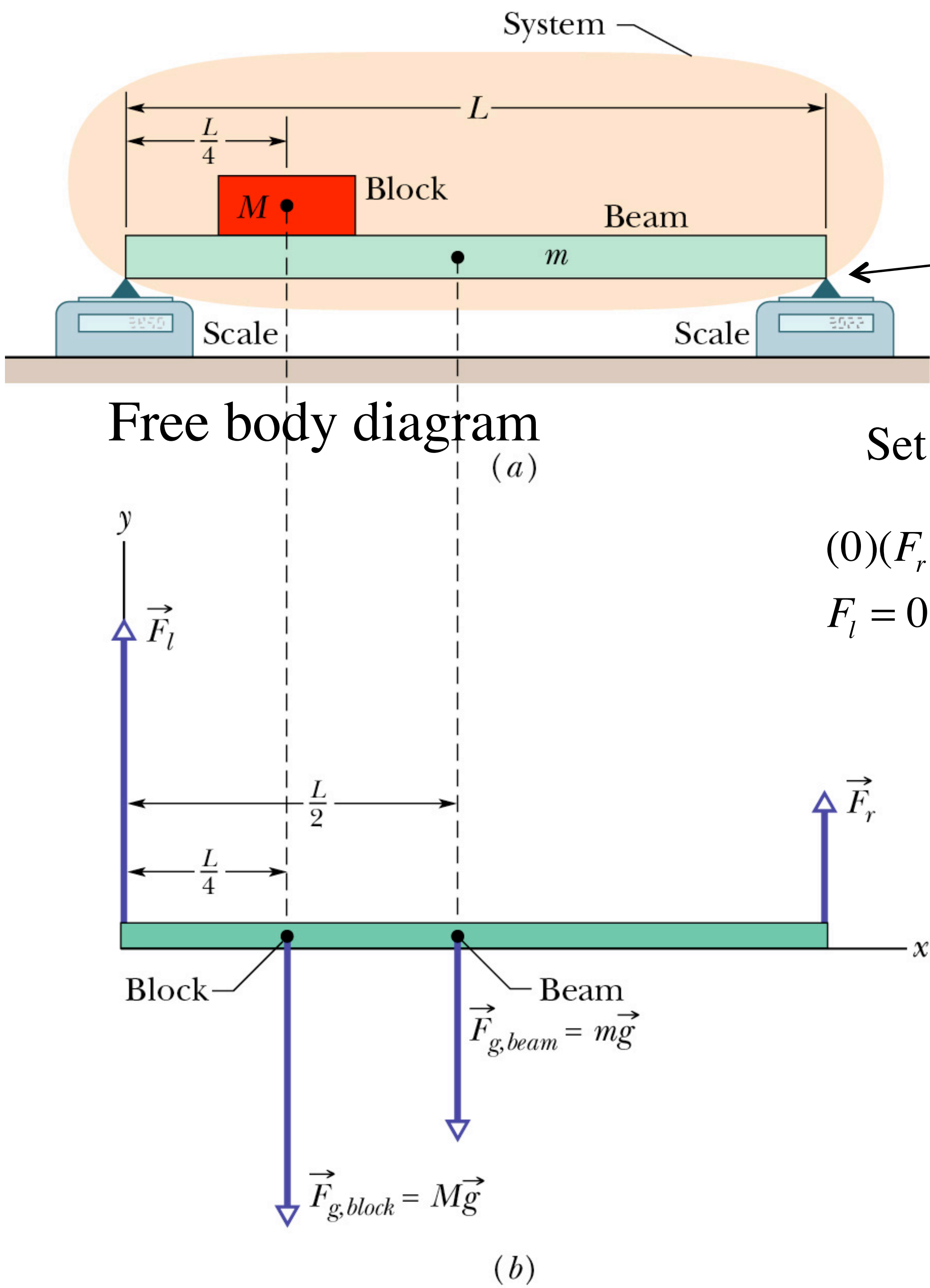
$$\sum F_x = 0 \qquad \sum \tau_x = 0$$

$$\sum F_y = 0 \qquad \sum \tau_y = 0$$

$$\sum F_z = 0 \qquad \sum \tau_z = 0$$

Gravity acts on a single point on a body called the center of gravity.
If g is the same for every point on the body, then cog =com.

m= 1.8 kg, L=1 meter
M=2.7 kg at L/4 from the left end. What do the scales read?



Normal Force=F_l and F_r

y-dir. $F_l + F_r - Mg - mg = 0$

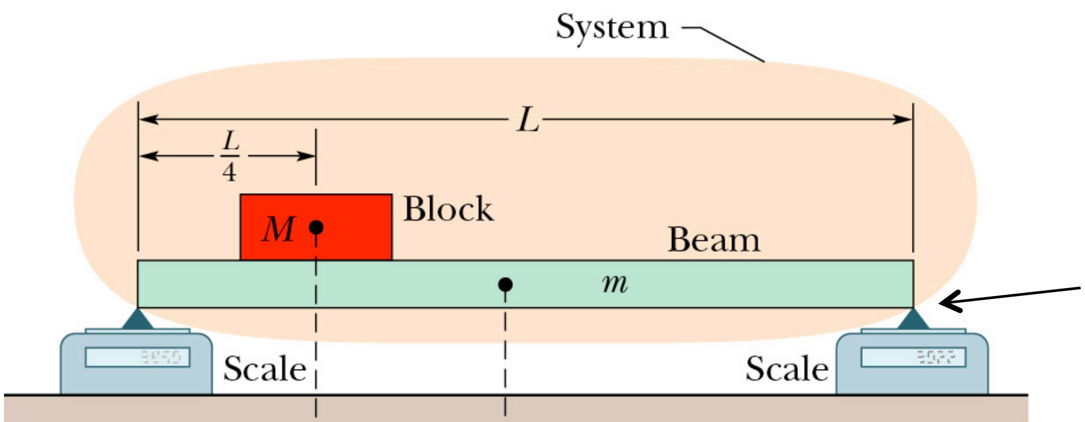
Set net torque = 0 about right end

$$(0)(F_r) + (3L/4)Mg) + (L/2)(mg) - (L)(F_l) = 0$$

$$F_l = 0.75(26.46) + 0.5(17.64) = 28.67N$$

$$F_r = Mg + mg - F_l$$

F_r = 15.43 N



m= 1.8 kg, L=1 meter

M=2.7 kg at L/4 from the left end. What do the scales read?

y-dir. $F_l + F_r - Mg - mg = 0$

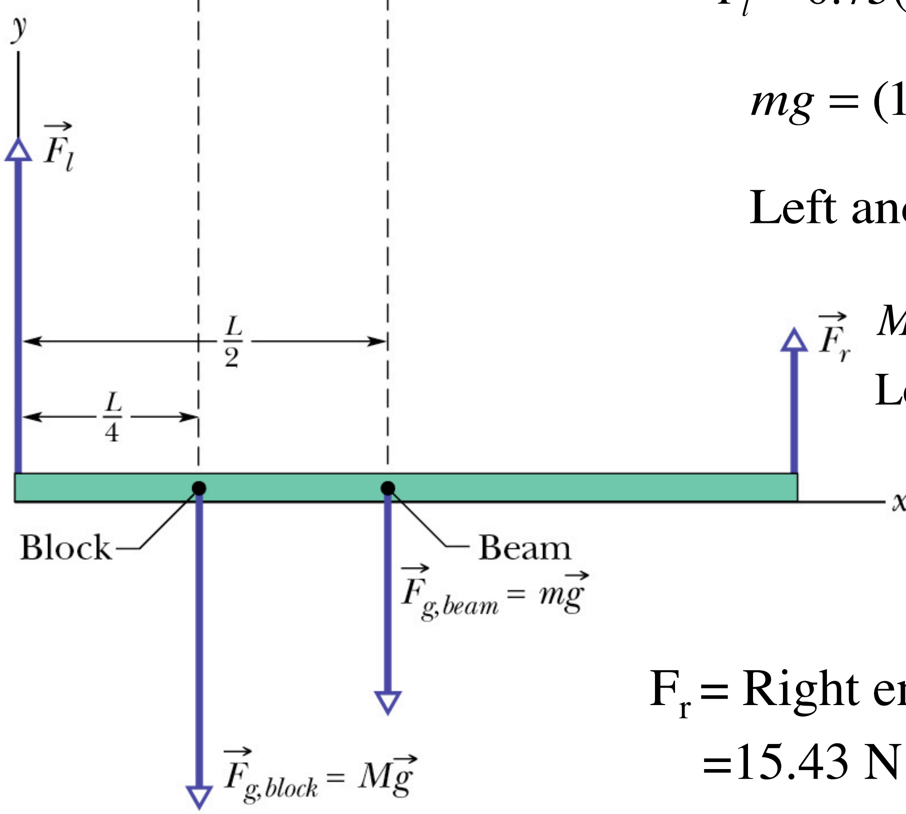
Set net torque = 0 about right end

Free body diagram

$$(0)(F_r) + (3L/4)Mg) + (L/2)(mg) - (L)(F_l) = 0$$

$$F_l = 0.75(26.46) + 0.5(17.64) = 28.67\,N$$

(a)



$$mg = (1.8)9.8 = 17.64$$

Left and right read $= \dfrac{mg}{2}$ = 8.82 N due to m.

$Mg = (2.7)(9.8) = 26.46$ N

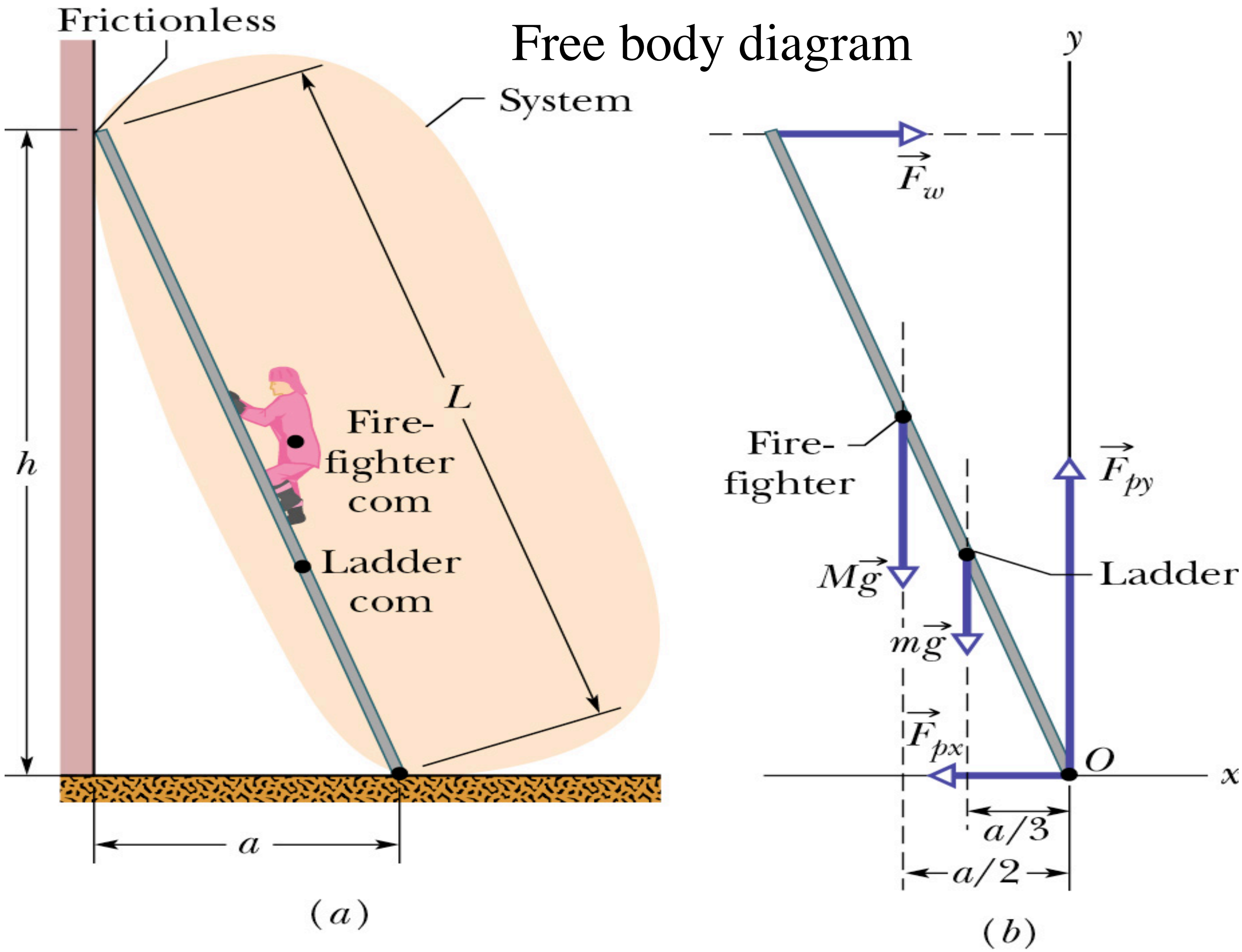
Left end reads .75(26.46) = 19.85 N due to M

F_l=3/4 Mg +1/2 mg= 28.67 N

F_r = Right end =mg+Mg-F_l=17.64+26.46-28.67

=15.43 N

(b)



(a) (b)

Given

$L = 12m$

$M = 72kg$

$m = 45kg$

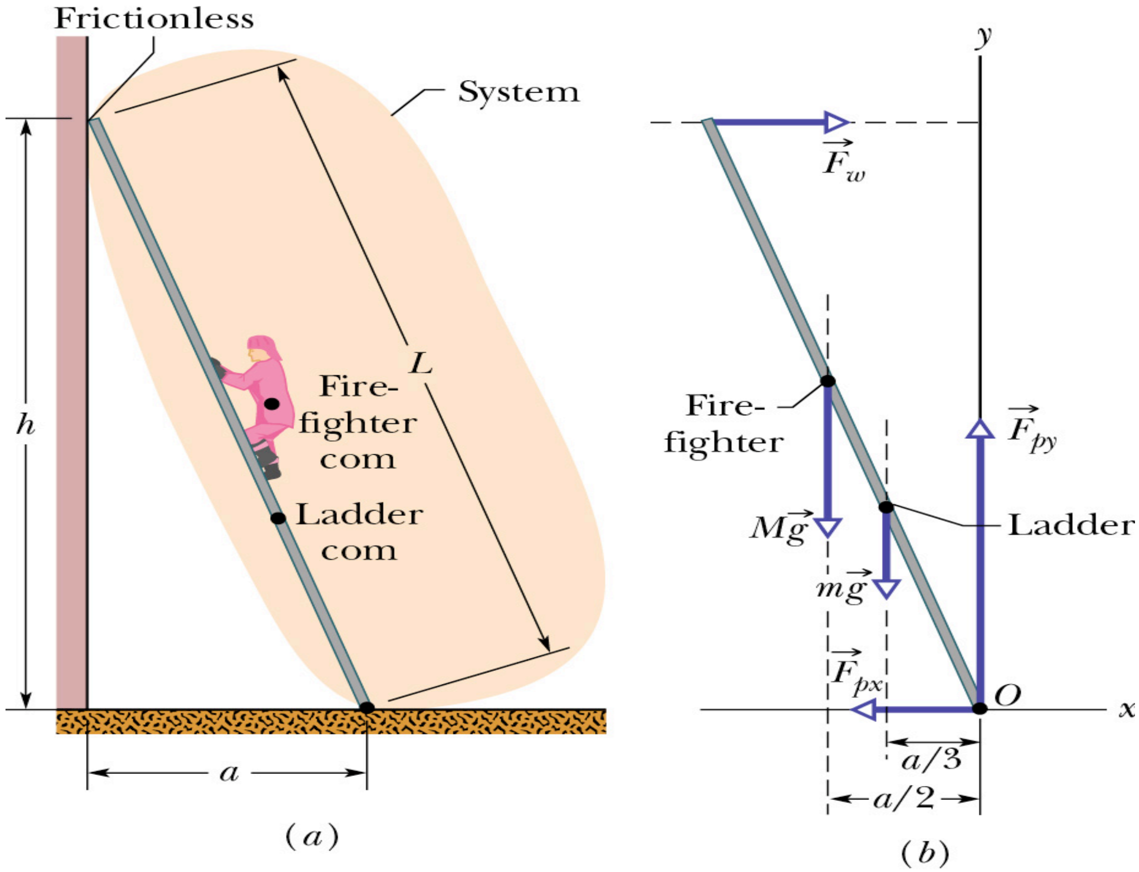
$h = 9.3m$

Find

F_W

F_{py}

F_{px}



Now use $F_{net,x}$ =0
and $F_{net,y}$ =0

$$F_w - F_{px} = 0$$
$$F_{px} = 407N$$
$$F_{py} - Mg - mg = 0$$
$$F_{py} = Mg + mg = (72 + 45)9.8 = 1146.6N$$

Take moments about which axis since net torque is 0

Choose O eliminate two variables

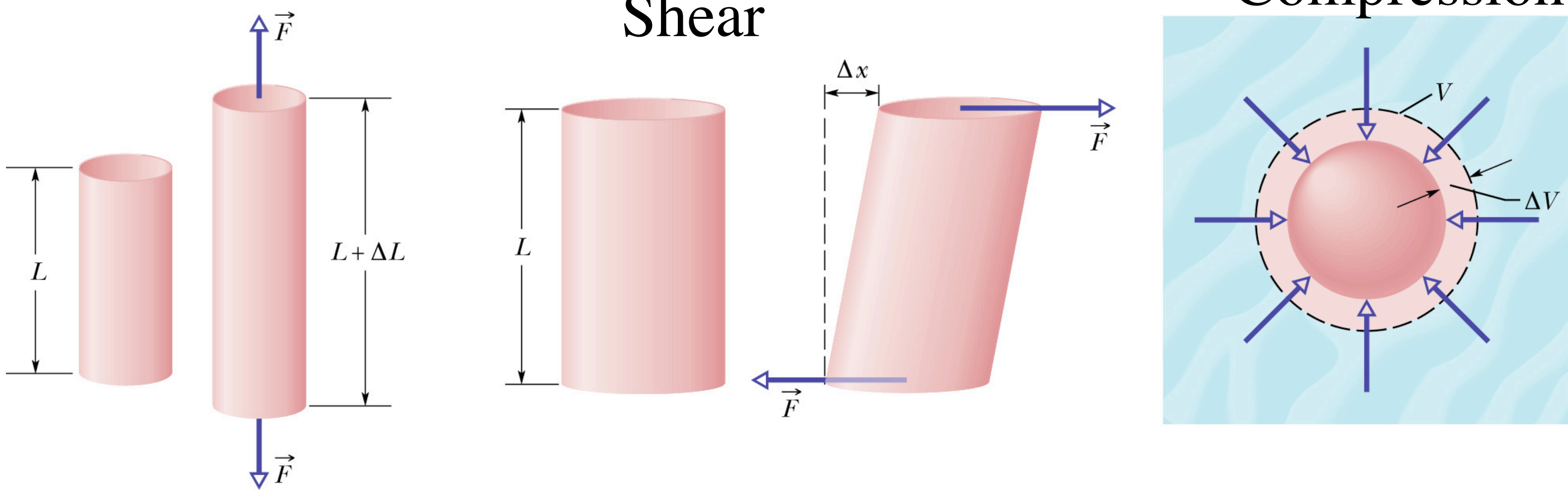
$$-(h)(F_W) + (a/2)(Mg) + (a/3)mg + (0)(F_{px}) + (0)(F_{py}) = 0$$
$$a = \sqrt{L^2 - h^2} = \sqrt{12^2 - 9.3^2} = 7.58m$$
$$F_w = ga(M/2 + m/3)/h$$
$$F_w = 9.8(7.58)(72/2 + 45/3)/9.3 = 407N$$

Elasticity

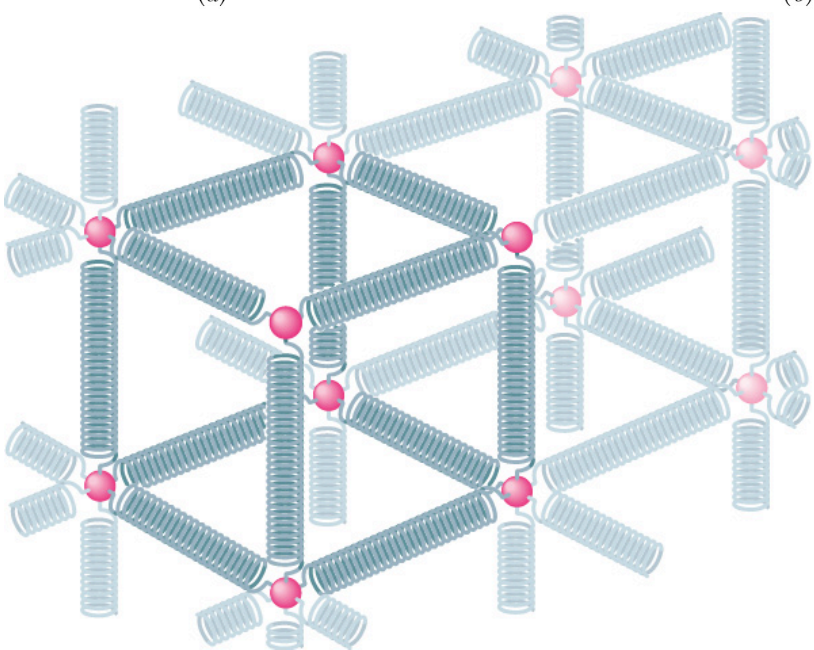
Strain

Shear

Compression



(a) (b) (c)



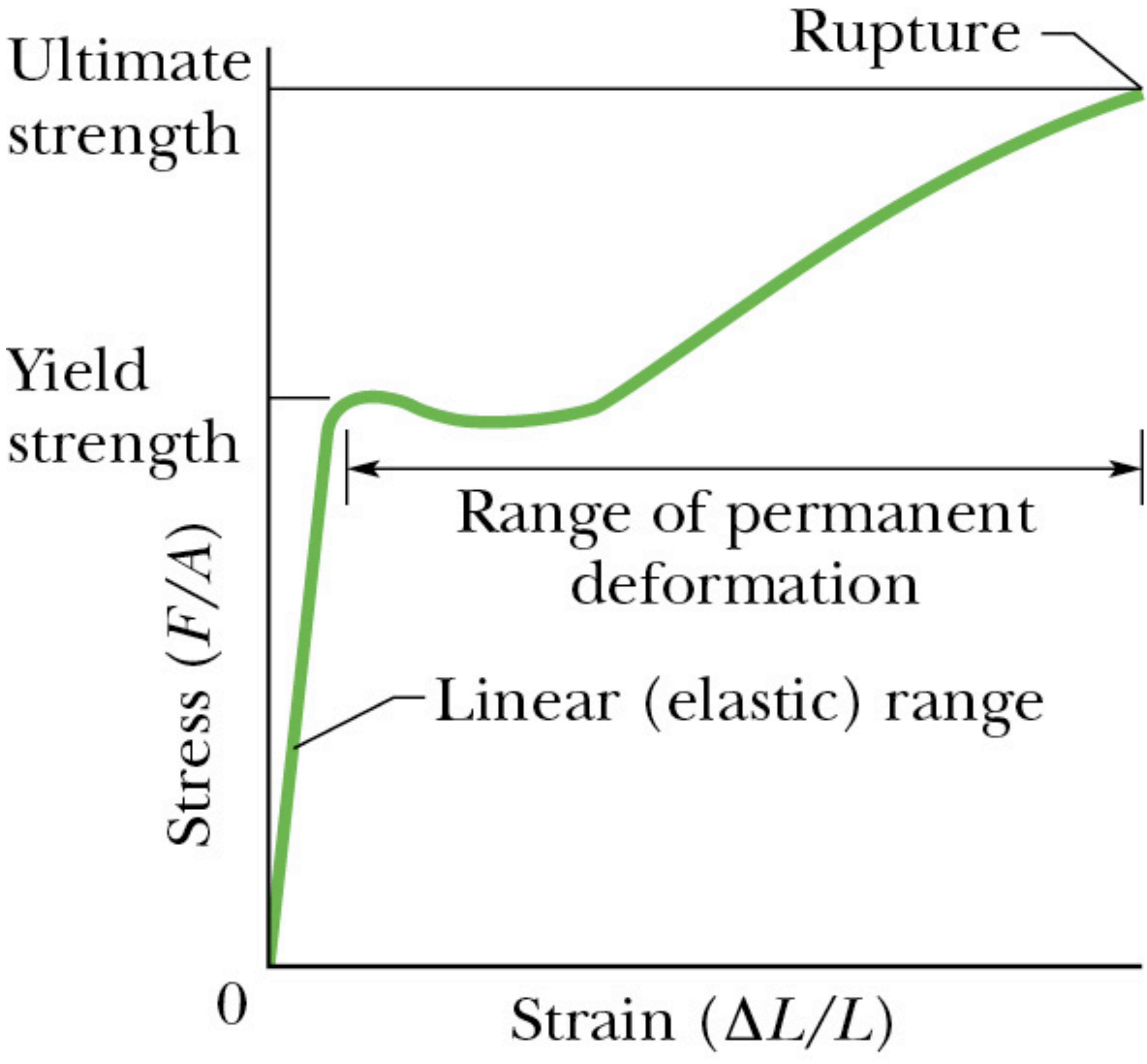
Strain

Consider a long wire with a mass hanging off the end of it.

A long wire stretches a distance ΔL. Divide by the length of the wire L. Call it the strain $= \Delta L/L$. Imagine all the atoms connected by springs with a force $F=k_s\ s$. Then the stretching is equivalent to all the springs stretching a little bit.

- Strain = $\Delta L / L$
- Stress = F_T / A
- Young's Modulus is the ratio of stress/strain

$$Y = \frac{F_T / A}{\Delta L / L}$$


Ultimate strength
Rupture
Yield strength
Range of permanent deformation
Stress (F/A)
Linear (elastic) range
0
Strain (ΔL/L)

Various materials subjected to a stretching and compression test demo

Material	Youngs Modulus E 10^9 N/m^2	Ultimate Strength S 10^6 N/m^2
Steel	200	400
Aluminum	70	110
glass	65	50 c
concrete	30	40 c
wood	13	50 c
bone	9 c	170 c
plastic	3	48

Sample problem

Suppose we have a steel ball of mass 10 kg hanging from a steel wire 3 m long and 3mm in diameter. Young's modulus is 2 x 10^{11} N/m^2. How much does the wire stretch?

$$\Delta L / L = \frac{F_T / A}{Y}$$

$$F_T = 9.8(10) = 98N$$

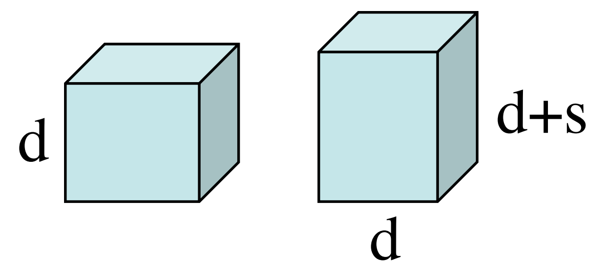
$$A = 3.14(0.003)(0.003) = 2.8 \times 10^{-5} m^2$$

$$\Delta L = \frac{L}{Y} F_T / A = \frac{3}{2 \times 10^{11}} (98 / 2.8 \times 10^{-5}) = 52.5 \times 10^{-6} m$$

Microscopic picture of Young's Modulus

For one atom the atomic stress is $k_s s/d^2$ and the strain for one atom is s/d where d is the size of the atom and s is the amount the atom is stretched. Y is the ratio of stress/ strain.

Then $Y = \frac{k_s s / d^2}{s / d} = \frac{k_s}{d}$ for one atom.



Chapter 14
Fluids (Liquids and Gases)

- What is a fluid
- Density, pressure, of gases and liquids
- Static fluids $p = p_0 + rgh$
- Mercury barometer,open tube manometer
- Pascal's principle
- Archimedes principle
- Surface tension and adhesive forces
- Fluids in motion
- Equation of continuity
- Bernoulli's Equation

http://www.tangenttoy.com/bubbleman/ about soap bubbbles

Fluids: liquids and gases

Fluids conform to the container that holds them and we normally talk about their density and pressure

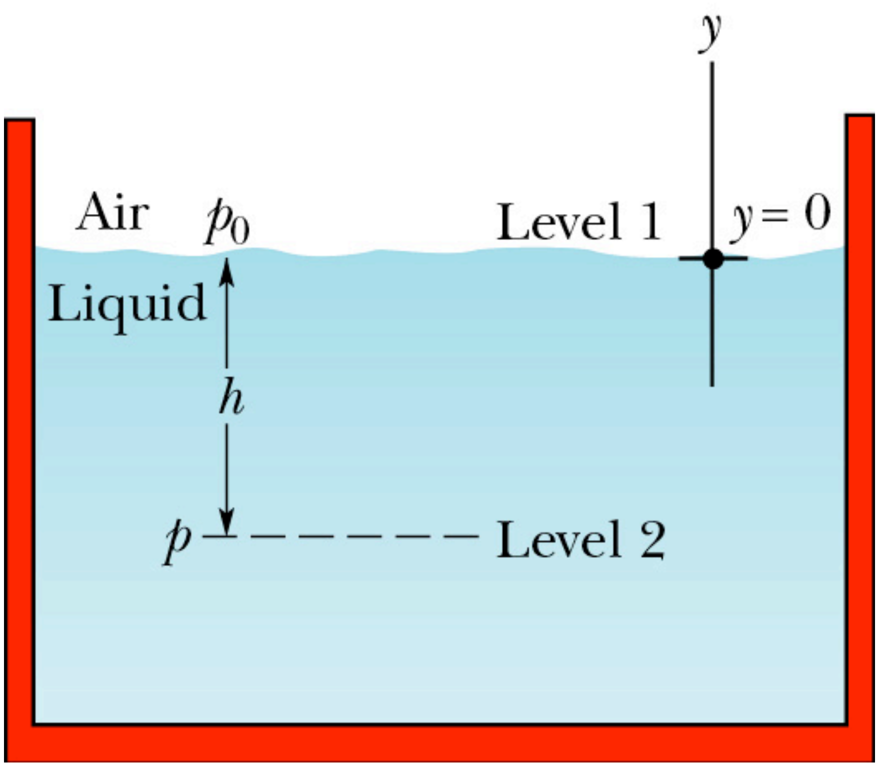
- Liquids are Incompressible
- Gases are very compressible
- Uniform Density

  $r = m/V$ kg/m^3
- Pressure (Static)
  - $P = F/A$ N/m^2 or Pa
  - Scalar- independent of orientation
- Atmospheric pressure
  - 1atm 1.01×10^5 N/m^2
  - 760 mm of Hg or Torr
  - 14.7 lb/in^2

What are all the forces acting on this book? Put a book on the table.

Static Pressure

What is the force acting downward at level 1?
at level 2 ?



- Level 1 F = Weight of Atmosphere
- Level 2 F = Weight of Atmosphere + Weight of water
- The pressure is just the weight of the water plus the weight of the air per square meter

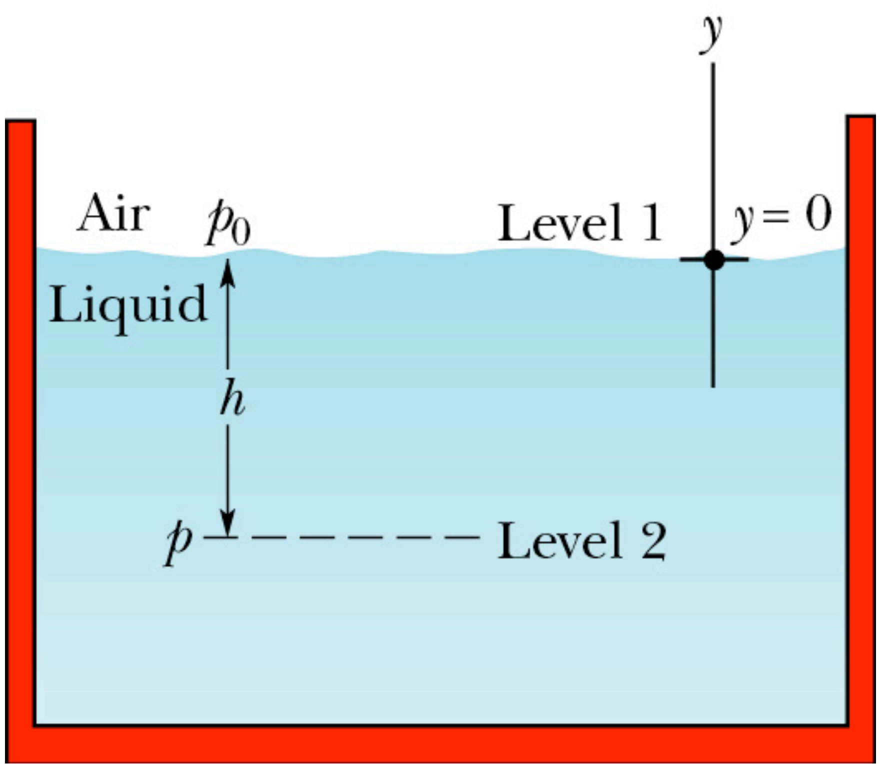
$$F_2 = F_1 + mg$$

$$AP = AP_0 + \rho hAg$$

$$P = \frac{F}{A}$$

$$P = P_0 + \rho gh$$

What is the absolute and gauge pressure at a 2 meter depth in the water?
First find the atmospheric pressure P_0.



$$P = P_0 + \rho g h$$

$$h = 2m$$

$$\rho = 1000 kg / m^3$$

$$g = 9.8 m / s^2$$

$$P_0 = 1.01 \times 10^5 Pa$$ Or N/m^2

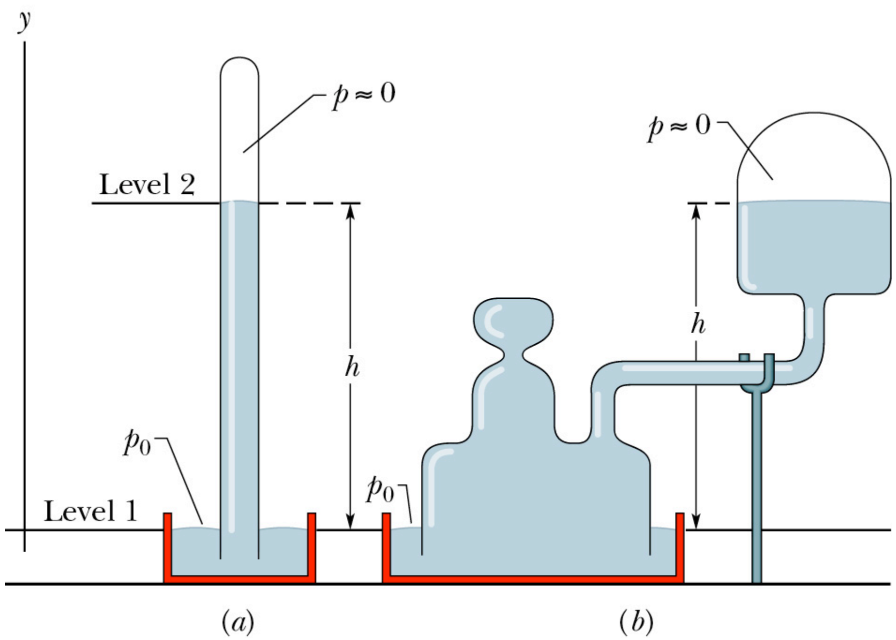
$$P = (1.01 \times 10^5) + (1000)(9.8)(2)$$

$$P = 1.21 \times 10^5 Pa$$

$$P - P_0 = 0.20 \times 10^5 Pa$$

(Gauge Pressure)

What is a Mercury Barometer?



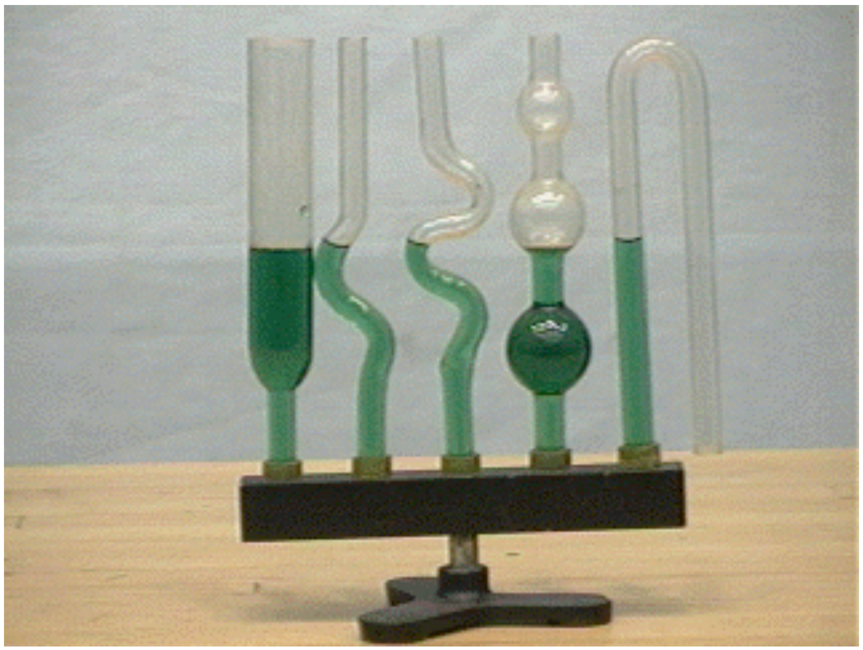
$$F = P_0 A$$

- This is the upward acting force due to the pressure at the bottom of the tube.
- This will support an amount of mercury equal to the weight $hA\rho g$
- So $P_0 A = hA\rho g$ or
- $P_0 = h\rho g$ Solving for h,
- We get $h = \frac{P_0}{\rho g}$ or

$$h = \frac{1.01\times10^5}{(13550)(9.8)} = 0.760m = 760mm$$

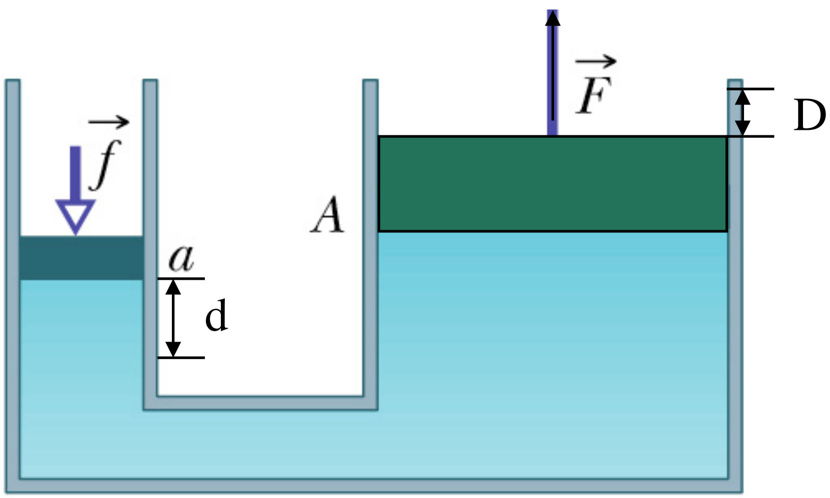
How high would a column of water go?

Pascal Vases



- Shows that the pressure in a liquid depends on the height of the liquid, not it's volume

Pascal's principle: A change in pressure applied to an enclosed incompressible fluid is transmitted undiminished to every portion of the fluid and to the walls of its container



Change in pressure = $\Delta P = \dfrac{f}{a} = \dfrac{F}{A}$ (1)

$V_f = V_F$ $V = AD = da,$ (2)

$$D = d\frac{a}{A}$$

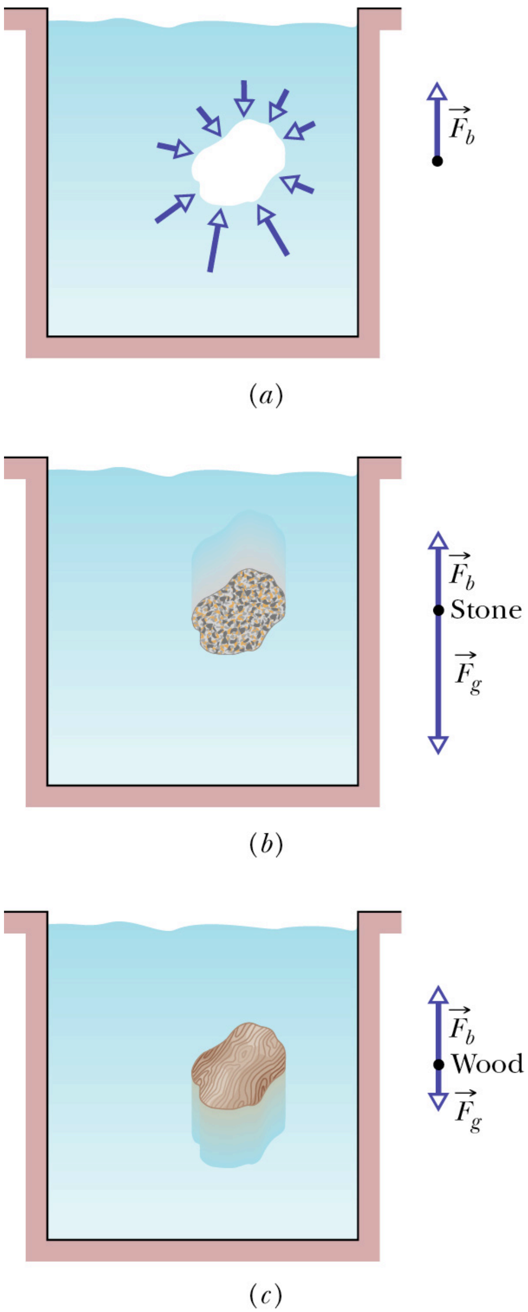
$$\frac{a}{A} = \frac{f}{F}$$ From (1)

Work done $FD = fd$

The work done by both systems is the same.
The advantage is that a given force over a long distance can be transformed to a larger force over a shorter distance.

Cartesian Diver

To illustrate the transmission of pressure in a liquid.



Archimedes principle

The stone displaces a volume of water equal to its volume.
The upwardly directed buoyant force is equal to the weight of the displaced fluid or water. This is the surprising thing.

Floating: If the buoyant force is equal to the weight mg of the object, Then it is said to float.

If it sinks then you can define its apparent weight.
apparent weight = actual weight - buoyant force

Archimedes Demo

Example 26 chpt 14 ed 7

Weight of Thumb

Stick thumb in the water and weigh it.

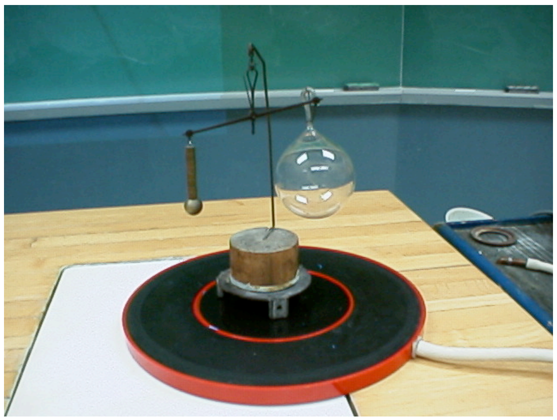
Remove thumb and stick it in the water and weigh it again .

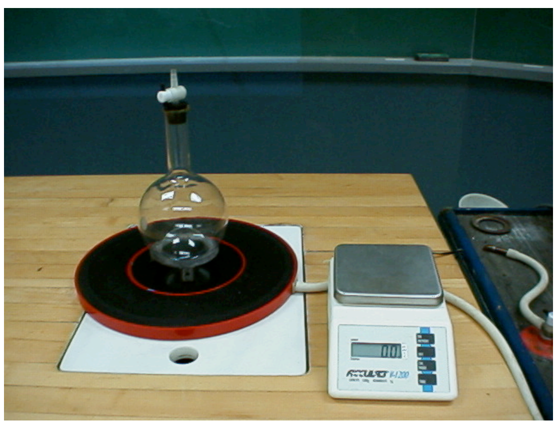
Can you account for the readings.

Archimedes principle

To show that the buoyant force exerted on an object is equal to the weight of the displaced water (or fluid).

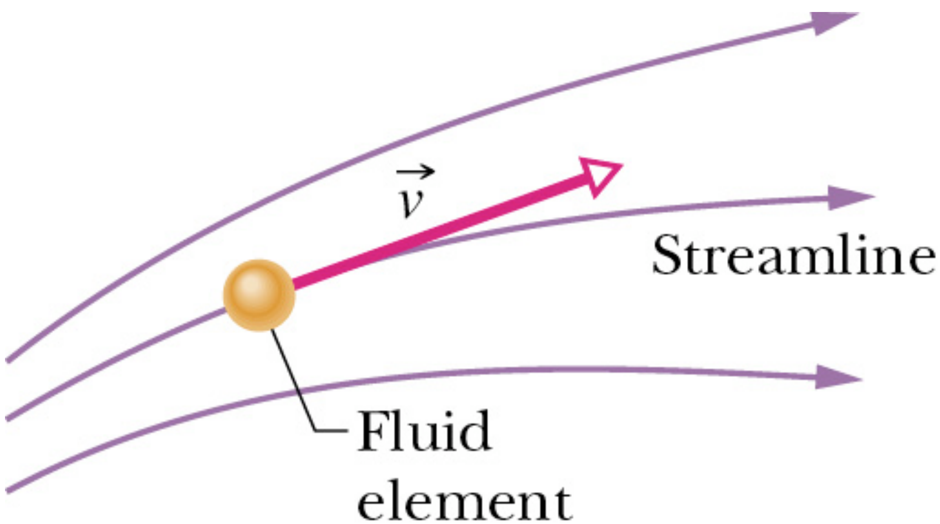
Air effects



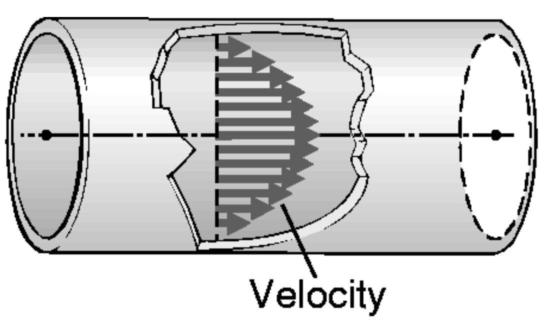


- Buoyancy of air
- Weight of Air

Laminar Flow



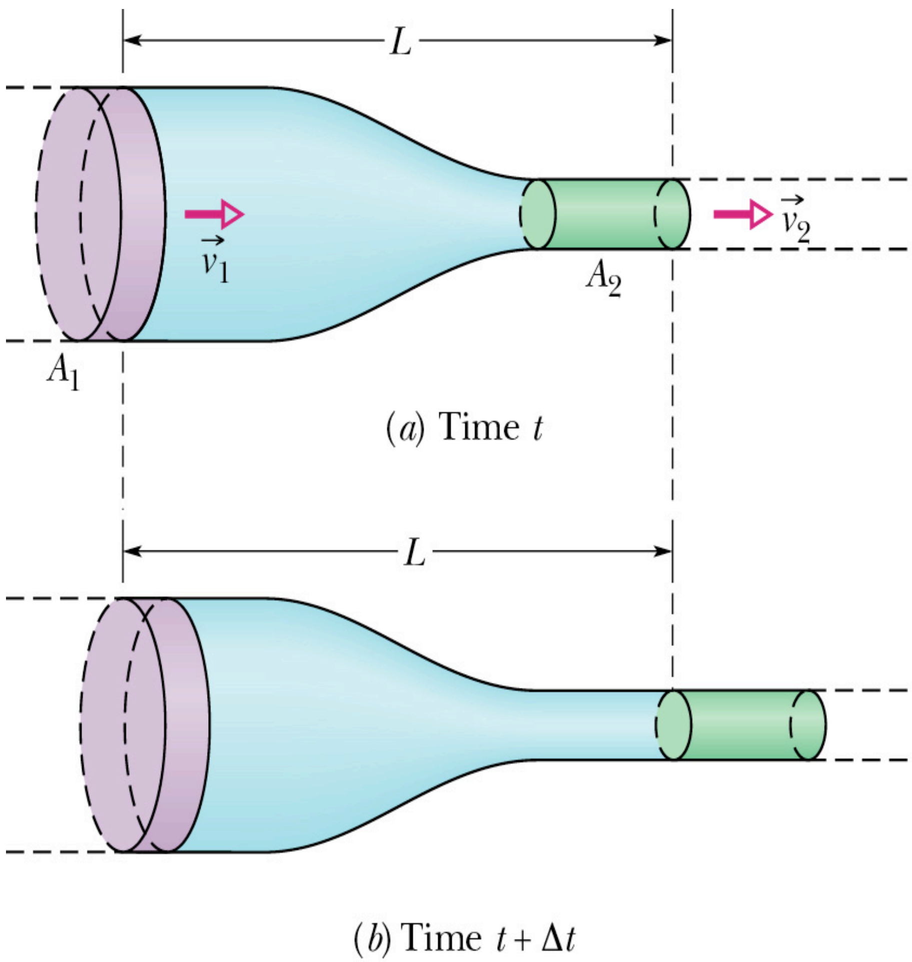
- Steady flow
- Incompressible flow
- Non-viscous flow
- Irrotational flow



An example of viscous flow

Equation of continuity:

$$A_1 v_1 = A_2 v_2$$ = Volume flow rate

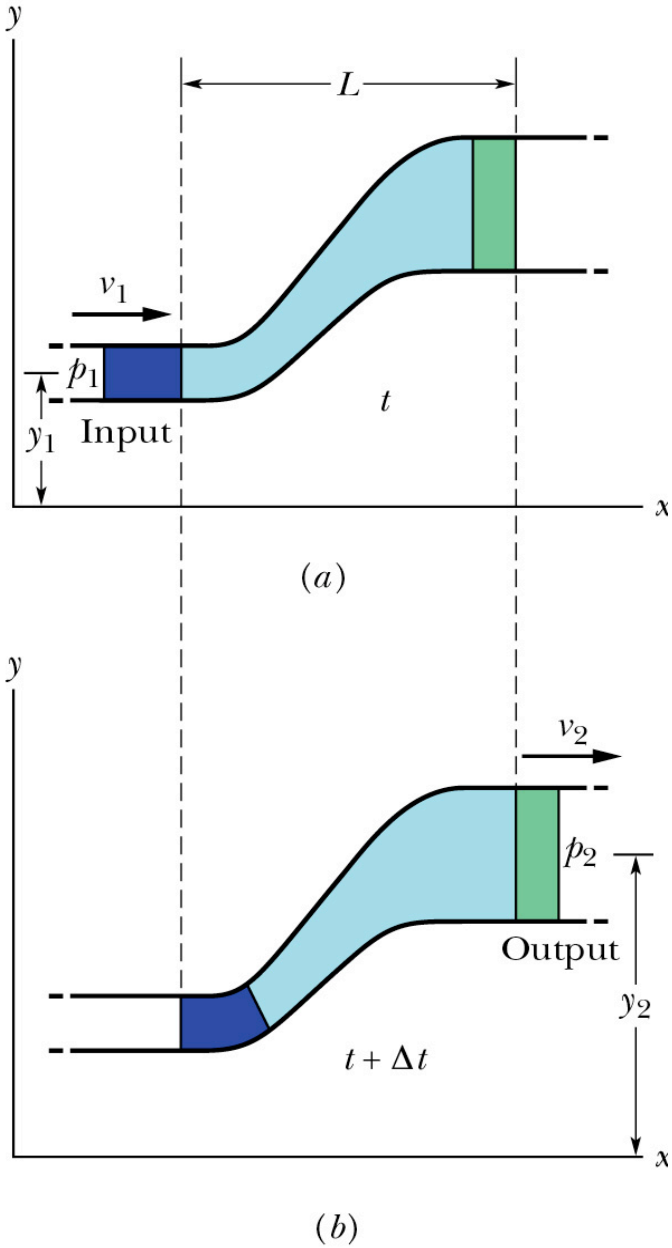


Mass flow rate= density x Volume flow rate.

$$\rho A v = \text{constant}$$

Show water stream narrowing in diameter as it falls.

Bernoulli's Equation



Conservation of energy in a volume of fluid

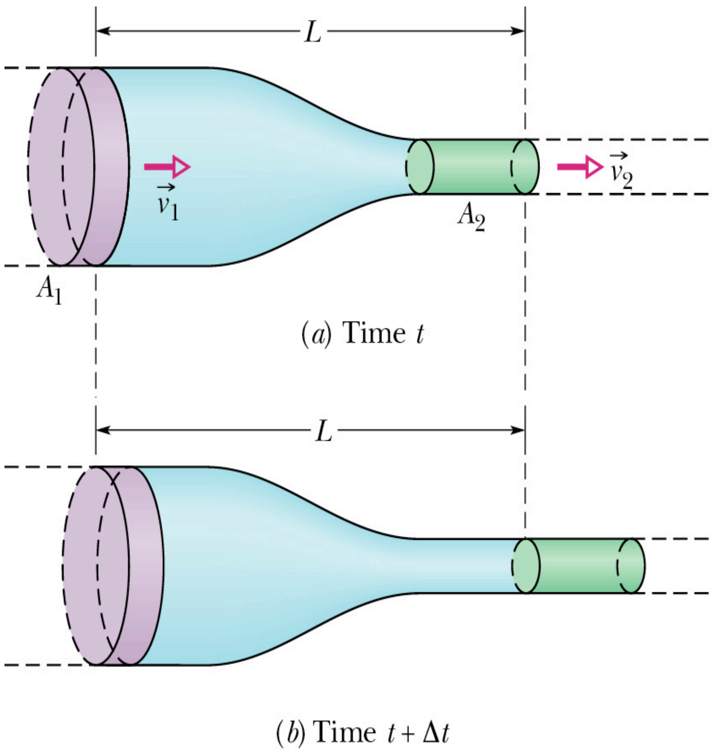
$$p_1 + \frac{1}{2}\rho v_1^{\ 2} + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^{\ 2} + \rho g y_2$$

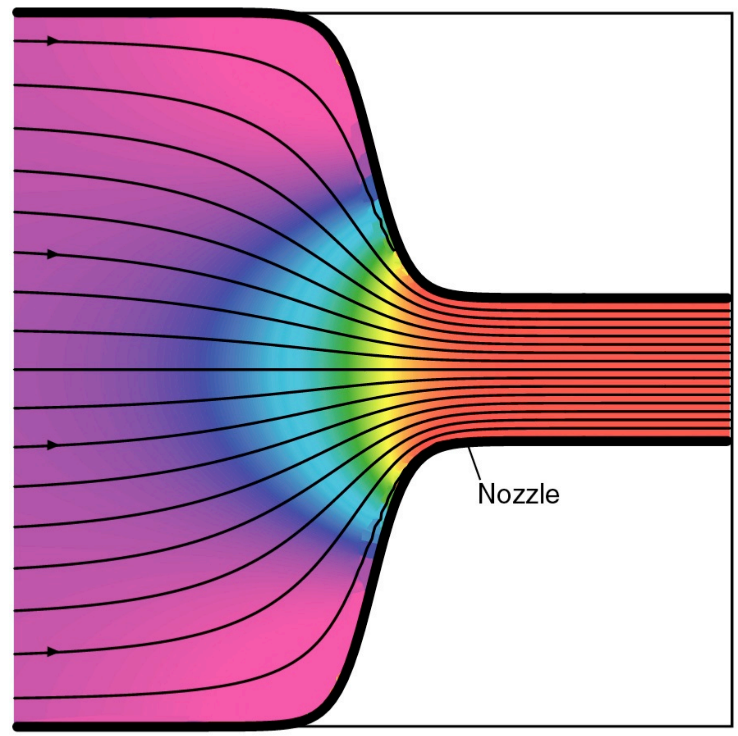
$$p + \frac{1}{2}\rho v^2 + \rho g y = \text{a constant along a streamline}$$

Special case y =0

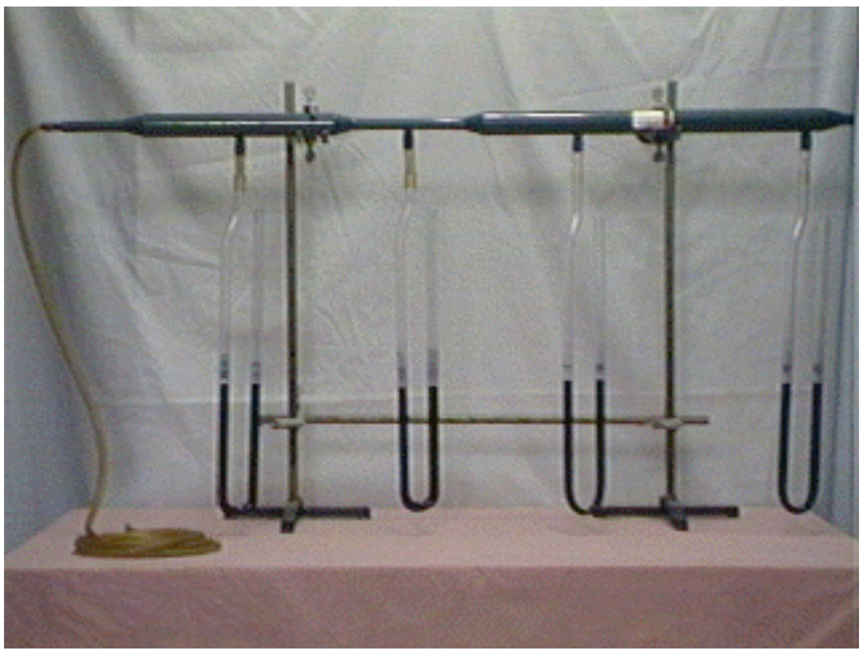
$$p + \frac{1}{2}\rho v^2 = \text{a constant along a streamline}$$

$$p + \frac{1}{2}\rho v^2 = \text{a constant along a streamline}$$



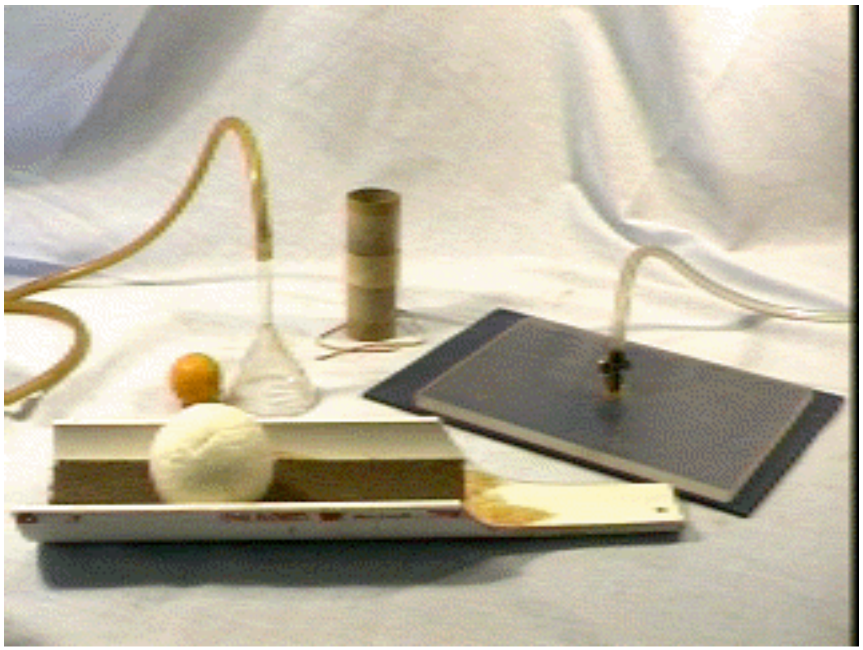


Venturi Tube and Qualitative Verification of Bernoulli's Equation



- To demonstrate operation of a venturi tube and to verify variation of gas velocity with area of tube.
- Show different pressures in flow tube by flowing air through system and noting pressure relationships by the level of colored liquid in the tubes.

Air Flow Around Different Objects



- Curve ball may be demonstrated by using the thrower and Styrofoam ball. The ball curves in the direction of the leading edge spin.

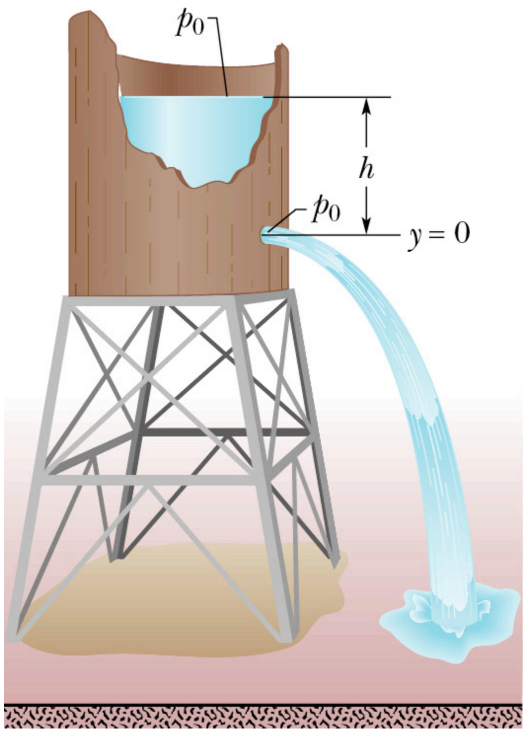
- Ping-pong ball may be supported on an air jet held vertical and also may be held in a funnel with an air jet.

- The cardboard cylinder is wrapped with a long length of rubber band and pulled back. It is released in such a manner that it spins out from the bottom. If it is released with enough energy, it will describe a loop in the air.

- The plexiglass plates can be held together with quite an impressive force by blowing air through the nozzle and bringing the plates together.

Water tank problem

What is the speed of the water through the narrow hole of area a?
The water tank as large area A.



Apply Bernoulli's equation to the top of the water level where y=h and at the hole y= 0 where the water is ejected.

$$p_0 + \frac{1}{2}\rho {v_0}^2 + \rho g h = p_0 + \frac{1}{2}\rho v^2 + \rho g(0)$$

Cancel out p_0

$$\frac{1}{2}\rho(v^2 - {v_0}^2) = \rho g h$$

$$v_0 = \frac{a}{A} v$$

From the continuity eq.

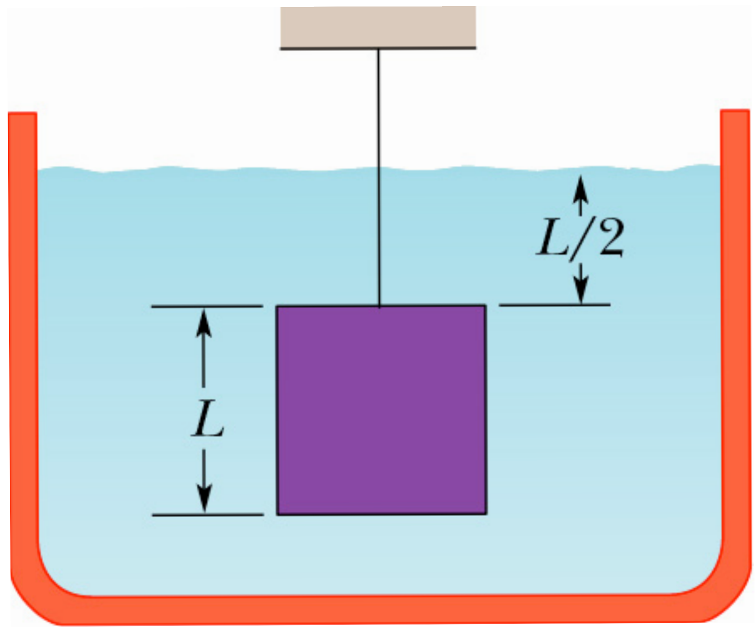
$$\frac{1}{2} v^2 (1 - \frac{a^2}{A^2}) = \rho g h$$

Since A/a >>1, neglect $(a/A)^2$ compared to 1

$$\frac{1}{2}\rho v^2 = \rho g h$$

$$v = \sqrt{2gh}$$

Chapter 14 Ed 7 Problem 26E



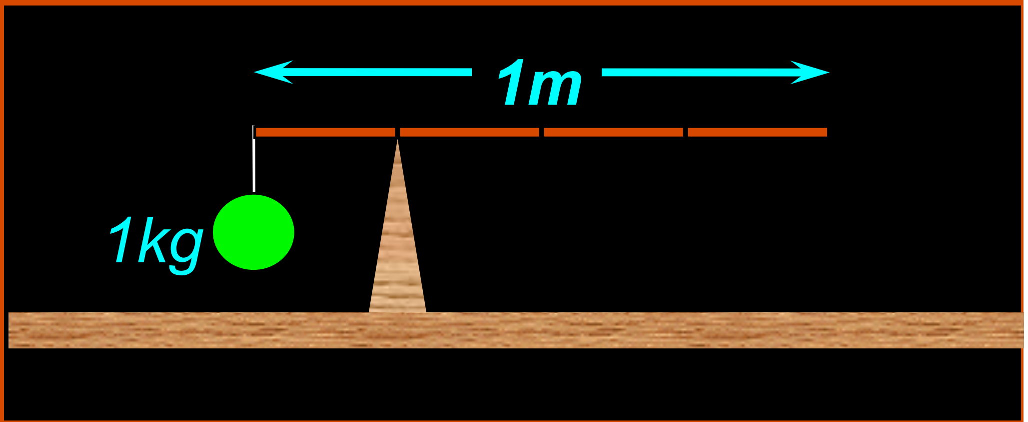
Problem 36 Ejected ball from water

Problem 20 “Sucking” up lemonade

ConcepTest 11.1 Balancing Rod

A **1 kg** ball is hung at the end of a rod **1 m** long. If the system balances at a point on the rod **0.25 m** from the end holding the mass, what is the mass of the rod ?

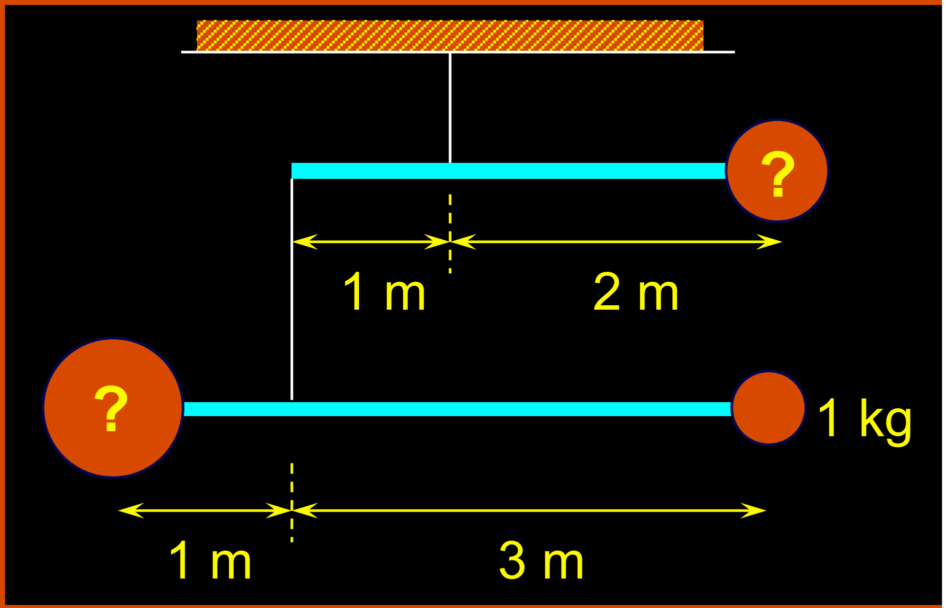
1) 1/4 kg
2) 1/2 kg
3) 1 kg
4) 2 kg
5) 4 kg



ConcepTest 11.2 Mobile

A (static) mobile hangs as shown below. The rods are massless and have lengths as indicated. The mass of the ball at the bottom right is 1 kg. What is the total mass of the mobile ?

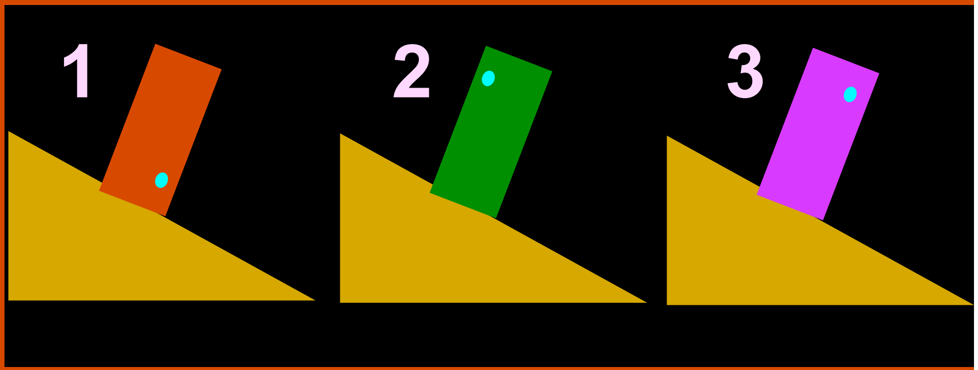
1) 5 kg
2) 6 kg
3) 7 kg
4) 8 kg
5) 9 kg



ConcepTest 11.3a Tipping Over I

A box is placed on a ramp in the configurations shown below. Friction prevents it from sliding. The center of mass of the box is indicated by a blue dot in each case. **In which case(s) does the box tip over ?**

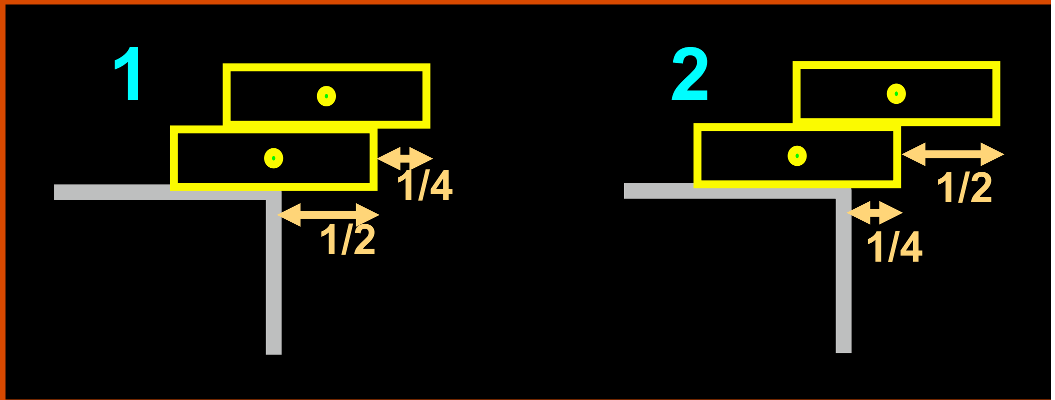
1) all
2) 1 only
3) 2 only
4) 3 only
5) 2 and 3



ConcepTest 11.3b Tipping Over II

Consider the two configurations of books shown below. **Which of the following is true?**

1) case 1 will tip
2) case 2 will tip
3) both will tip
4) neither will tip



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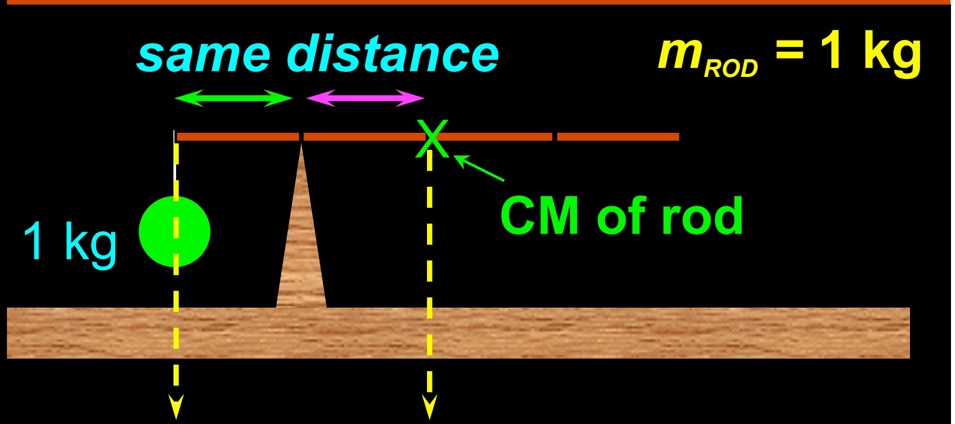
2) 1/2 kg
3) 1 kg
4) 2 kg
5) 4 kg

- The total torque about the pivot must be zero !! The CM of the rod is at its center, 0.25 m to the right of the pivot. Since this must balance the ball, which is the same distance to the left of the pivot, the masses must be the same !!



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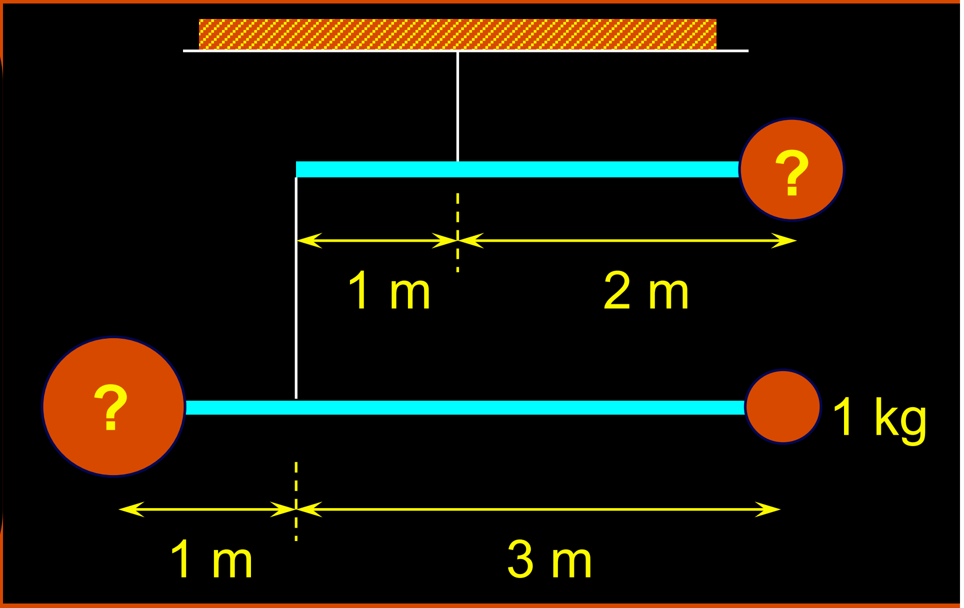
1) 5 kg
2) 6 kg
3) 7 kg
4) 8 kg
5) 9 kg

Use torques in two steps: (1) find the big mass on the bottom left (lower rod only). (2) use the entire lower rod assembly (with two masses) to find the mass on top right. Finally, add up all the masses.



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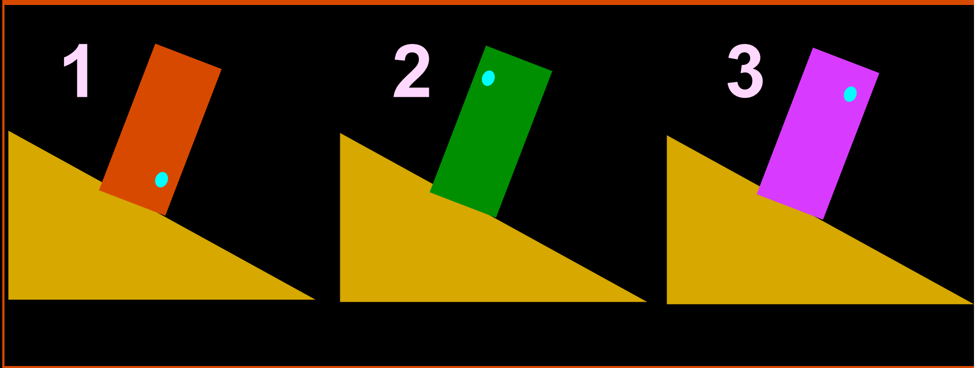
1) all
2) 1 only
3) 2 only
4) 3 only
5) 2 and 3

The torque due to gravity acts like all the mass of an object is concentrated at the CM. Consider the bottom right corner of the box to be a pivot point. **If the box can rotate such that the CM is lowered, it will !!**



ConcepTest 11.3b Tipping Over II

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3) both will tip
4) neither will tip

The CM of the system is midway between the CM of each book. Therefore, the CM of case #1 is not over the table, so it will tip.