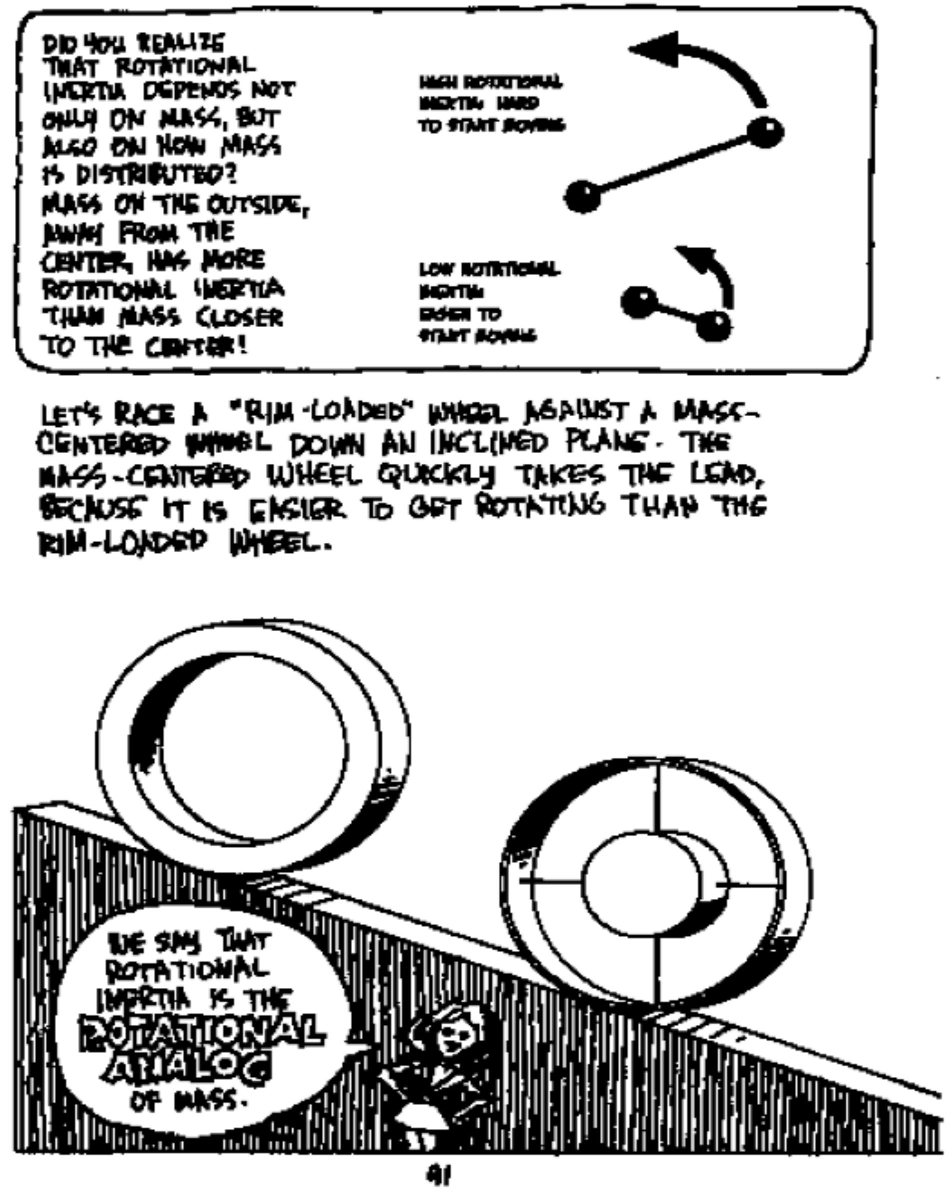
Lecture 7
Chapter 10,11

Rotation, Inertia, Rolling, Torque, and Angular momentum



Demos

July 14 2009 ROTATION ,TORQUE, ANGULAR MOMENTUM CHAPTER 10 & 11
Falling rod accelerates faster than g
Objects rolling down inclined in plane
Student on stool with barbells demoing angular momentum
Student on stool turning spinning bicycle wheel over
Demo rolling motion using a wheel
Stick receiving a blow on one end. Point where translation and rotation cancel out. Center of percussion
Baseball bat pendulum
A pulley with string wrapped around it and a weight.
Atwoods machine with a large pulley.
Screw driver with different size handles

Summary of Concepts to Cover from chapter 10 Rotation

- Rotating cylinder with string wrapped around it: example
- Kinematic variables analogous to linear motion for constant acceleration
- Kinetic energy of rotation
- Rotational inertia
- Moment of Inertia
- Parallel axis theorem

ROTATION ABOUT A FIXED AXIS

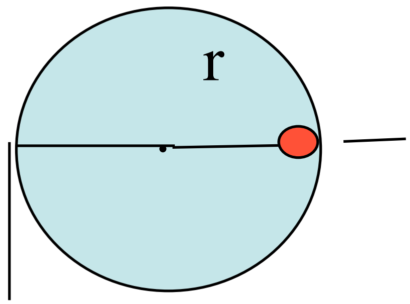
- Spin an rigid object and define rotation axis.
- Define angular displacement, angular velocity and angular acceleration.
- Show how angle is measured positive (counterclockwise).
- Interpret signs of angular velocity and acceleration.
- Point out analogy to 1D motion for the variables.
- Point out that omega and alpha are vectors that lie along the axis of rotation for a fixed axis of rotation Angular displacements are not vectors. Show the figure with two angular displacements of a book.

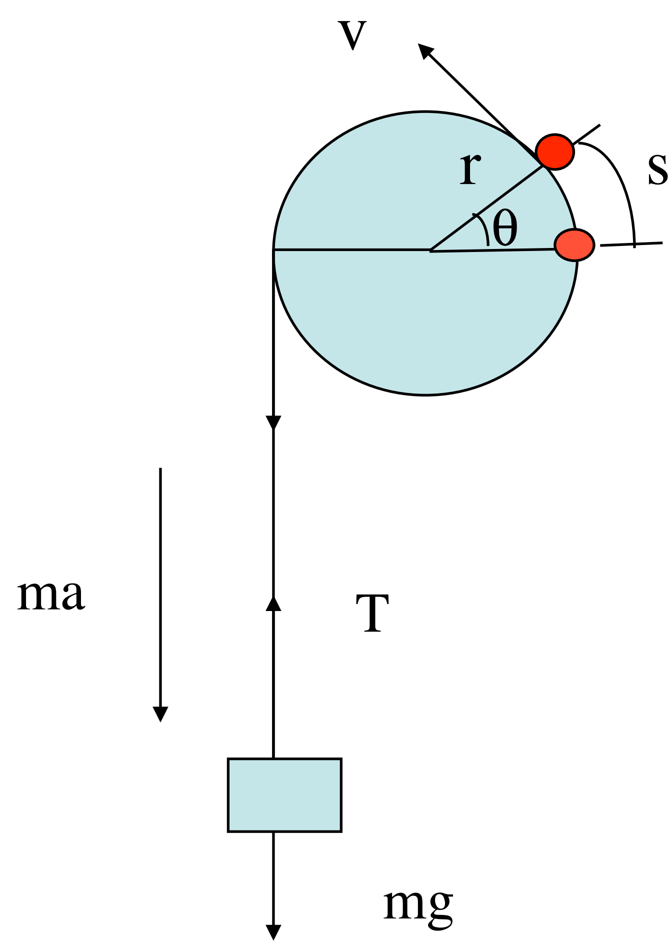
ROTATION WITH CONSTANT ANGULAR ACCELERATION

- Restrict discussion to a fixed axis of rotation but also applies if the axis is in translation as well.
- Write down or point out the analogy of the angular kinematic equations with linear motion. See Table 11.1 in text
- Same strategies are used to solve rotational problems as linear ones.

Rotation with constant angular acceleration: Consider some string wound around a cylinder.
There is no slippage between string and cylinder.
Red dot indicates a spot on the cylinder
that is rotating as I apply a force to the massless string

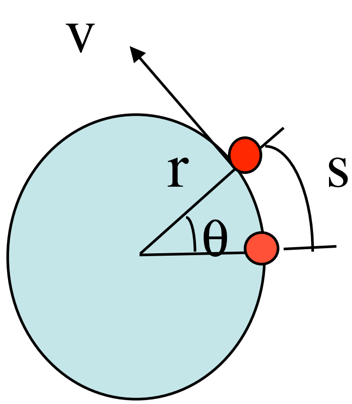
Front view



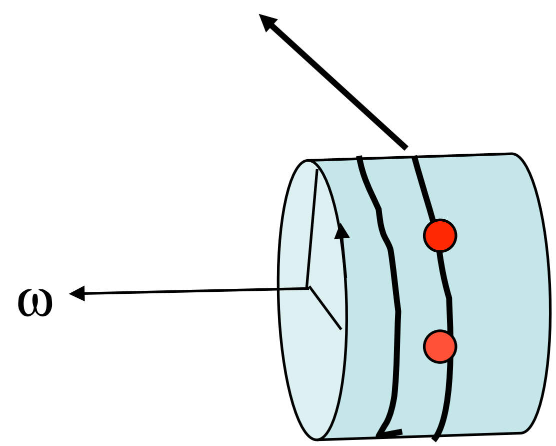


Red dot indicates a spot on the cylinder
that is rotating as I apply a force to the massless string

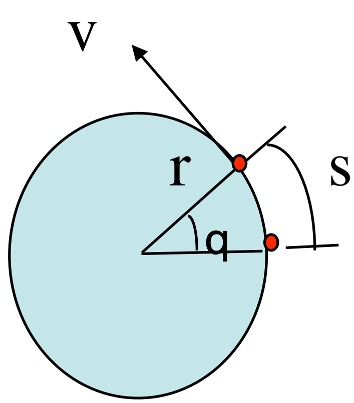
Front view



Isometric view



Define radians



For θ =360 degrees

$$\theta = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi \text{ radians in a circle}$$

Conversion from degrees to radians
is 0.0174 radians per degree
or 57.3 degrees per radian

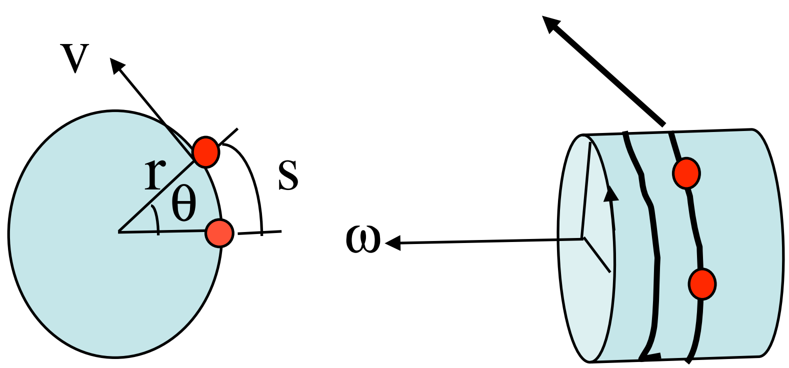
s and θ are not vectors

Define angular velocity

$$s = r\theta$$

Take derivative holding r constant

$$\frac{ds}{dt} = r\frac{d\theta}{dt}$$



$$v = \frac{ds}{dt}$$ Tangential velocity

$$\omega = \frac{d\theta}{dt}$$

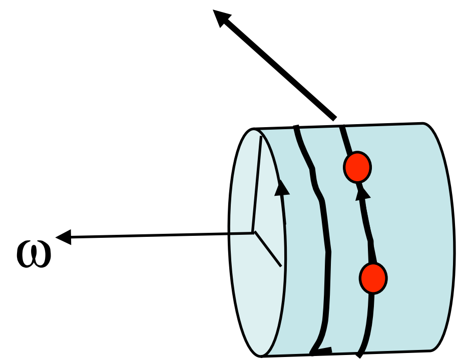
Angular velocity:
Vector
Magnitude
Direction
Units

$v = r\omega \qquad \vec{v} = \vec{\omega} \times \vec{r}$

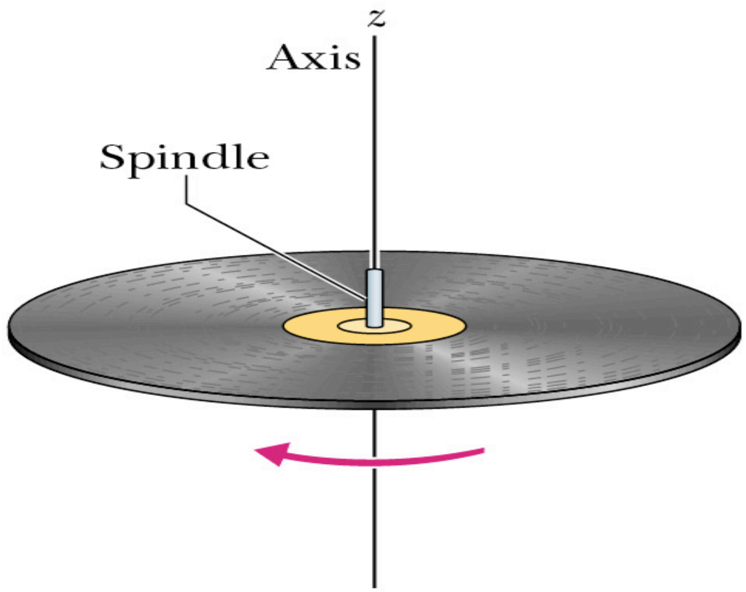
where ω is in radians per sec

and θ is in radians

Use Right hand rule to get direction of **w**

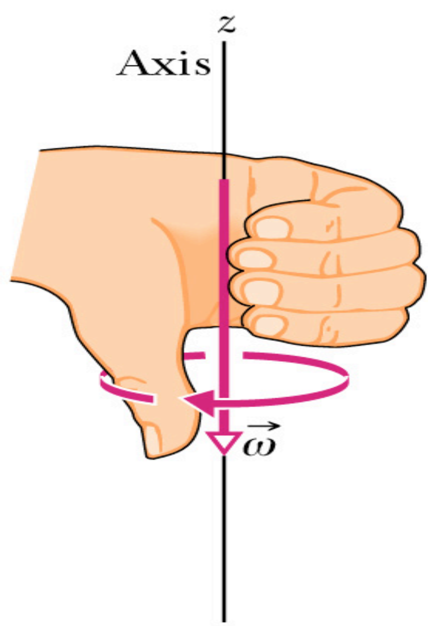


Counterclockwise is + for angular displacement θ
and angular velocity **ω.**



(*a*)

(*b*)



(*c*)

Define Angular or Rotational Acceleration

$$v = r\omega$$

$$a_t = \frac{dv}{dt} = r\frac{d\omega}{dt}$$

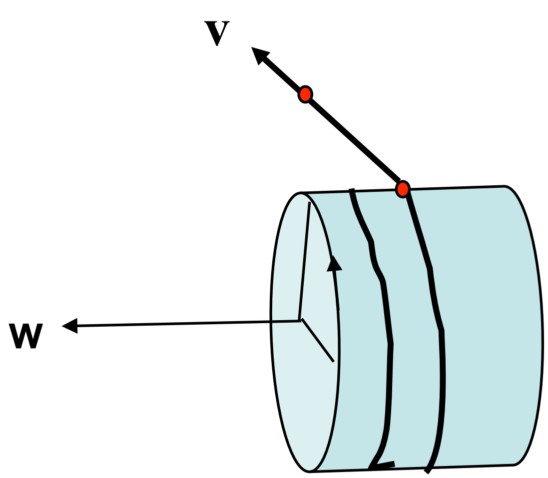
$$a_t = \alpha r$$ Also called the tangential acceleration

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt}(rad / s^2)$$ is called the angular acceleration

α is in the same or opposite direction as ω

Recall there is also the radial acc. $$a_r = \frac{v^2}{r} = \omega^2 r$$

Two Kinds of Acceleration

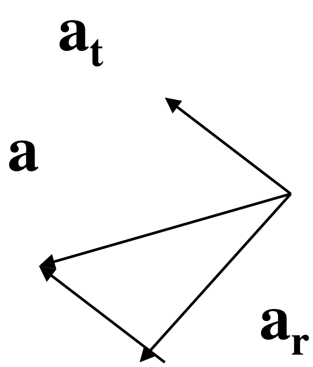


Tangential acceleration

$$a_t = \alpha r = \frac{d\omega}{dt} r$$

Radial acceleration

$$a_r \frac{v^2}{r} = \omega^2 r$$



in radial direction all the time

$$\vec{a} = \vec{a}_t + \overrightarrow{a_r}$$

$a_t + a_r$ are perpendicular to each other

For constant acceleration

$$x - x_0 = \frac{1}{2}(v_0 + v)t \qquad \theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$$

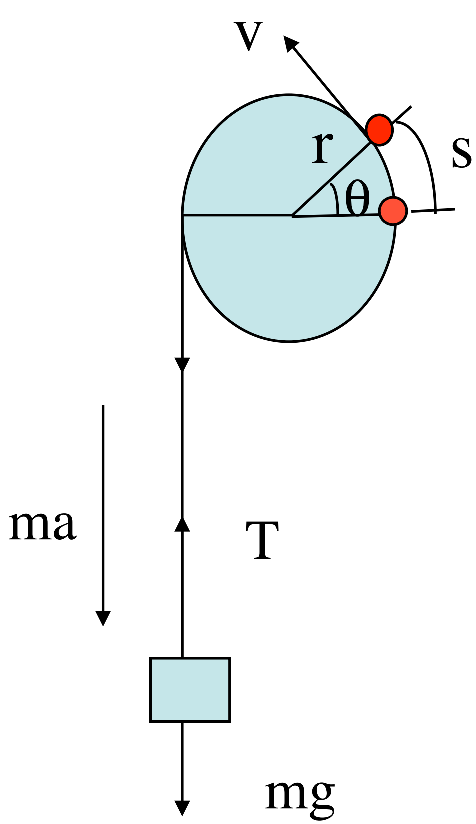
$$x - x_0 = v_0 t + \tfrac{1}{2} a t^2 \qquad \theta - \theta_0 = \omega_0 t + \tfrac{1}{2}\alpha t^2$$

$$v = v_0 + at \qquad \omega = \omega_0 + \alpha t$$

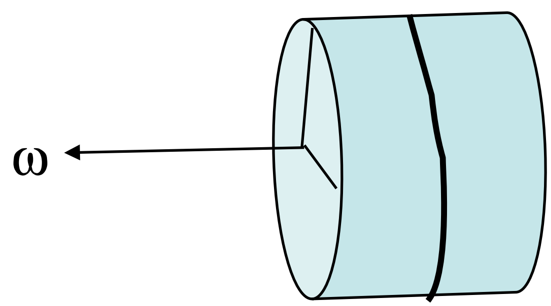
$$v^2 = {v_0}^2 + 2a(x - x_0) \qquad \omega^2 = {\omega_0}^2 + 2\alpha(\theta - \theta_0)$$

We have an analogous set of formulas for angular variables

What is the acceleration of the mass? How do we take into account the rotation of the pulley?



Free body diagram



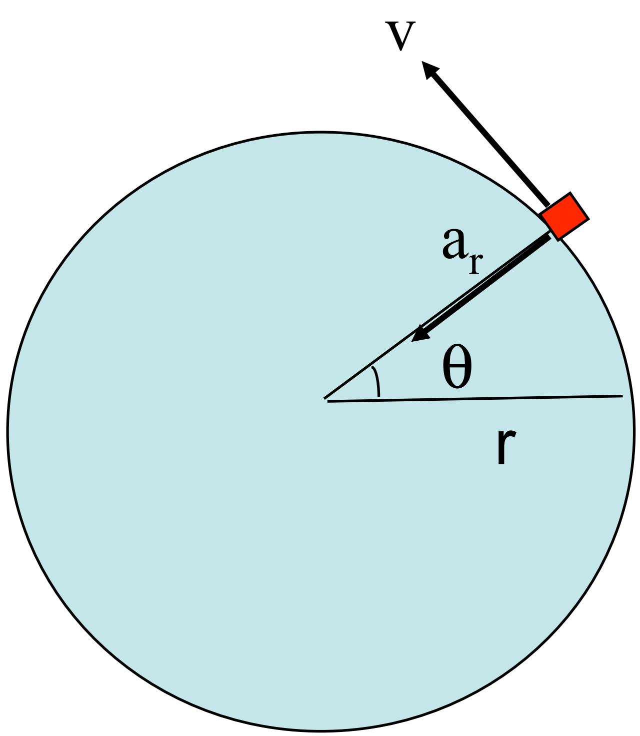
$$\sum F_y = T - mg = -ma$$

Need more information

How do we define kinetic energy of a rotating body?

Kinetic energy of rotation and rotational inertia I.

Consider what is the kinetic energy of a small rigid object moving in a circle?



$$K = \frac{1}{2}mv^2$$

$$v = \omega r$$

$$K = \frac{1}{2}mr^2\omega^2$$ Looks strange

We call mr^2 the moment of inertia I.

$$K = \frac{1}{2}I\omega^2$$

Kinetic energy of rotation

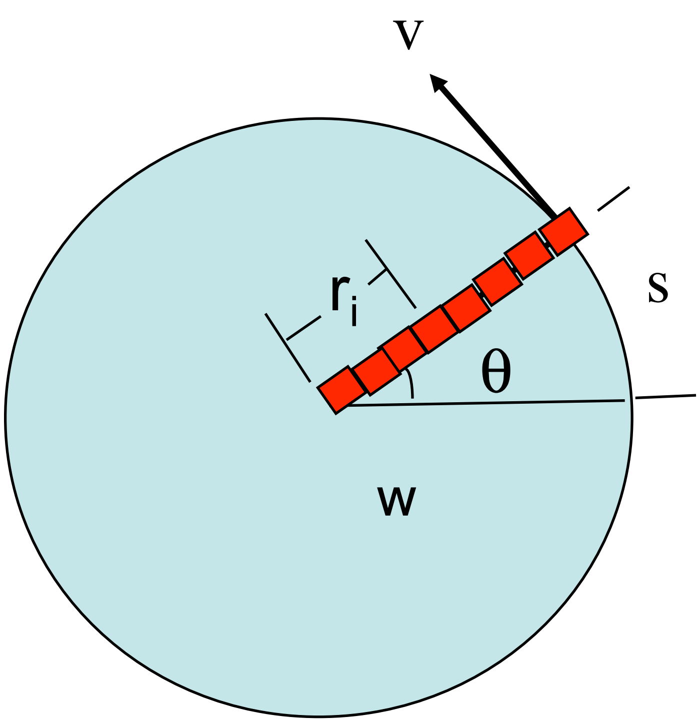
It is very important that we can define such a variable I. Here is why

Suppose our rotating body is a rigid rod. Now how do we define the kinetic energy?

$$K = \frac{1}{2} m r^2 \omega^2$$

$$K = \frac{1}{2} \sum m_i r_i^2 \omega^2$$

$$K = \frac{1}{2} I \omega^2$$



$$I = \sum_{i=1}^{i_{\max}} r_i^2 \Delta m_i = \int_0^M r^2 dm \quad = \tfrac{1}{3} ML^2$$

I is called the moment of inertia about an axis through the end of the rod.

$$K = \frac{1}{6} ML^2 \omega^2$$

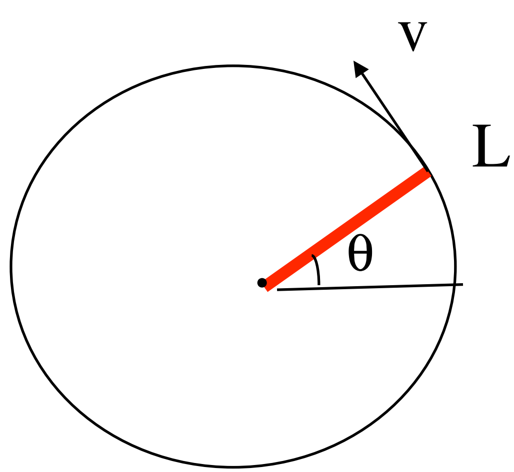
Evaluation of the rotational inertia

$$I = \sum_{i=1}^{i_{\max}} r_i^2 \Delta m_i = \int_0^M r^2 dm \qquad dm = \rho dr$$

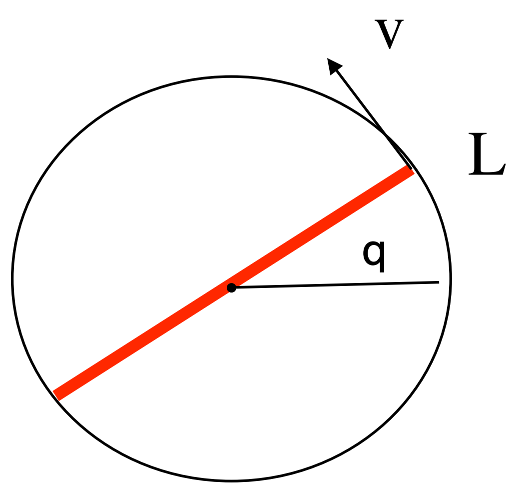
$$= \rho \int_0^L r^2 dr$$

$$= \rho \tfrac{1}{3} r^3 \Big|_0^L = \rho \frac{L^3}{3} = \frac{M}{L}\frac{L^3}{3} = \tfrac{1}{3} ML^2 = \mathrm{I}$$

for a rod rotating about an axis through the end perpendicular to the length

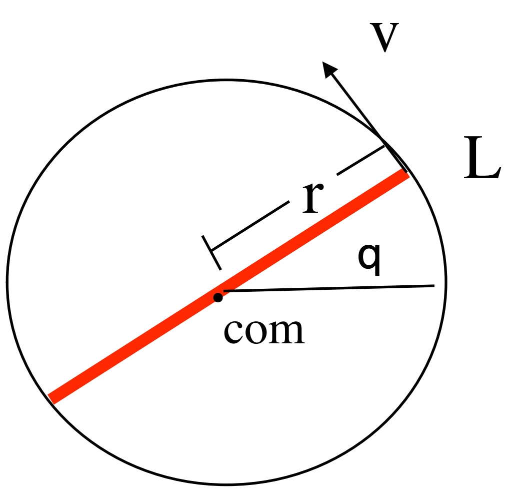


Now consider rod rotating about an axis through the center of mass of the rod



$$I_{com}=\int_0^M r^2 dm = 2\rho\int_0^{\frac{L}{2}} r^2 dr = 2\rho \tfrac{1}{3} r^3\Big|_0^{\frac{L}{2}} = 2\frac{M}{L}\tfrac{1}{3}\frac{L^3}{8} = \tfrac{1}{12}ML^2$$

Parallel Axis Theorem



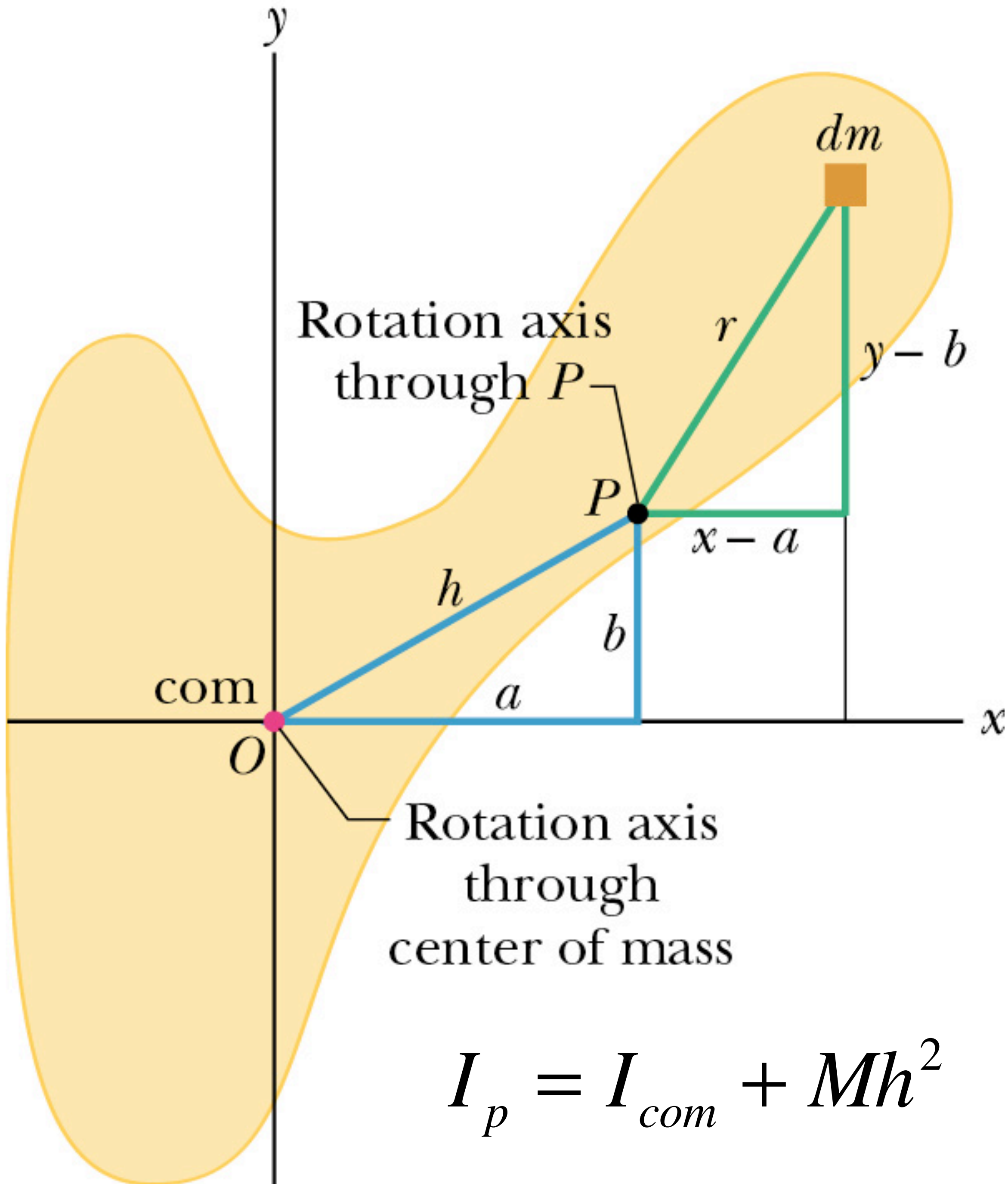
Notice that the difference

$$I_{end} - I_{com} =$$

$$\frac{1}{3}ML^2 - \frac{1}{12}ML^2 = \frac{1}{4}ML^2 = M(\frac{L}{2})^2 = Mr^2$$

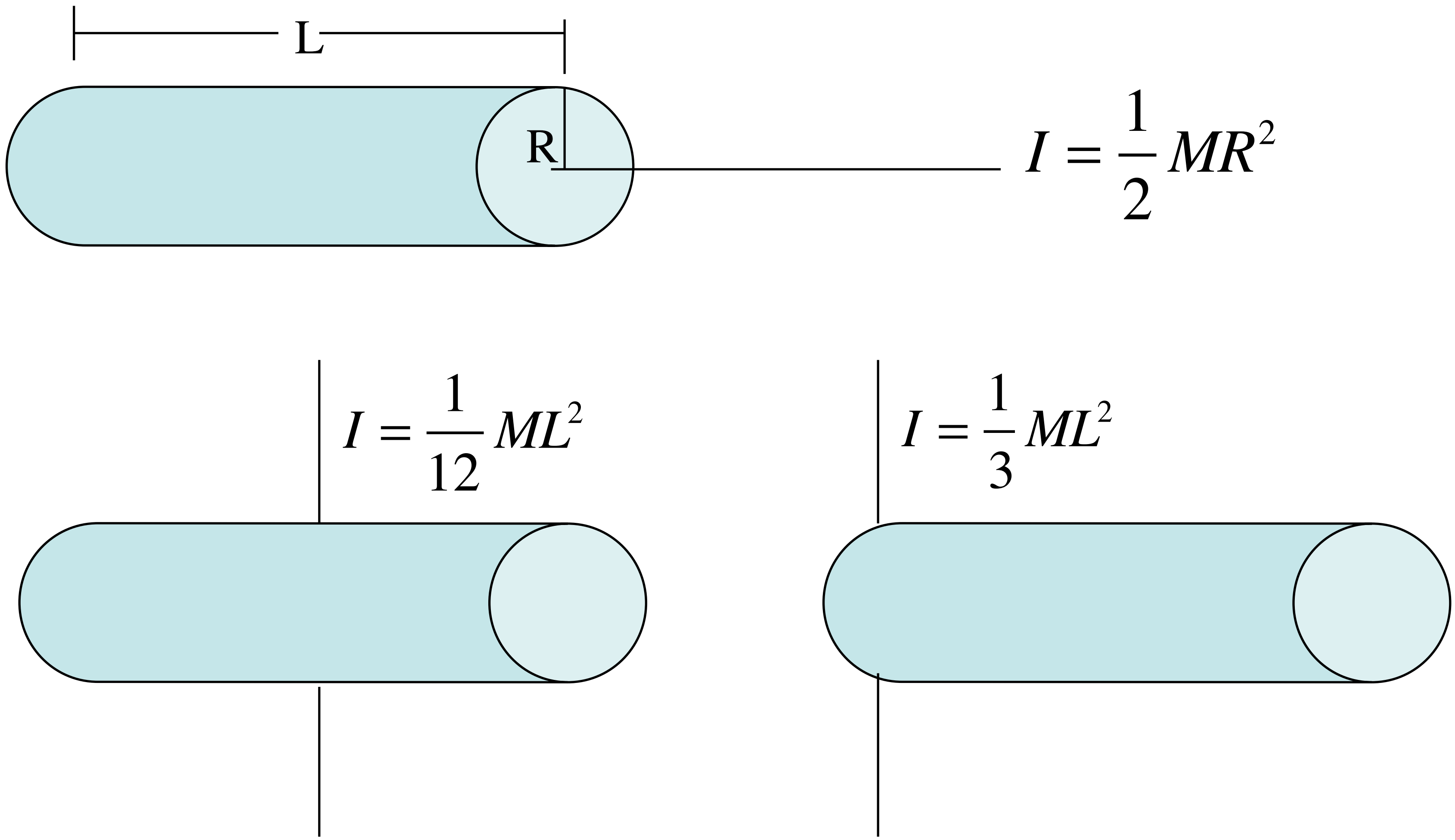
$$I_{end} = I_{com} + Mr^2$$

$$I_p = I_{com} + Mr^2$$ General relation


y
dm
Rotation axis
through P
r
y − b
P
x − a
h
b
com
a
x
O
Rotation axis
through
center of mass

$$I_p = I_{com} + Mh^2$$

Moment of inertia of long thin rod of same radius and length rotating about different axes.



$$I = \frac{1}{2}MR^2$$

$$I = \frac{1}{12}ML^2$$

$$I = \frac{1}{3}ML^2$$

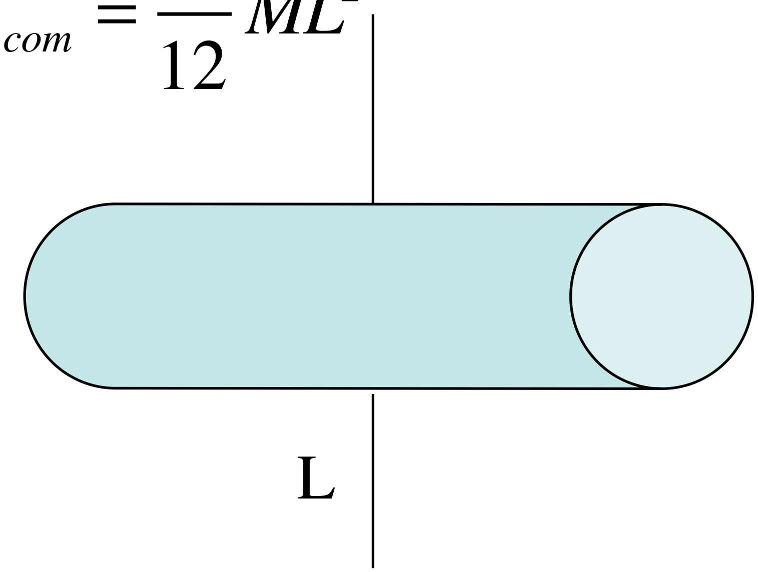
Parallel Axis Theorem

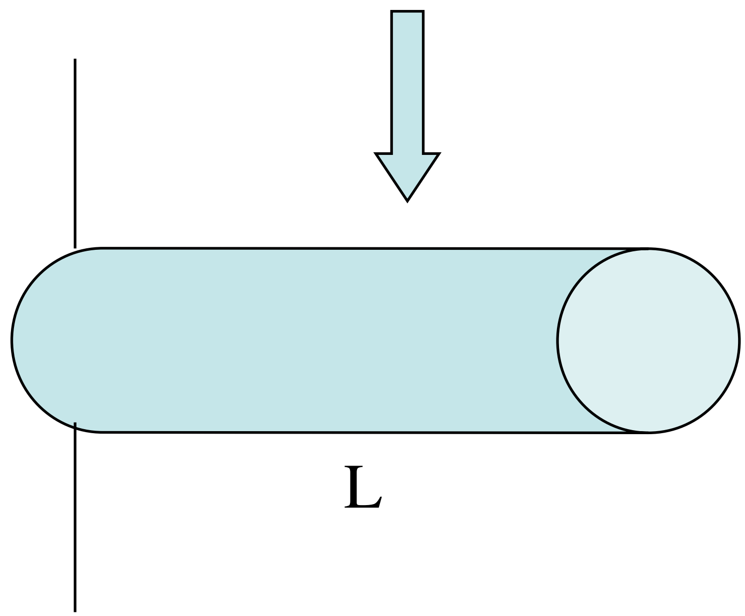
$$I_p = I_{com} + Mh^2$$

h=L/2

$$I = \frac{1}{12}ML^2 + M(\frac{L}{2})^2 = \frac{1}{12}ML^2 + \frac{1}{4}ML^2 = \frac{1}{3}ML^2$$

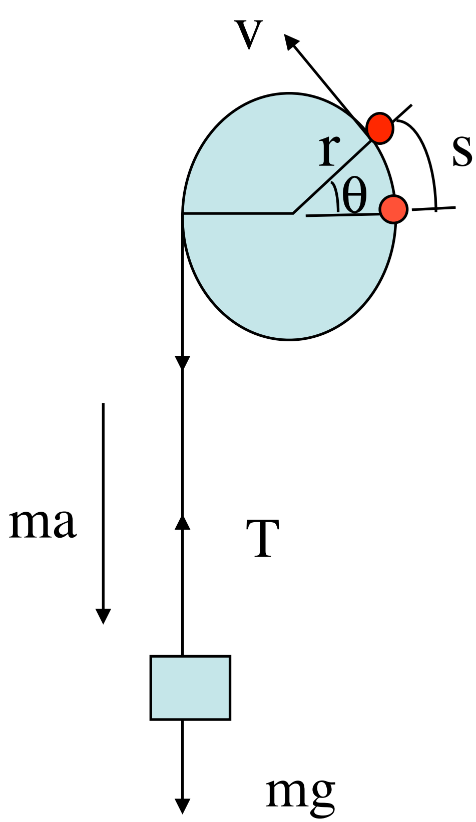
$$I_{com} = \frac{1}{12}ML^2$$



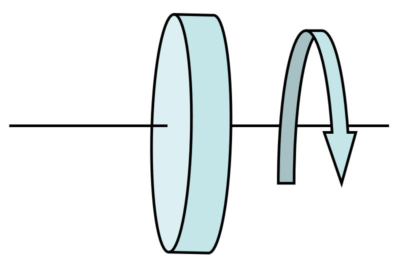


MOMENT OF INERIA FOR A PULLEY

$$I = \frac{1}{2} M r^2$$



Rotating about central axis

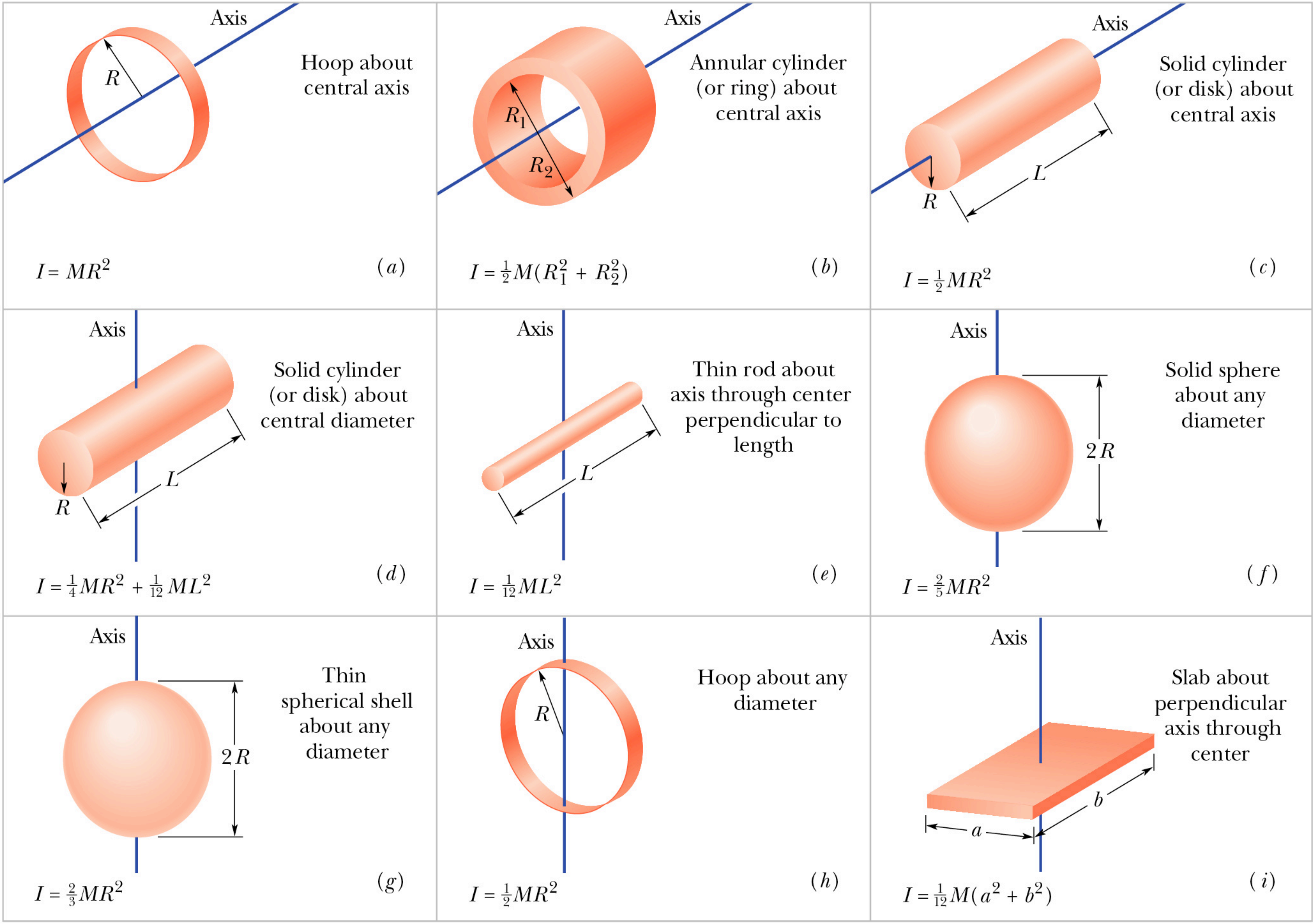


Still need more information to find T

$$\sum F_y = T - mg = -ma$$

Demo for moment of inertia

Rotate hoops and cylinders down an incline plane



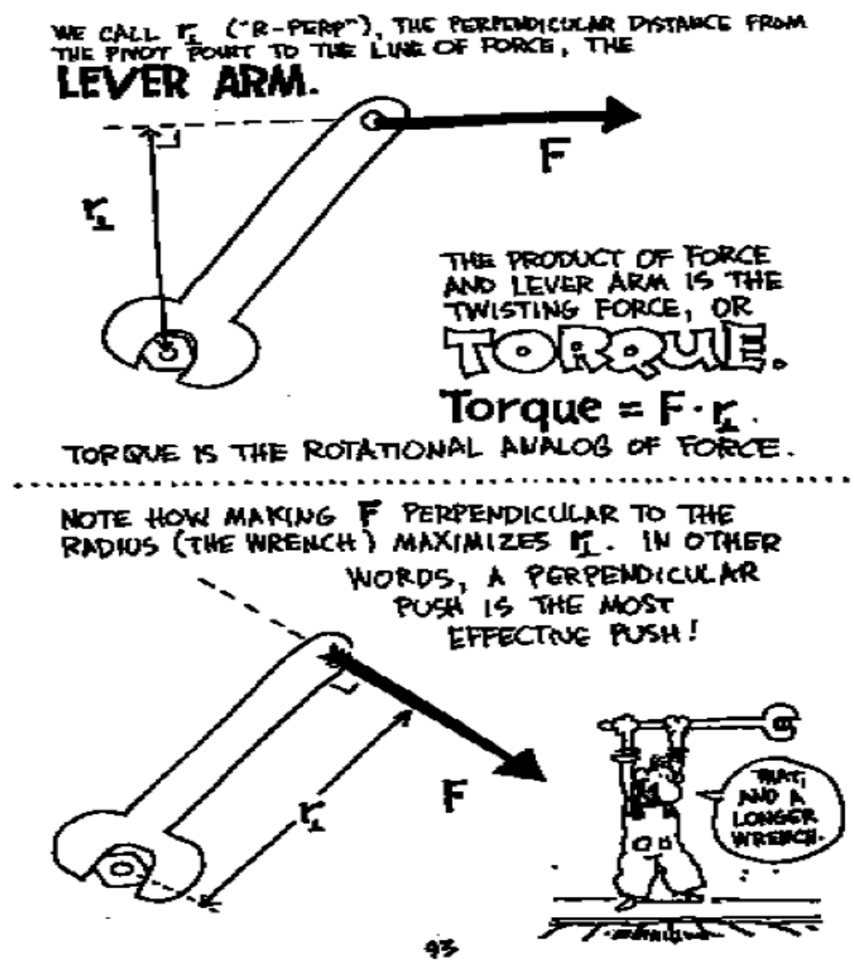
Axis
Hoop about central axis
R
$I = MR^2$
(a)
Axis
Annular cylinder (or ring) about central axis
R_1
R_2
$I = \frac{1}{2}M(R_1^2 + R_2^2)$
(b)
Axis
Solid cylinder (or disk) about central axis
R
L
$I = \frac{1}{2}MR^2$
(c)
Axis
Solid cylinder (or disk) about central diameter
R
L
$I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$
(d)
Axis
Thin rod about axis through center perpendicular to length
L
$I = \frac{1}{12}ML^2$
(e)
Axis
Solid sphere about any diameter
$2R$
$I = \frac{2}{5}MR^2$
(f)
Axis
Thin spherical shell about any diameter
$2R$
$I = \frac{2}{3}MR^2$
(g)
Axis
Hoop about any diameter
R
$I = \frac{1}{2}MR^2$
(h)
Axis
Slab about perpendicular axis through center
a
b
$I = \frac{1}{12}M(a^2 + b^2)$
(i)

Concepts in Chapter 11
Rolling, Torque, and Angular Momentum

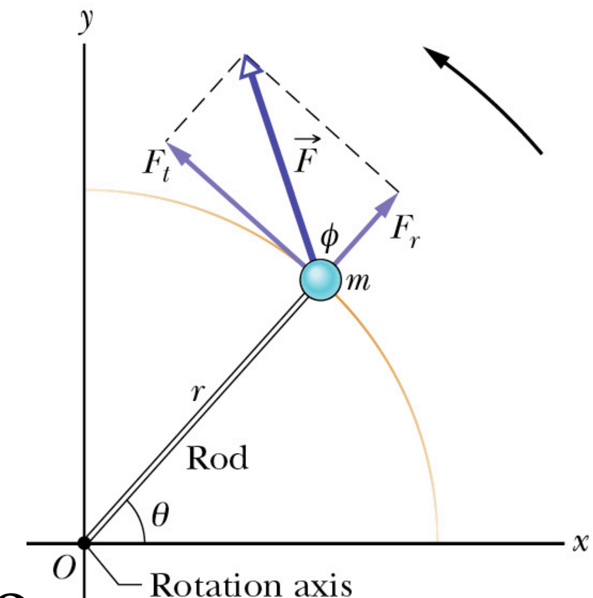
- Torque
- Newton's Law for rotations
- Work Energy theorem for rotations
- Atwood machine with mass of pulley
- More on rolling without sliding
- Center of percussion: baseball bat and hammer
- Sphere rolling down an inclined plane
- Yo -yo
- Angular momentum
- Conservation of angular momentum
- Breaking a stick balanced on a wine glass

Torque. It is similar to force but it also depends on the axis of rotation. Why do we have to define torque?

- Use towel to open stuck cap on jar
- Door knob far from hinge
- Screw driver with large fat handle
- Lug wrench to unscrew nuts on rim for tires



Torque

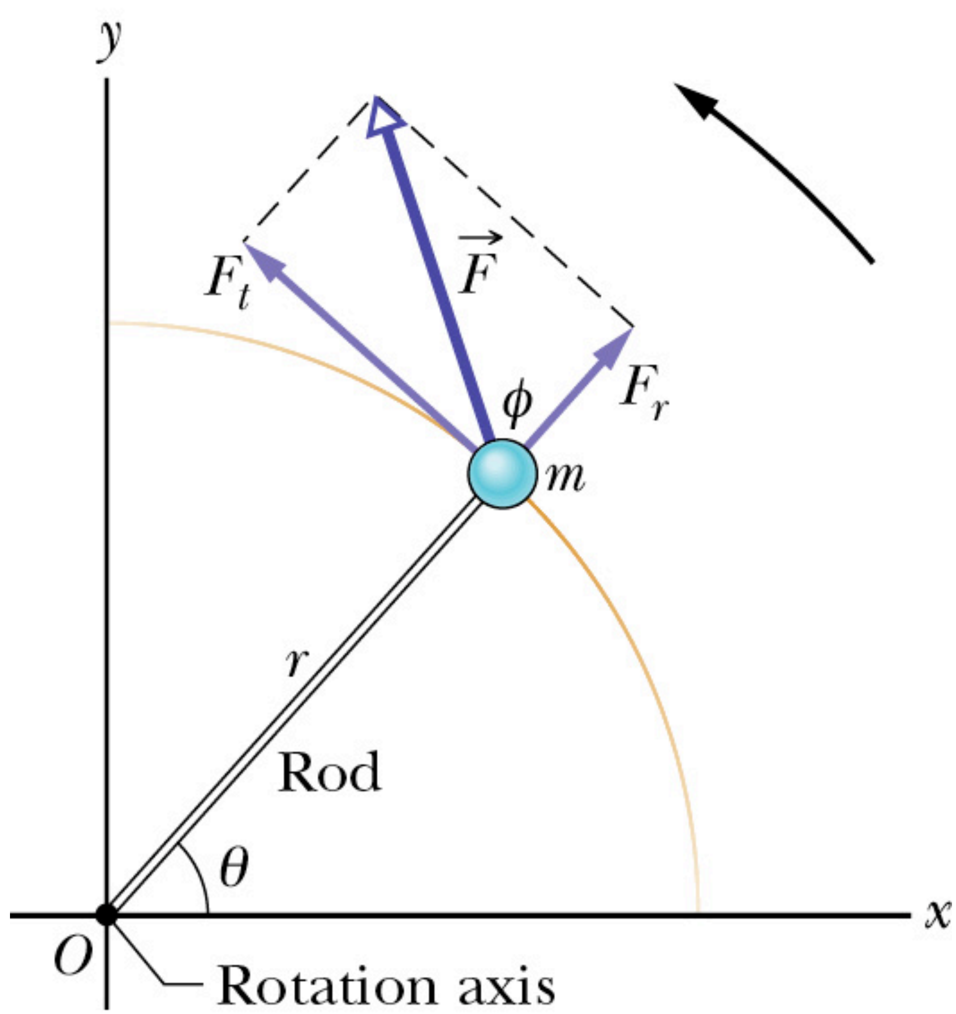


$$\text{Torque} = rF\sin\phi = r \times F = \tau$$

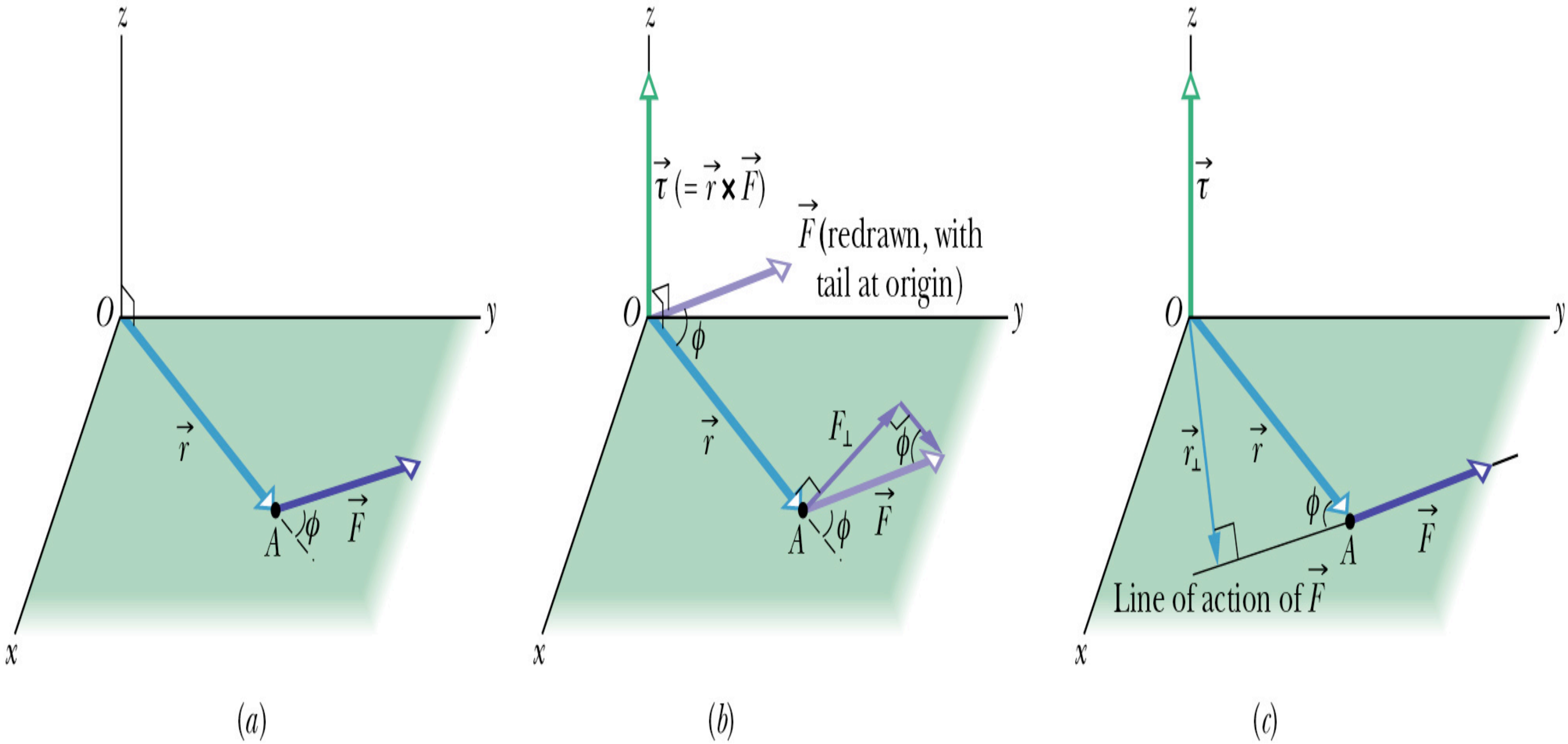
- Also Torque = $r_{perp}F = rF_t$
- r is a vector perpendicular to the rotation axis.

ϕ is the angle between r and F when the tails are together.

- If F is along r, the torque is 0. Only component of F that is perpendicular to r contributes to torque or $a_{t.}$ Parallel component contributes to $a_{r.}$
- Increase r or F to get more torque.
- Positive torque corresponds to counterclockwise rotation
- Long handled wrench, door knob far from the hinge.



$$\text{Torque} = rF\sin\phi = r \times F = \tau$$



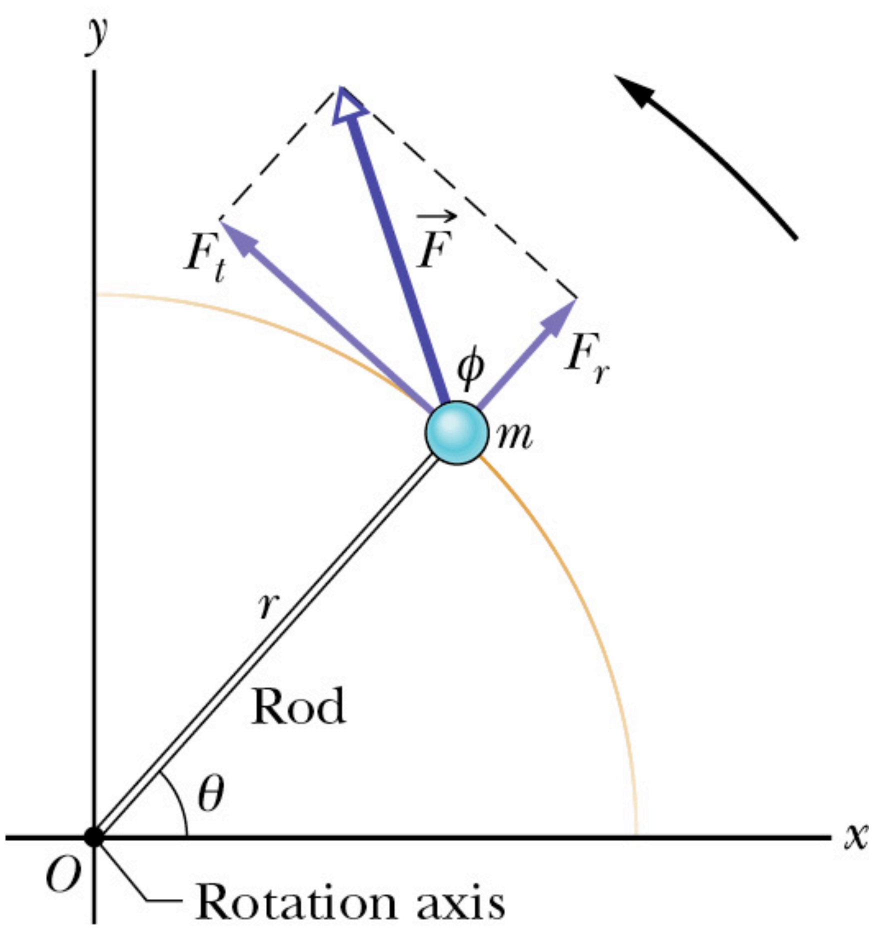
$$\text{Torque} = rF\sin\phi = r \times F = \tau$$

Newton's 2nd law for rotation

$$\tau_{net} = I\alpha$$

Consider the consistency argument below

Suppose I have small mass at the end of a massless rod



$$F_t = ma_t$$

$$rF_t = mra_t$$

$$= \frac{mr^2 a_t}{r} = I\alpha$$

$$I = mr^2$$

$$\alpha = \frac{a_t}{r}$$

$$rF_t = rF\sin\phi$$

$$\vec{r} \times \vec{F} = I\alpha$$

$$\tau = I\alpha$$

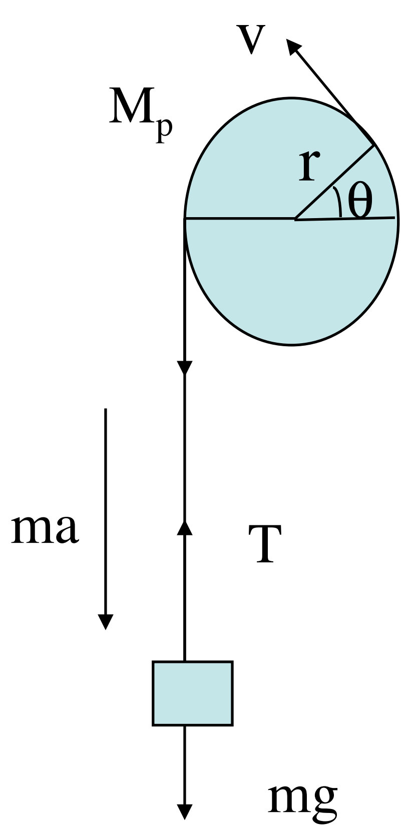
What is the acceleration of the mass? Now we can take into account the rotation of the pulley?

$$r = radius$$

$$F = T$$

$$I = \frac{1}{2} M_p r^2$$

$$\alpha = a/r$$



$$\sum F_y = T - mg = -ma$$

$$\vec{r} \times \vec{F} = I\vec{\alpha}$$

$$rT = \frac{1}{2} M_p r^2 \, a/r$$

$$T = \frac{M_p a}{2}$$

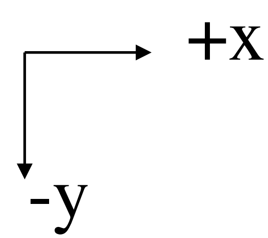
$$\frac{Ma}{2} - mg = -ma$$

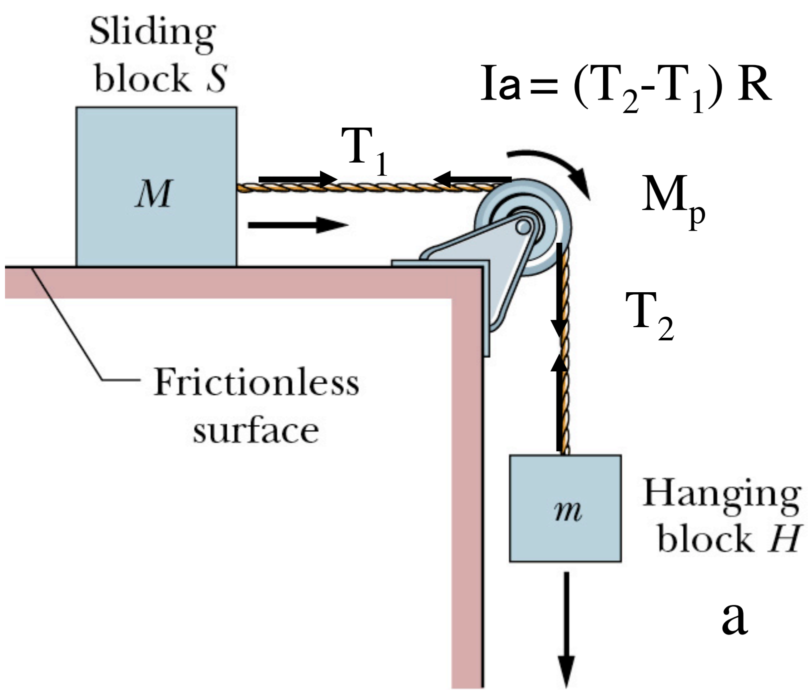
$$a = \frac{2mg}{(M_p + 2m)}$$

$$T = \frac{M_p m}{(M_p + 2m)}$$

Frictionless Sideways Atwood machine with a pulley with mass

Now take into account the rotation of the pulley.





$$T_1 = Ma$$

$$T_2 - mg = -ma$$

$$I\alpha = (T_2 - T_1)R$$

new equation

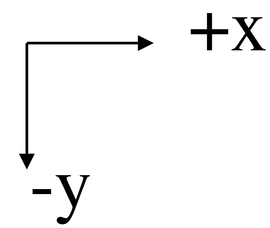
$$a = \alpha R$$

$$\frac{1}{2}M_p R^2 \frac{a}{R} = (T_2 - T_1)R$$

$$\frac{1}{2}M_p R^2 \frac{a}{R} = (mg - ma - Ma)R$$

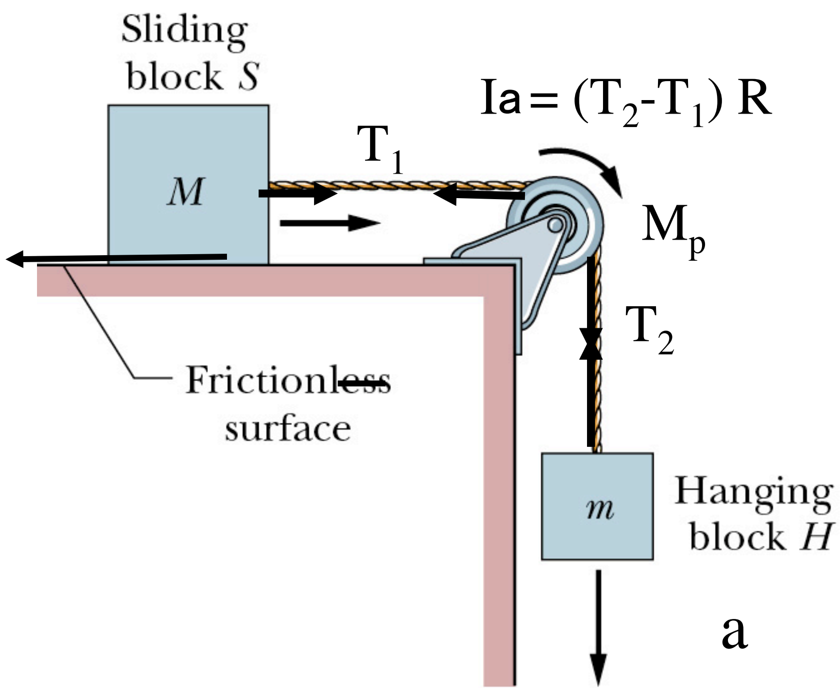
$$a = \frac{m}{(m + M + \frac{M_p}{2})} g$$

Now include friction between block M and surface

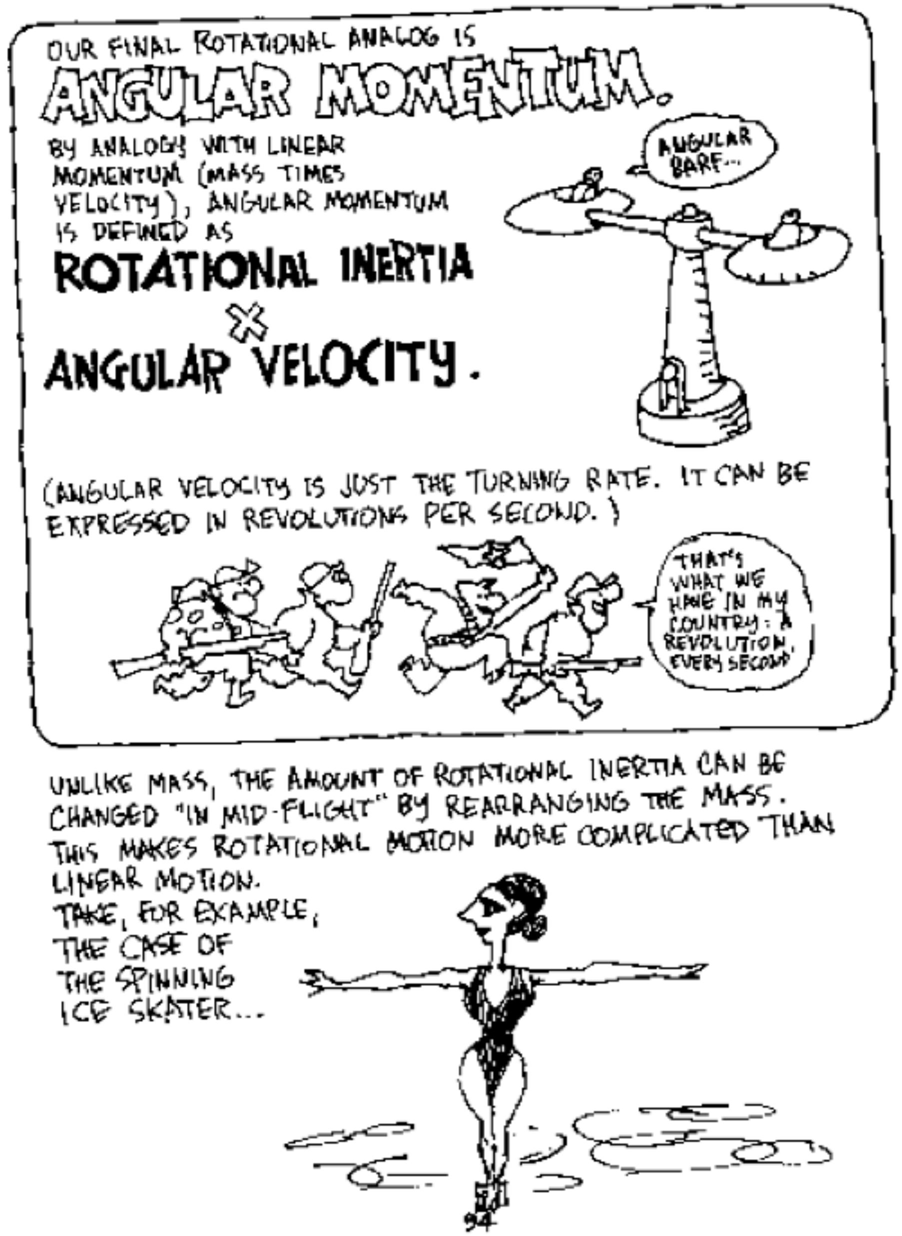


$$T_1 - \mu Mg = Ma$$
$$T_2 - mg = -ma$$

$$I\alpha = (T_2 - T_1)R \qquad \text{new equation}$$
$$a = \alpha R$$
$$\frac{1}{2}M_p R^2 \frac{a}{R} = (T_2 - T_1)R$$



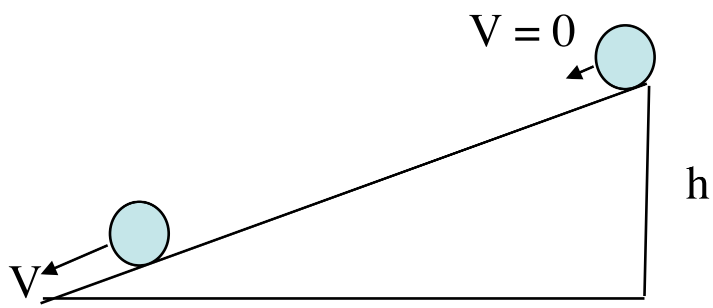
$$\frac{1}{2}M_p R^2 \frac{a}{R} = (mg - ma - Ma - \mu Mg)R$$
$$a = \frac{m(1-\mu)}{(m + M + \frac{M_p}{2})} g$$



$$\vec{L} = I\vec{\omega}$$

Inclined plane rolling demo. Which object gets to the bottom first and why?

Now we want to understand why objects accelerated at different rates down the inclined plane. What is its total kinetic energy of the object at the bottom of the inclined plane?

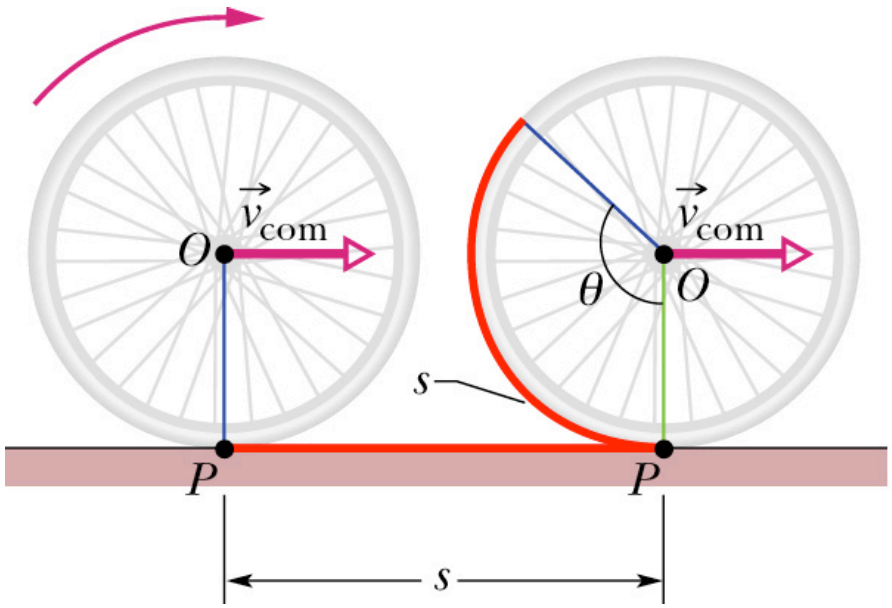


Case I: Frictionless plane. Pure translation, No rotation. Then K =1/2 MV^2 at the bottom of the plane

Case II: Slipless plane, Translation and rolling down inclined plane. Then your guess might be $K = \frac{1}{2} M V_{com}{}^{2} + \frac{1}{2} I_{com} \omega^{2}$ and you would be right.

First we have to ask what is rolling without slipping?

Linear speed of the center of mass of wheel is ds/dt



without slipping

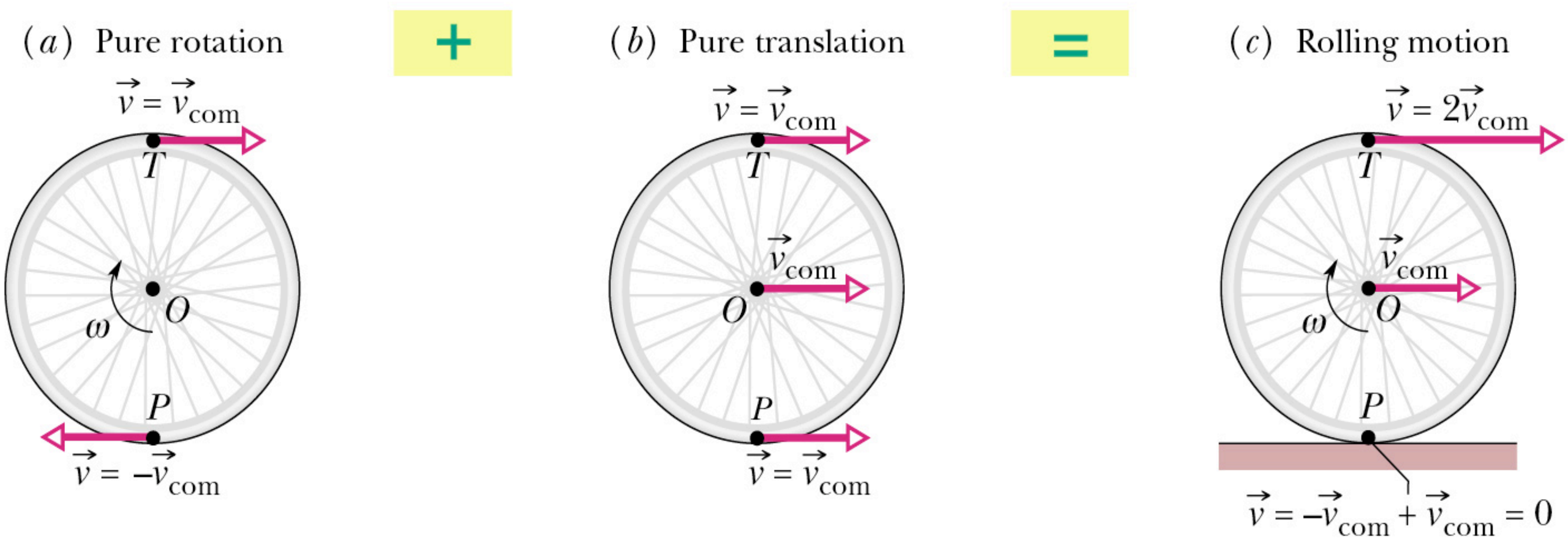
The angular speed ω about the com is $d\theta/dt$.

From $s=\theta R$ we get $ds/dt = d\theta/dt\ R$ or $v_{com} = \omega R$ for smooth rolling motion

Rolling can be considered rotating about an axis through the com while the center of mass moves.

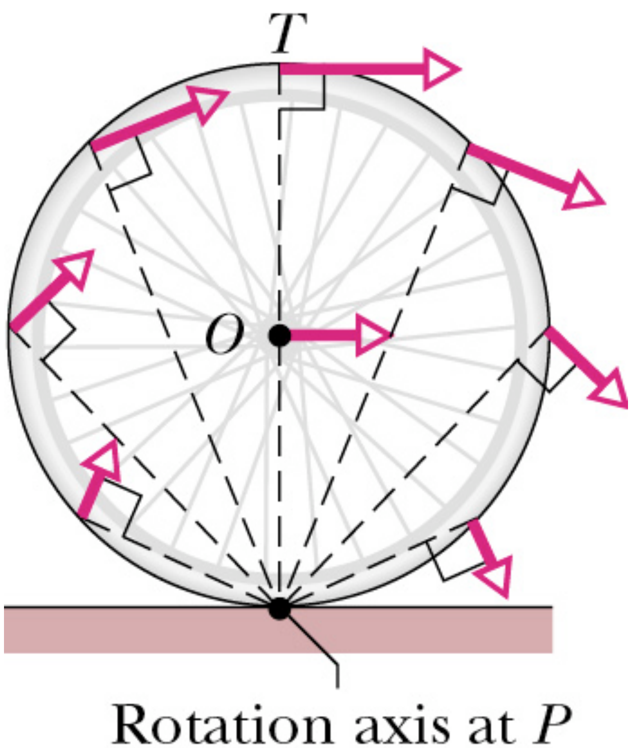
At the bottom P is instantaneously at rest. The wheel also moves slower at the bottom because pure rotation motion and pure translation partially cancel out

See photo in Fig 12-4. Also tire tracks are clear in the snow and are not smudged.



Rolling as pure rotation

Consider rolling as pure rotation with angular velocity w about an axis passing through point P. The linear speed at the top is $v_T = \omega(2R) = 2\ v_{com}$ (same as before)



The kinetic energy now is

$$K = \frac{1}{2} I_p \omega^2$$

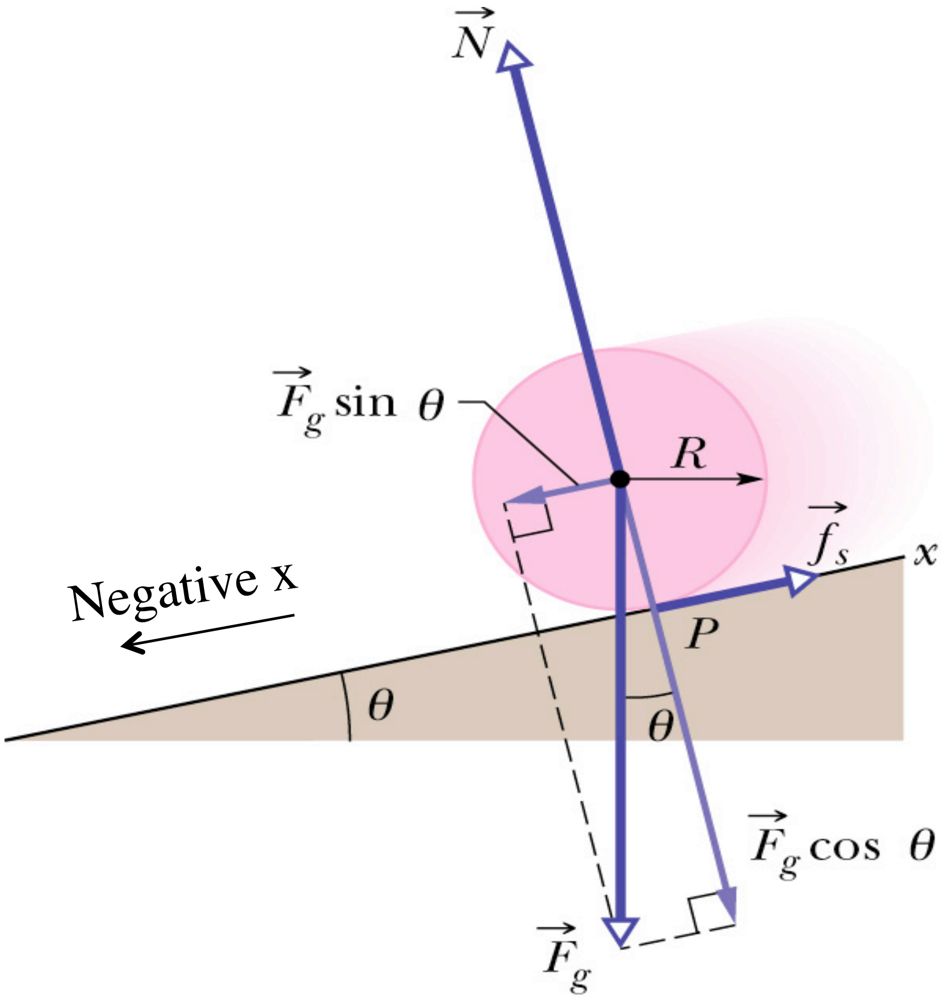
What is I_P?

$$I_p = I_{com} + MR^2$$

$$K = \frac{1}{2} I_{com}\omega^2 + \frac{1}{2} MR^2\omega^2$$

$$= \frac{1}{2} I_{com}\omega^2 + \frac{1}{2} Mv_{com}{}^2$$

What is the acceleration of a sphere smoothly rolling down an inclined plane?



a) Drop an object? $a = -g$

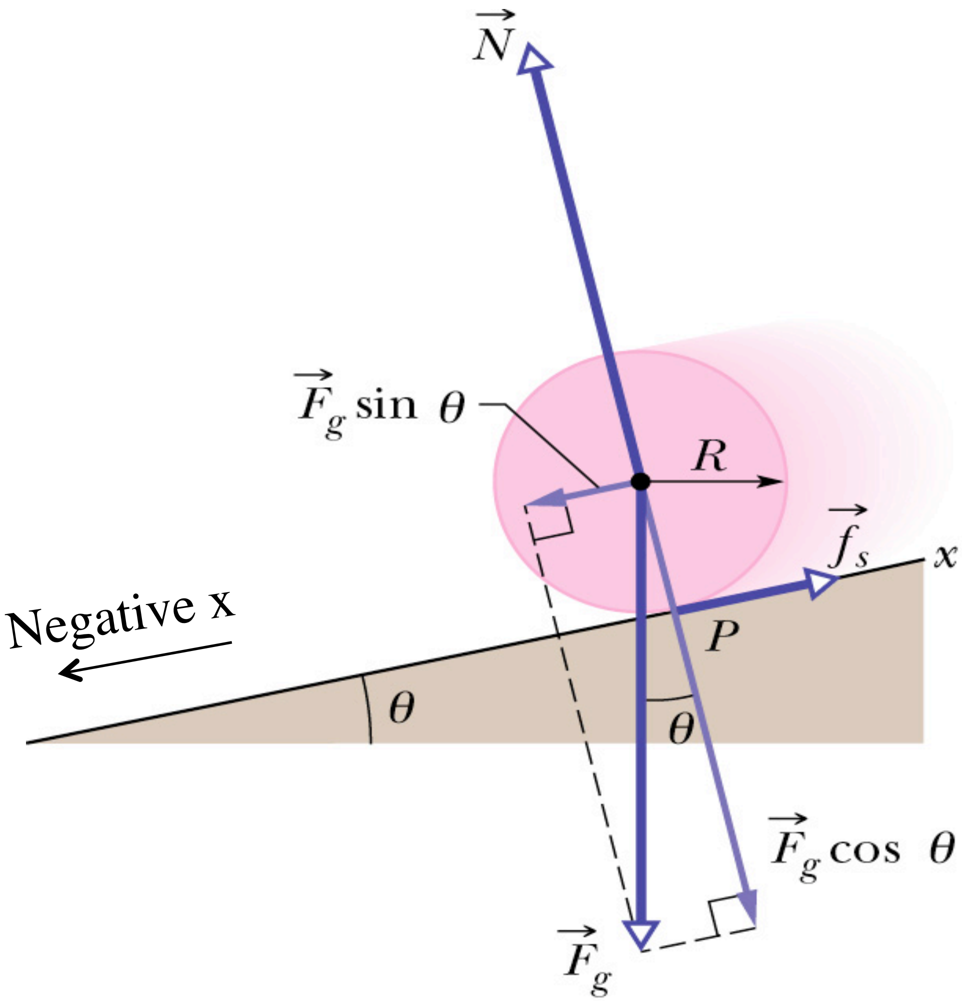
b) Block sliding down a frictionless inclined plane? $a = -g\sin\theta$

c) With friction?
$a = -g(\sin\theta - \mu\cos\theta)$

d) Sphere rolling down an inclined plane? ????

$$a_{com} =$$

What is the acceleration of a sphere smoothly rolling down an inclined plane?



$$F_{net} = Ma_{com}$$

x component Newtons Law

$$f_s - Mg\sin\theta = Ma_{com}$$

Find torque about the com

$$\tau_{net} = I\alpha$$

$$Rf_s = I_{com}\alpha$$

$$\alpha = \frac{-a_{com}}{R}$$

Solve for f_s

$$f_s = I_{com}\alpha / R$$

$$\alpha = \frac{-a_{com}}{R}$$

$$f_s = -I_{com}a_{com} / R^2$$

$$-I_{com}a_{com} / R^2 - Mg\sin\theta = Ma_{com}$$

Solve for

$$a_{com} = -\frac{g\sin\theta}{1+\dfrac{I_{com}}{MR^2}}$$

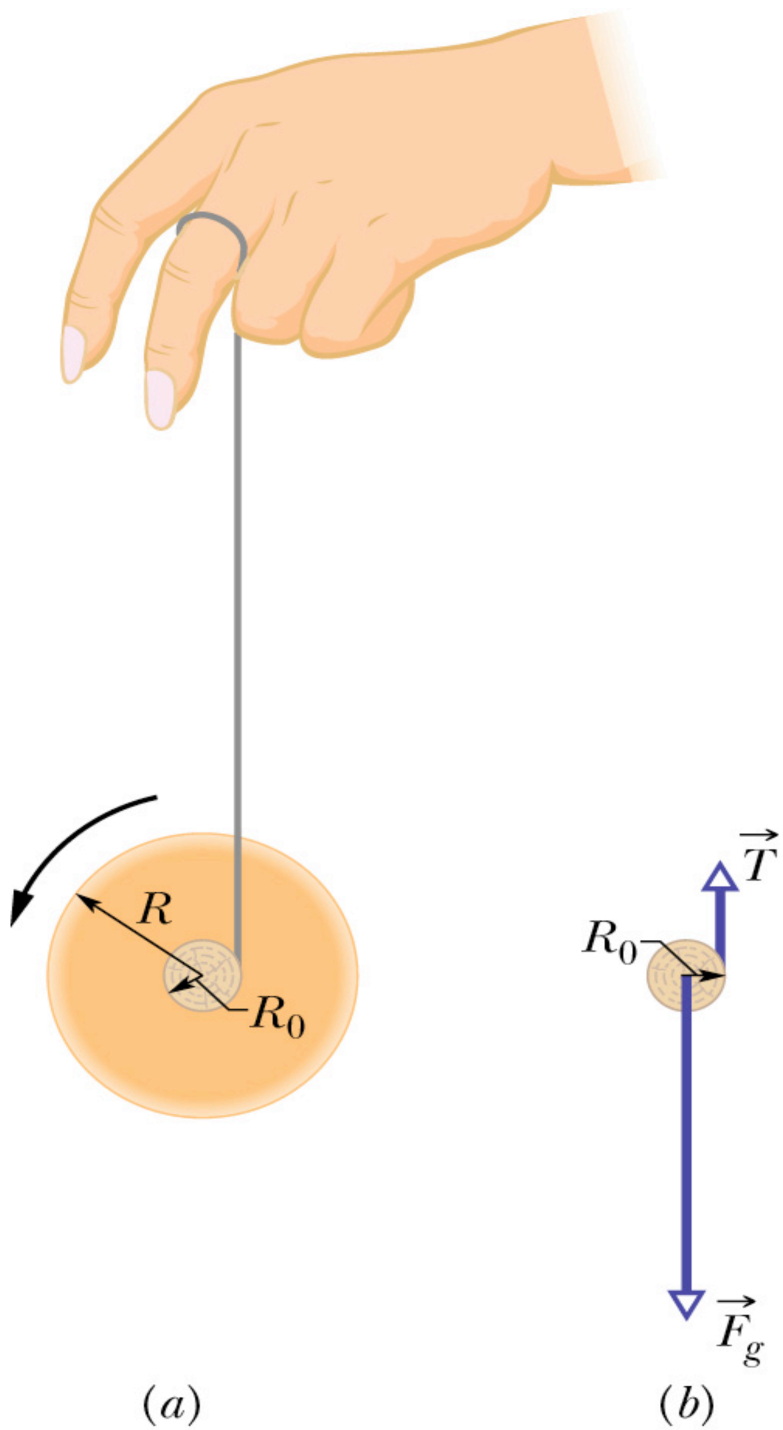
This will predict which objects will roll down the inclined faster.

$$a_{com} = -\frac{g\sin\theta}{1+\frac{I_{com}}{MR^2}}$$

Let θ= 30 deg

Sin 30 = 0.5

shape	I_{com}	$1+ I_{com}/MR^2$	a_{com}
sphere	$2/5\ MR^2$	1.4	0.71x g/2
disk	$1/2MR^2$	1.5	0.67 x g/2
pipe	MR^2	2.0	0.50 x g/2



$$a_{com} = -\frac{g\sin\theta}{1+\frac{I_{com}}{MR^2}}$$

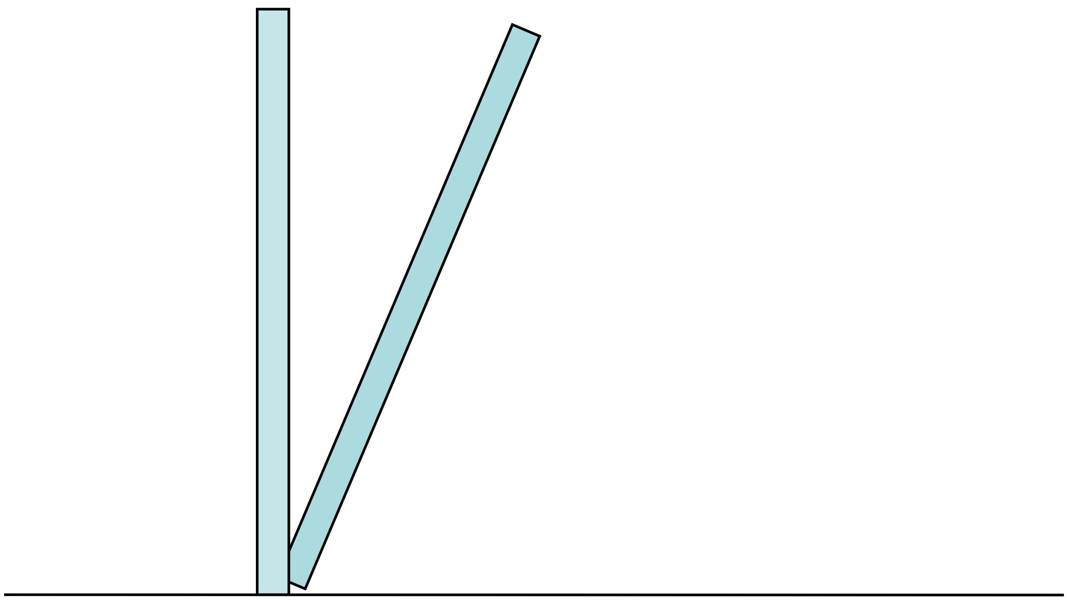
Yo-yo rolls down the string as if it were inclined plane at 90 degrees

Instead of friction, tension in the string holds it back

$$a_{com} = -\frac{g}{1+\frac{I_{com}}{MR_0^{\ 2}}}$$

The moment of inertia I_{com} is that of the yo-yo itself.

Can objects fall with a greater acceleration than gravity? Work out with in class.



Things to consider

- Angular momentum of a rigid body

$$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$$

- Conservation of angular momentum

$$I_i\vec{\omega}_i = I_f\omega_f$$

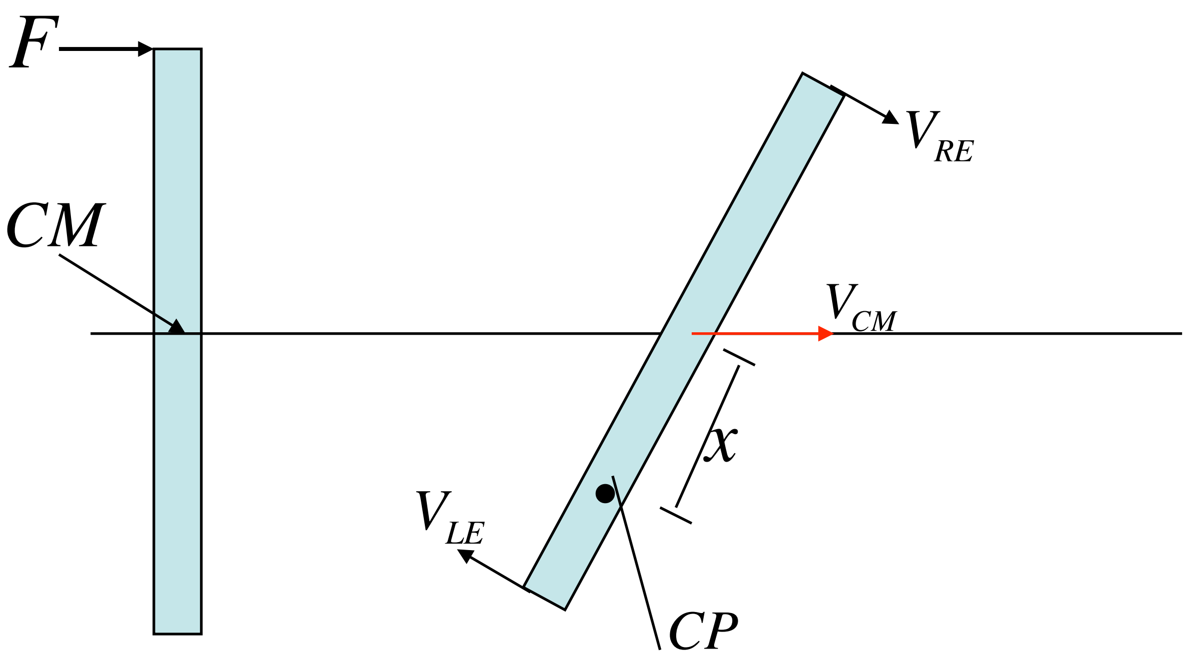
Another Stick Problem

A Stick resting on ice is struck at one end with
an impulsive force F perpendicular to its length

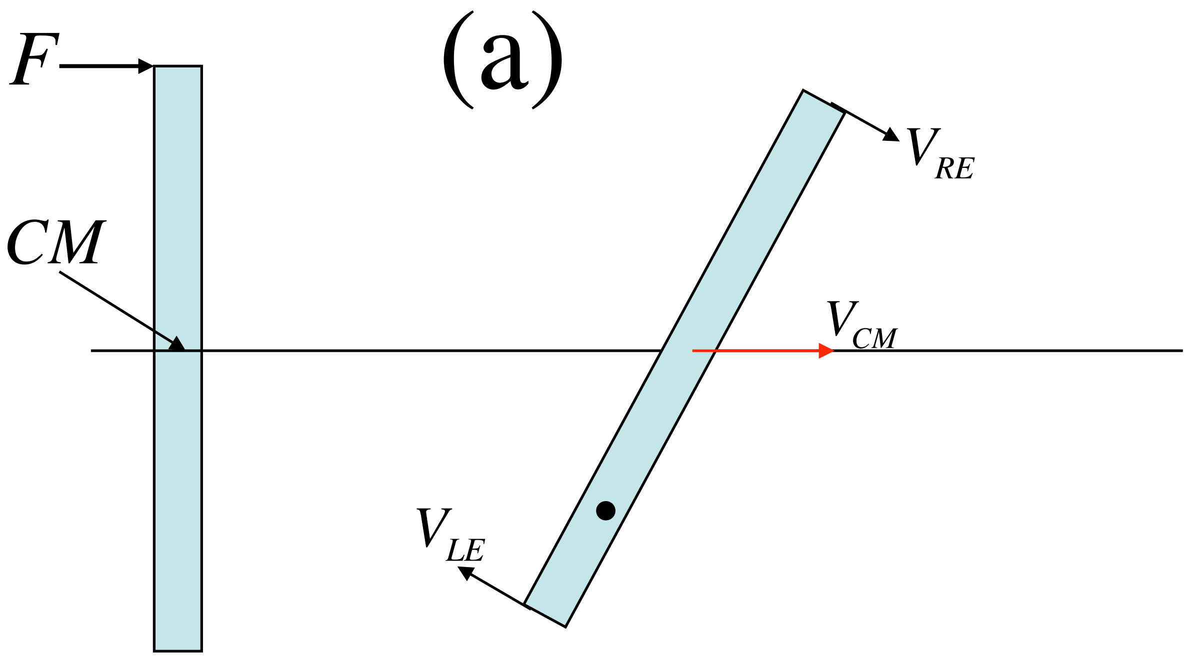
The stick has mass M and length L

(a) Find the acceleration of the center of mass (CM)
(b) Find the angular velocity of the stick about the CM
(c) Find the velocity of each end of the stick in terms of the velocity of the CM
(d) Find the center of percussion (CP), which is where the rotational and translational motion cancel.

Diagram



Δt is the time over which the force lasted



Find the acceleration of the center of mass (CM)

$$F = Ma_{CM}$$

$$a_{CM} = \frac{F}{M} \Rightarrow F = \frac{\Delta P}{\Delta t} = \frac{M(V_{CM} - 0)}{\Delta t} = \frac{MV_{CM}}{\Delta t}$$

$$a_{CM} = \frac{MV_{CM}}{M\Delta t}$$

$$\boxed{a_{CM} = \frac{V_{CM}}{\Delta t}}$$

(b)

Find the angular velocity of the stick about the CM

Use $\vec{L} = I\omega = \vec{r} \times \vec{p}$ and conservation of angular momentum

$$\Delta L = I\omega = r \times \Delta p$$

$I = \frac{1}{12} ML^2 =$ Moment of inertia about center of stick

$\Delta p = MV_{CM} =$ Just after the blow

$r = \frac{L}{2} =$ Moment arm

$$\frac{1}{12} ML^2 \omega = \frac{L}{2} MV_{CM}$$

$$\boxed{\omega = \frac{6V_{CM}}{L}}$$

(c)

Find the velocity of each end of the stick in terms of the velocity of the CM

$$V_{RE} = V_{CM} + \frac{L}{2}\omega$$

$$V_{RE} = V_{CM} + \frac{L}{2}(\frac{6V_{CM}}{L})$$

$$V_{RE} = V_{CM} + 3V_{CM}$$

$$V_{RE} = 4V_{CM}$$

$$V_{LE} = V_{CM} - \frac{L}{2}\omega$$

$$V_{LE} = V_{CM} - \frac{L}{2}(\frac{6V_{CM}}{L})$$

$$V_{LE} = V_{CM} - 3V_{CM}$$

$$V_{LE} = -2V_{CM}$$

(d)

Find the center of percussion (CP), which is where the rotational and translational motion cancel

$$V(x) = V_{CM} - x\omega = V_{CM} - x(\frac{6V_{CM}}{L}) = 0$$

$$x = \frac{L}{6}$$

The Breaking Broomstick Demo

(1) "Experiment to demonstrate Inertia"

(2) First published in 1881

(3) Dramatic-Why does the stick break so violently and leave the glass intact?

(4) Results somewhat counterintuitive

(5) Example of Newton's First Law

(6) Notion of Impulse

(7) Modeling

Physics Points

(1) Motion of each half behaves similarly to a single rod receiving A blow at one end. Observe end of rod. What happens?

(2) $v_{LE} = -2v_{CM}$

$v_{RE} = 4v_{CM}$

(3) Center of mass almost falls in a straight line under gravity.

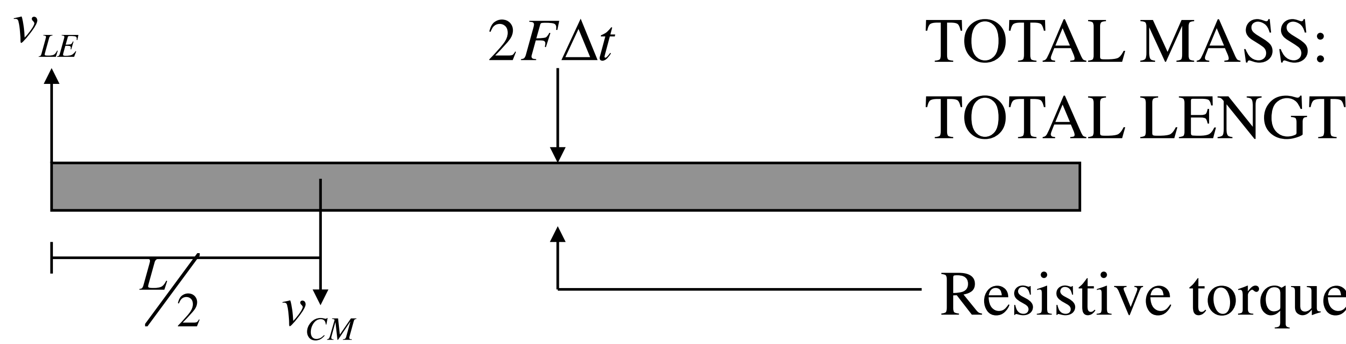
(4) Observe point on stick 2/3 from CM where the rotational Motion is canceled by the vertical motion.(Instantaneous axis of rotation)

(5) Slight sideways constant velocity due to blow

(6) Measure v_{LE}, v_{CM}, v_{RE} and compare to simple model

(7) Estimate Δt and deduce F.

Breaking Broomstick Demo



TOTAL MASS: $2m$

TOTAL LENGTH: $2L$

(1) $v_{CM} = \dfrac{F\Delta t}{m}$

(3) $v_{LE} = v_{CM} - \dfrac{L}{2}\omega$

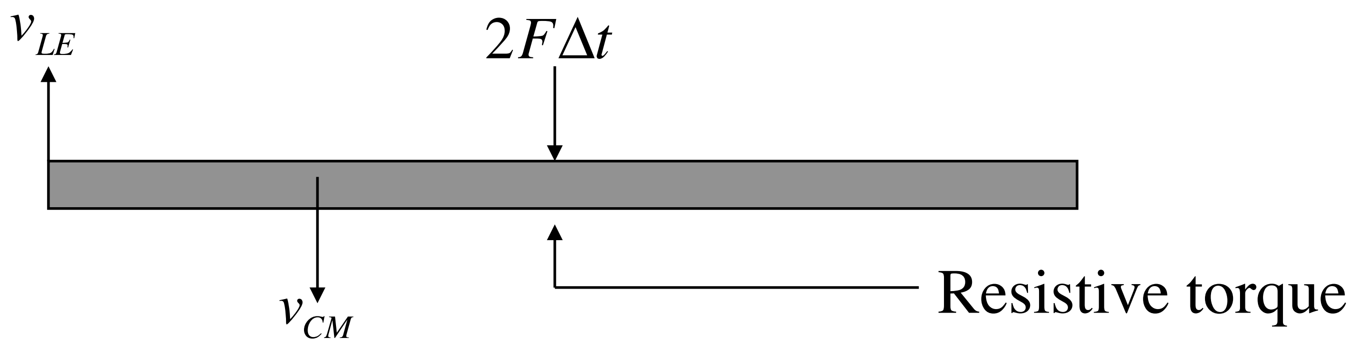
(2) $I\omega = \vec{r} \times \vec{p}$

$$L = \ I\omega = \frac{L}{2}(F\Delta t) - [\tau\Delta t]$$ (Moment taken about V_{CM}

$$I\omega = \frac{L}{2} m v_{CM} - [\tau\Delta t]$$

Angular momentum about L/2

Broomstick Breaking Cont.



LEFT SIDE

$$v_{LE} = v_{CM} - \frac{L}{2}\left(\frac{L}{2}\frac{mv_{CM}}{\frac{1}{12}mL^2}\right) - \left[-\frac{\tau\Delta t}{\frac{1}{12}mL^2}\right]$$

$$v_{LE} = v_{CM} - 3v_{CM} + \frac{6\tau\Delta t}{mL}$$

$$\boxed{v_{LE} = -2v_{CM}}$$

w/out resistive torque

$$\boxed{v_{LE} = -2v_{CM} + \frac{6\tau\Delta t}{mL}}$$

w/ resistive torque

RIGHT SIDE

$$v_{RE} = v_{CM} + \frac{L}{2}\omega$$

$$v_{RE} = v_{CM} + 3v_{CM}$$

$$\boxed{v_{RE} = 4v_{CM}}$$

w/out resistive torque

$$\boxed{v_{RE} = 4v_{CM} - \frac{6\tau\Delta t}{mL}}$$

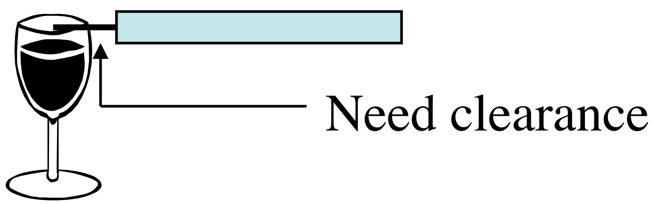
w/ resistive torque

Apparatus

-4 ft long, 7/8in diameter white pine, cedar, or hickory dowel rod or broomstick

-Stick pins in each end; cut off heads

-Support each pin with a wine glass, coke can, block of wood, etc.



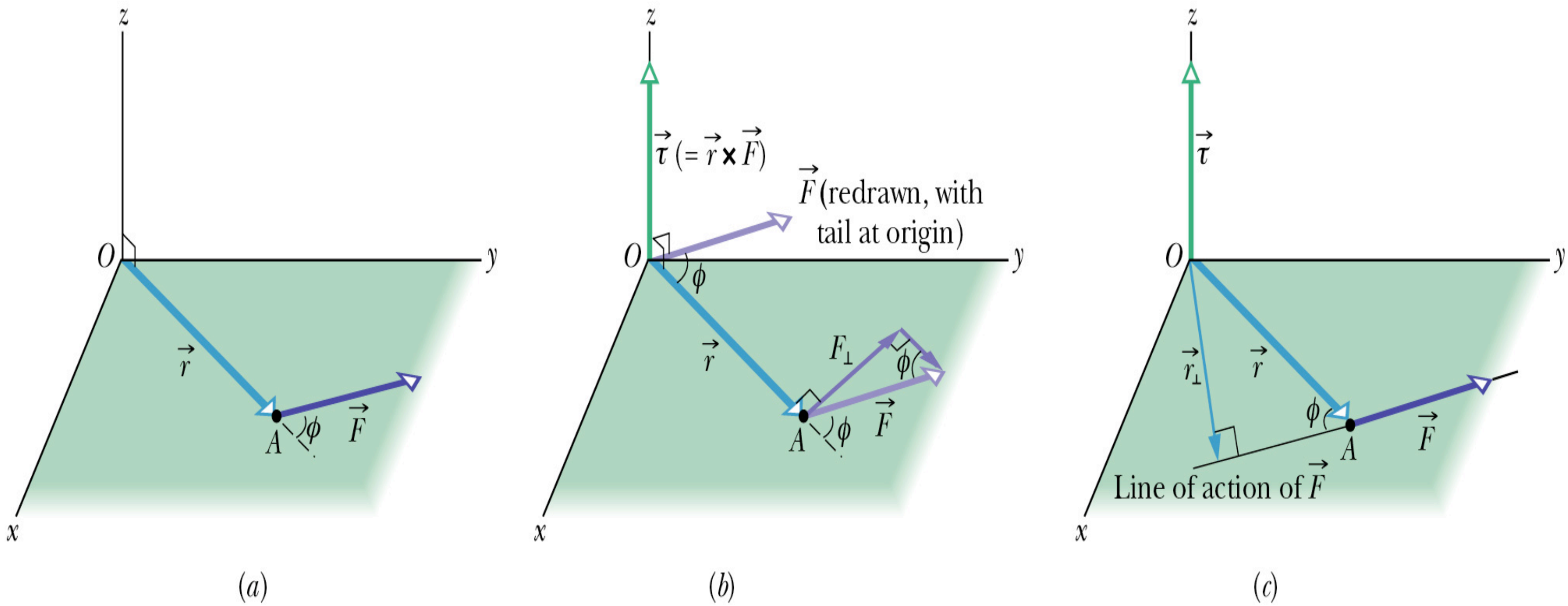
-Striking stick: Steel 1/2” in diameter and 2ft long

-Mark the halfway point of stick so you know where to strike it

-Use a hacksaw to etch it around the circumference; avoid stick fracturing due to other weakness.

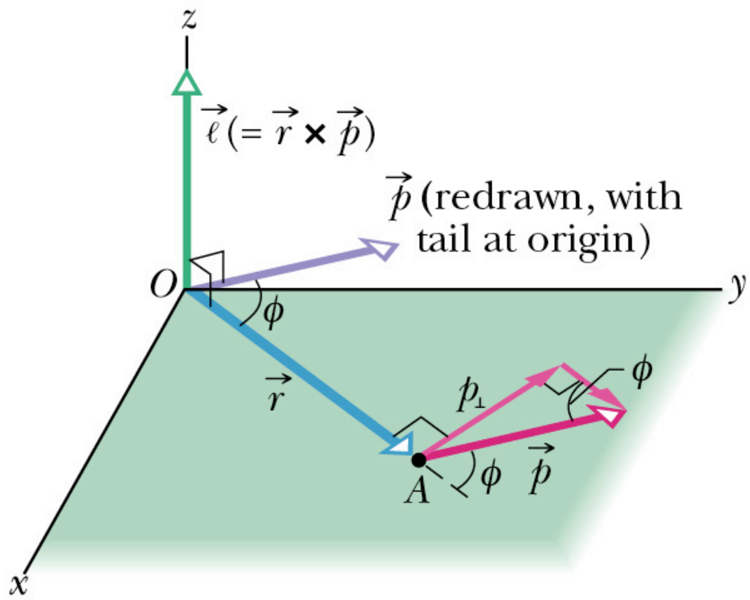
-Raise striking stick and hit the center as hard as you can; follow through

Torque: magnitude and direction

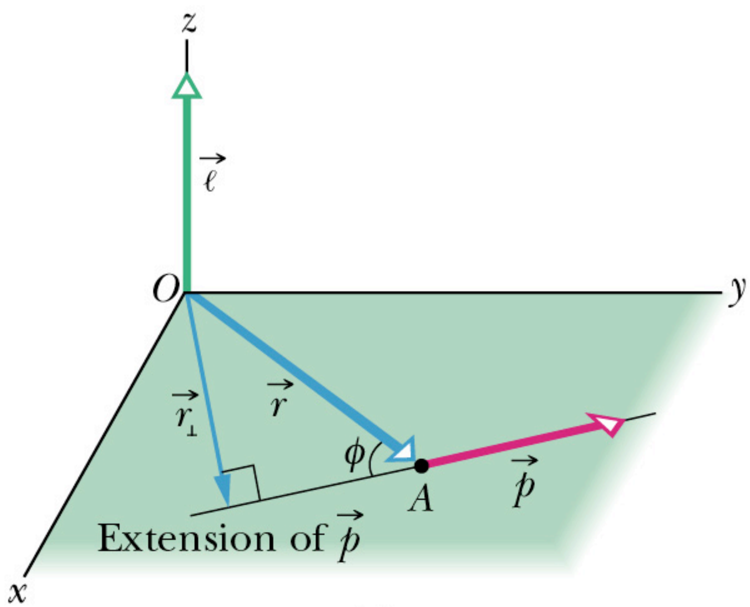


$$\vec{T} = \vec{r} \times \vec{F}$$

Angular momentum : magnitude and direction



(a)



(b)

$$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$$

Work-energy theorem for rotations

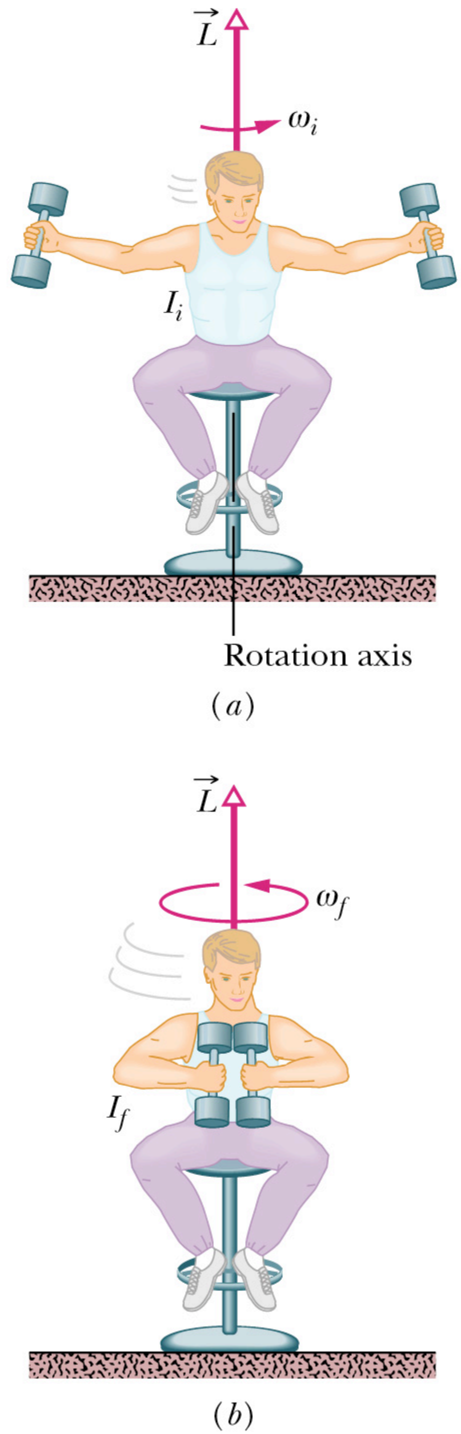
$$W = \frac{1}{2} I \omega_f^{\,2} - \frac{1}{2} I \omega_i^{\,2}$$

$$W = \tau(\theta_f - \theta_i)$$

$$P = \tau\omega$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

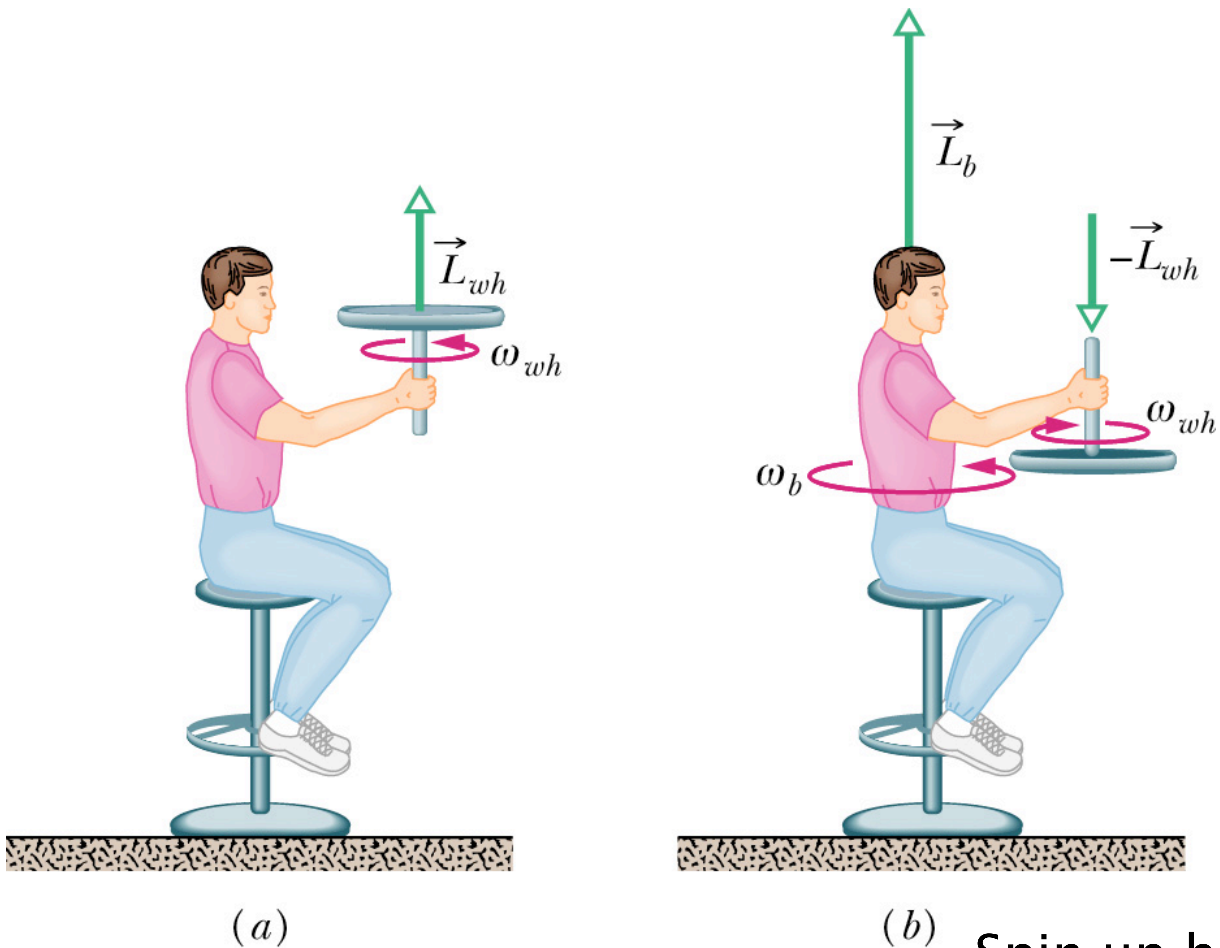
$$\vec{L} = \vec{r} \times \vec{p}$$

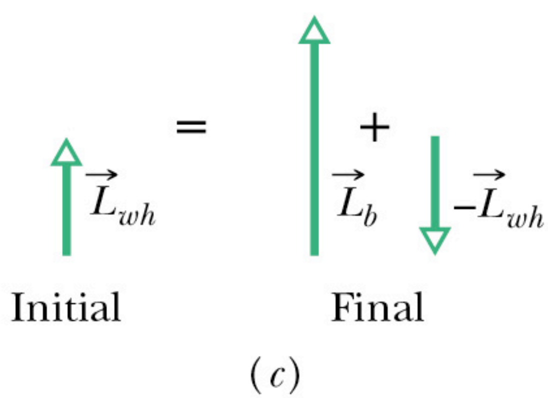


Conservation of Angular momentum

$$I\vec{\omega}_i = I\vec{\omega}_f$$

Sit on stool with dumbells held straight out 180 degrees apart. Slowly rotate the stool then bring arms in towards body. Angular velocity will increase drastically.





Spin up bicycle wheel on buffer and hold its axis vertical while sitting on the stool. Slowly rotate the wheel axis 180 degrees and the stool will rotate in accordance with the law of conservation of angular momentum.

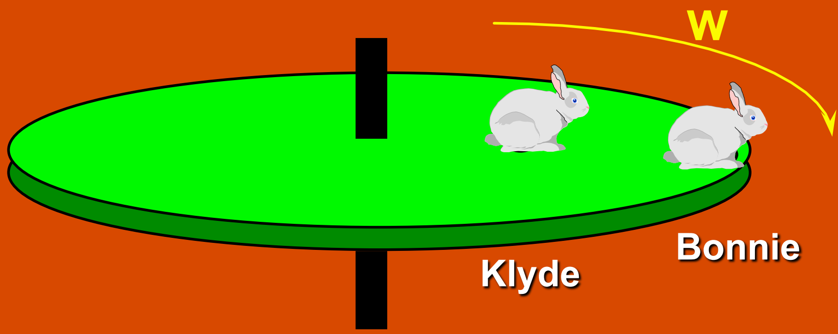
Precession of a Bicycle Wheel Gyroscope Under Gravitational Force

In each of the lecture halls there is a string which hangs from the ceiling and has a hook attached to its free end. Spin the bicycle wheel up with the buffer wheel and attach the axle of the wheel to the hook on the string. With the axis of the bicycle wheel held horizontal, it is released. The axis remains horizontal while the wheel precesses around a vertical axis.

ConcepTest 9.1b Bonnie and Klyde II

Bonnie sits on the outer rim of a merry-go-round, and Klyde sits midway between the center and the rim. The merry-go-round makes one revolution every two seconds. Who has the larger linear (tangential) velocity?

1) **Klyde**
2) **Bonnie**
3) **both the same**
4) **linear velocity is zero for both of them**



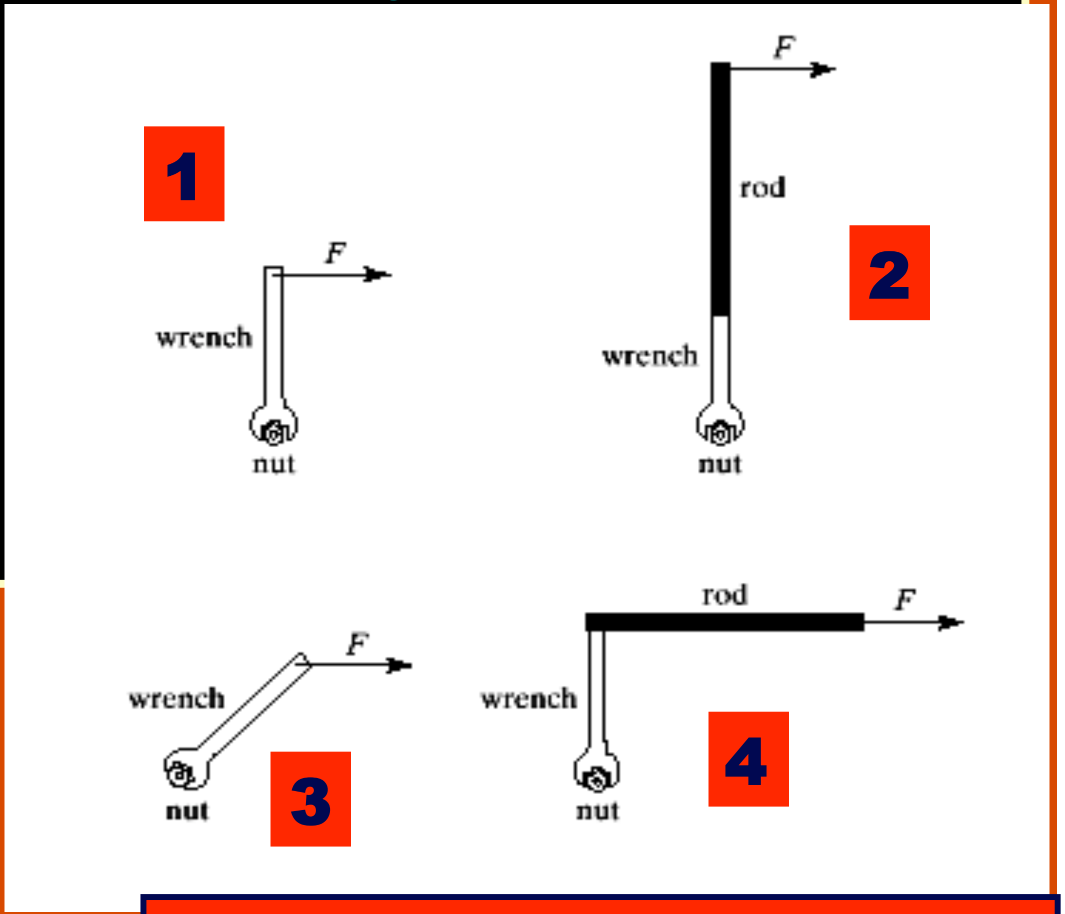
ConcepTest 9.3a Angular Displacement I

An object at rest begins to rotate with a constant angular acceleration. If this object rotates through an angle θ in the time *t*, through what angle did it rotate in the time 1/2 *t*?

1) 1/2 θ
2) 1/4 θ
3) 3/4 θ
4) 2 θ
5) 4 θ

ConcepTest 9.4 Using a Wrench

You are using a wrench to loosen a rusty nut. Which arrangement will be the most effective in loosening the nut?



5) all are equally effective

ConcepTest 9.9 Moment of Inertia

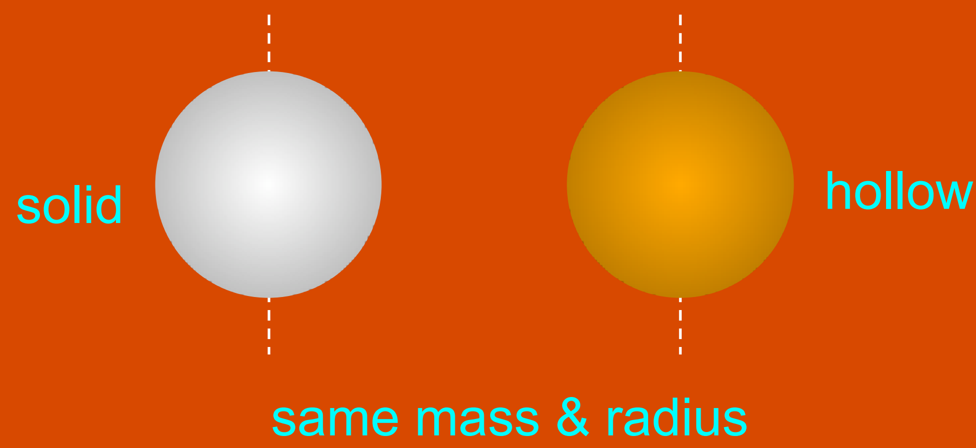
Two spheres have the same radius and equal masses. One is made of solid aluminum, and the other is made from a hollow shell of gold.

Which one has the bigger moment of inertia about an axis through its center?

a) solid aluminum

b) hollow gold

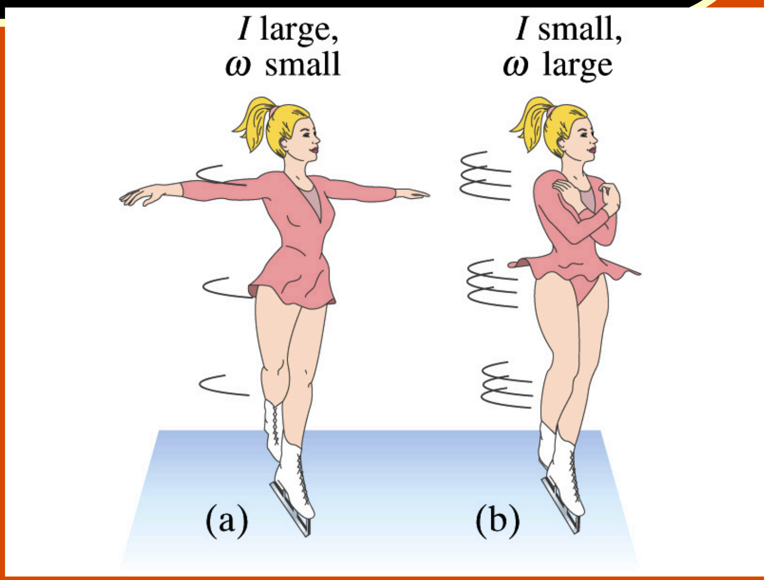
c) same



ConcepTest 9.10 Figure Skater

A figure skater spins with her arms extended. When she pulls in her arms, she reduces her rotational inertia and spins faster so that her angular momentum is conserved. Compared to her initial rotational kinetic energy, her rotational kinetic energy after she pulls in her arms must be

1) the same
2) larger because she's rotating faster
3) smaller because her rotational inertia is smaller



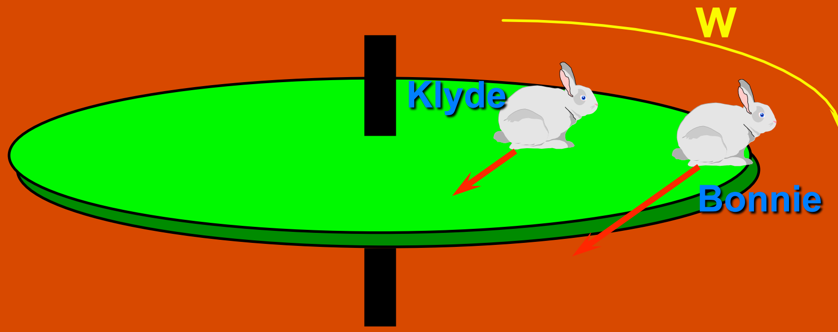
ConcepTest 9.1b Bonnie and Klyde II

Bonnie sits on the outer rim of a merry-go-round, and **Klyde** sits midway between the center and the rim. The merry-go-round makes one revolution every two seconds. **Who has the larger linear (tangential) velocity?**

1) Klyde
2) Bonnie
3) both the same
4) linear velocity is zero for both of them

Their **linear speeds v** will be different since $v = R\text{w}$ and **Bonnie is located further out** (larger radius R) than Klyde.

[redacted]



Follow-up: Who has the larger centripetal acceleration?

ConcepTest 9.3a Angular Displacement I

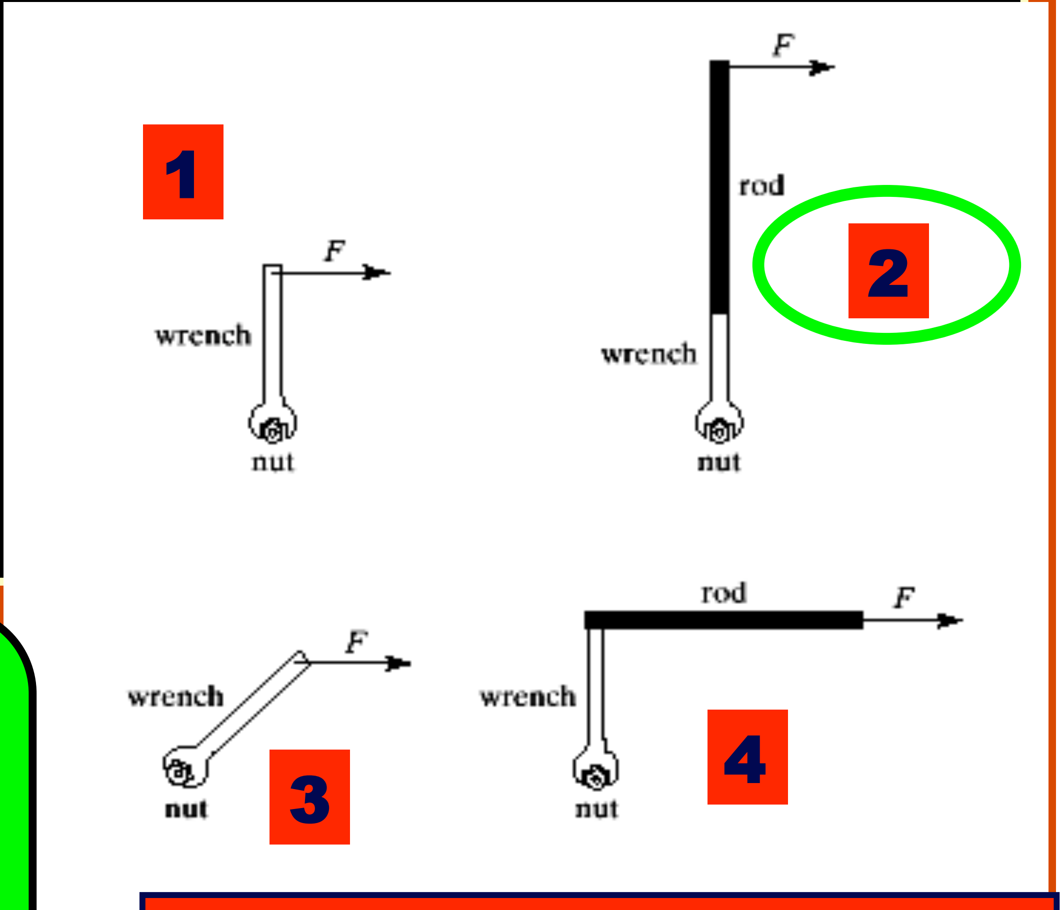
An object at rest begins to rotate with a constant angular acceleration. If this object rotates through an angle q in the time *t*, through what angle did it rotate in the time 1/2 *t*?

1) 1/2
2) 1/4
3) 3/4
4) 2
5) 4

The angular displacement is = 1/2 t^2 (starting from rest), and there is a quadratic dependence on time. Therefore, in **half the time**, the object has rotated through **one-quarter the angle**.

ConcepTest 9.4 Using a Wrench

You are using a wrench to loosen a rusty nut. Which arrangement will be the most effective in loosening the nut?



5) all are equally effective

Since the forces are all the same, the only difference is the lever arm. The arrangement with the largest lever arm (case #2) will provide the largest torque.

Follow-up: What is the difference between arrangement 1 and 4?

ConcepTest 9.9 Moment of Inertia

Two spheres have the same radius and equal masses. One is made of solid aluminum, and the other is made from a hollow shell of gold.

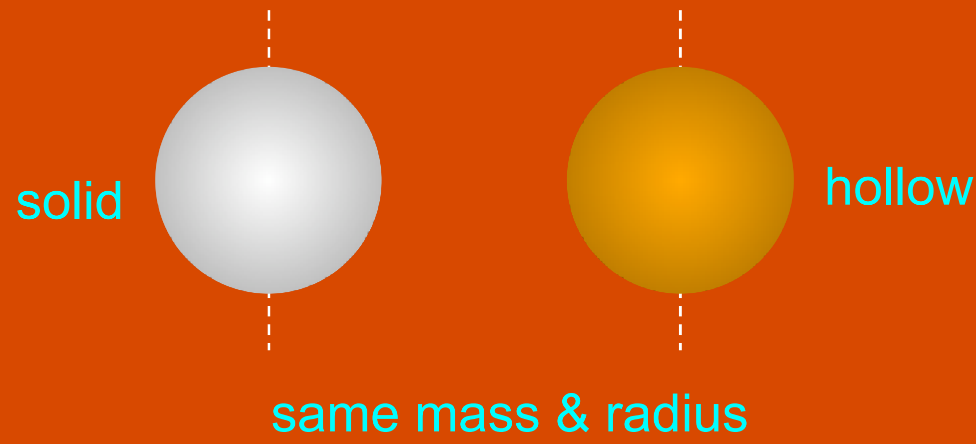
Which one has the bigger moment of inertia about an axis through its center?

a) solid aluminum

b) hollow gold

c) same

Moment of inertia depends on mass and distance from axis squared. It is bigger for the shell since its mass is located farther from the center.

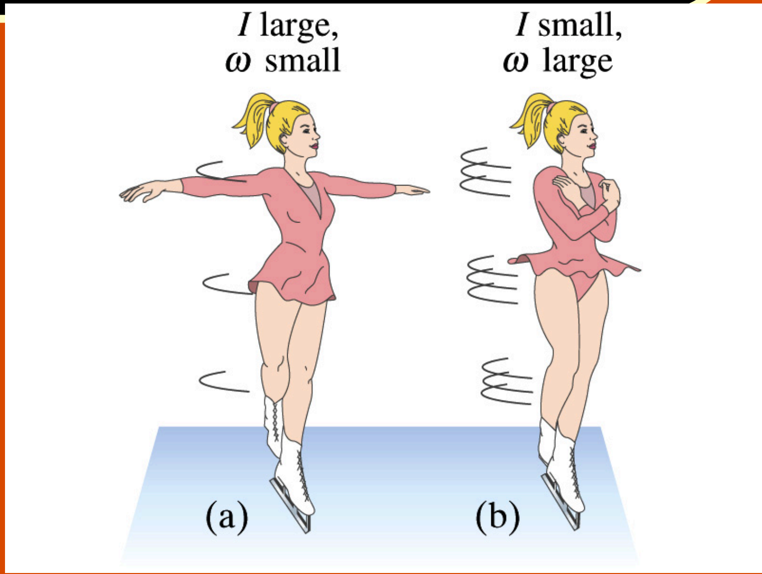


ConcepTest 9.10 Figure Skater

A figure skater spins with her arms extended. When she pulls in her arms, she reduces her rotational inertia and spins faster so that her angular momentum is conserved. Compared to her initial rotational kinetic energy, her rotational kinetic energy after she pulls in her arms must be

1) the same
2) larger because she's rotating faster
3) smaller because her rotational inertia is smaller

KE_{rot}=1/2 I ω^2 = 1/2 *L* ω (used *L*= *I*ω). Since *L* is conserved, larger ω means larger KE_{rot}. The "extra" energy comes from the work she does on her arms.



Follow-up: Where does the extra energy come from?