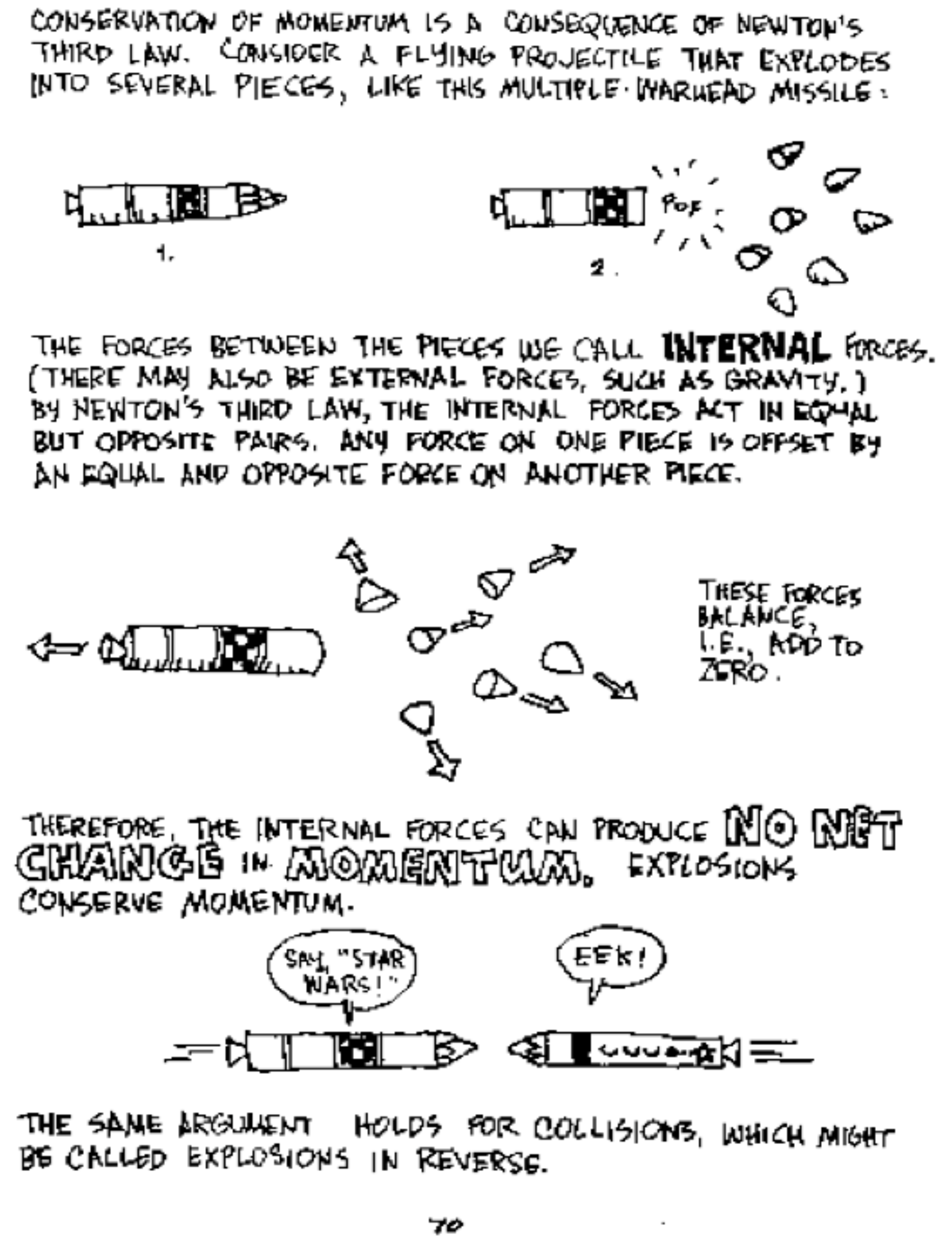
Lecture 6
Chapter 9
Systems of Particles

Warm-up problem

Puzzle Question:
Cite one reason why
basket ball players and dancers
have a greater hang time.

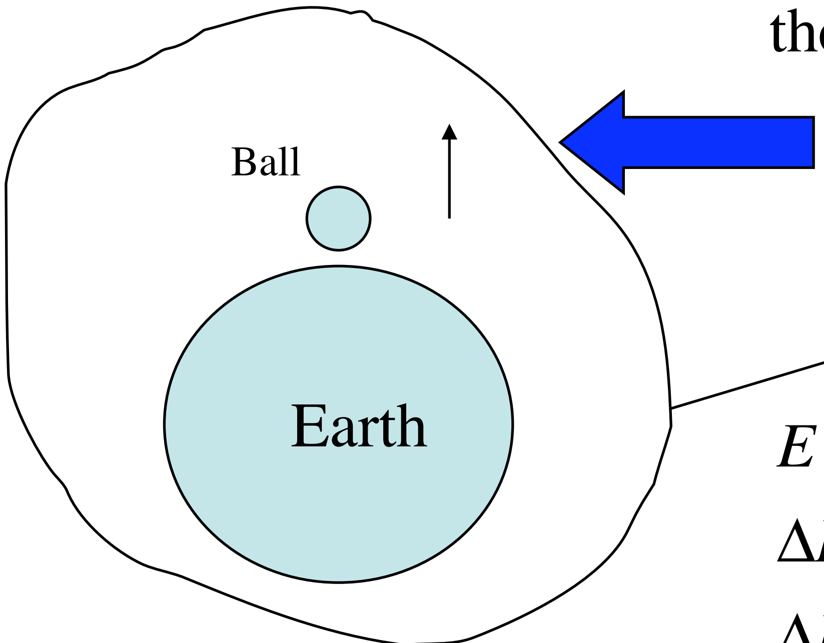


Turn on recorder

Linear Momentum form of Newtons second Law
Review 2D collision problem
Impulses
Machine gun force
Center of mass again examples
- virginia
- Walking along flat car
- Shooting a gun on ice

Hockey puck collisions
Billiard ball example
Rocket equation from flat car problem
- example

External Work or Force (No Friction)



W (eg. I could lift the ball, then I am an external force)

Ball-Earth system

$$E = K_B + K_E + U_{BE}$$

$$\Delta E = \Delta(K_B + K_E) + \Delta U_{BE}$$

$$\Delta E = \Delta K + \Delta U$$

If an external force does work, then the conservation of energy equation becomes

$$W = \Delta K + \Delta U$$

Example lifting a ball

External Work or Force and Friction Present

$$W = \Delta K + \Delta U + f_k d$$

$$\Delta E_{th} = f_k d$$

$$W = \Delta K + \Delta U + \Delta E_{th} + \Delta E_{\mathrm{int}}$$

Here we have included an internal energy change to account for Other processes than friction.

Show example of what I mean. Pushing a block across floor with friction or up an inclined plane.

System isolated from environment
Friction and other internal forces can be present

$$W = \Delta K + \Delta U + \Delta E_{th} + \Delta E_{\text{int}}$$

For a system isolated from the environment where no energy transfers can take place between the system and the environment, then W= 0. and we have

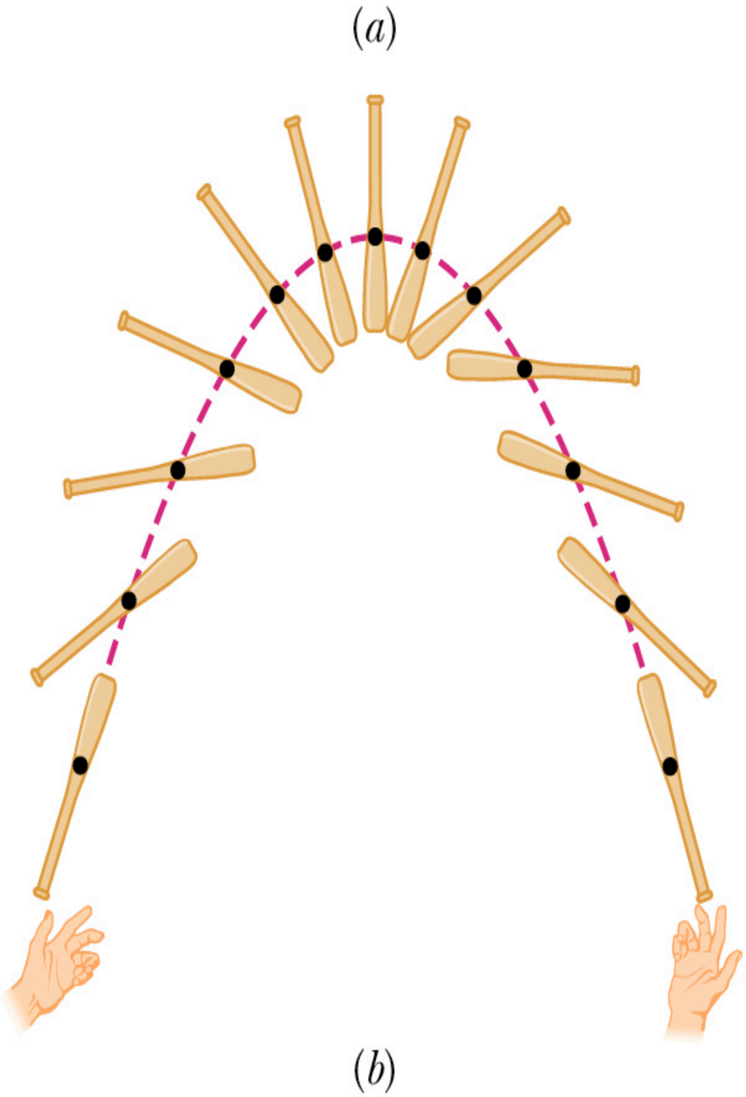
$$\Delta K + \Delta U + \Delta E_{th} + \Delta E_{\text{int}} = 0$$

Example: Block sliding down inclined plane with gravity present and friction without me touching it What about sound the block makes as it moves?

Topics

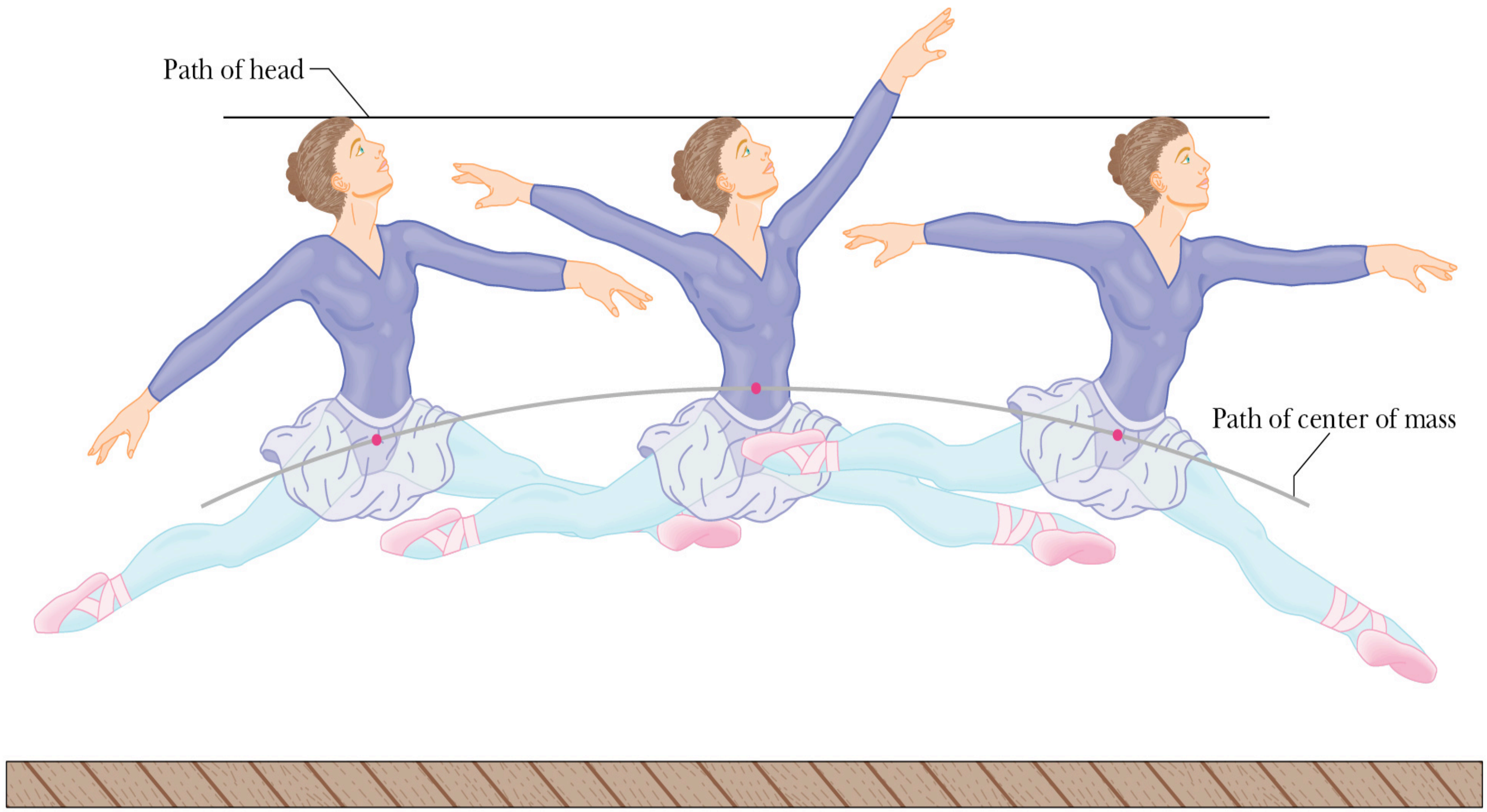
- Center of mass
- Linear momentum = P,
- Newton's 2nd law in terms of P
- Conservation of Linear Momentum
- Collisions and Collision time
- Conservation of momentum
- Conservation of kinetic energy leads to relationship among the variables.
  - One dimensional elastic and inelastic collisions (Air Track and basketball-super ball)
  - Two dimensional elastic collisions (Pool Table)

Center of mass (special point in a body)



- Why is it important? For any rigid body the motion of the body is given by the motion of the cm and the motion of the body around the cm.
- The motion of the cm is given by Newton's second Law, F=ma.
- Under gravity, the motion of the cm of the rigid body (bat) is parabolic like a point projectile

What happens to the ballet dancers head when she raises her arms at the peak of her jump? Note location of cm relative To her waist. Her waist is lowered at the peak of the jump.
How can that happen?



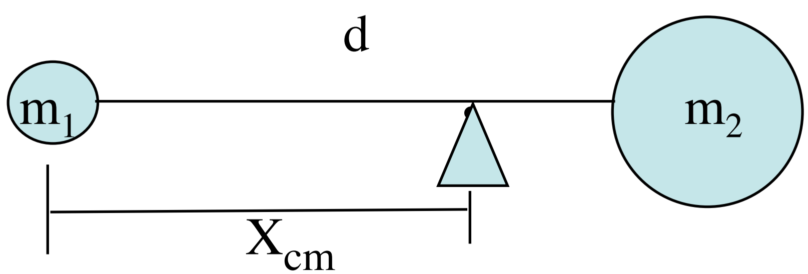
How do you find the center of mass of an arbitrary shape?

Show how you would find it for the state of Virginia.

How do you find it analytically?

Center of Mass

As an example find the center of mass of the following system analytically. Note that equilibrium is achieved when the distance between the mass and the fulcrum (called lever arm) times the weight on the left equals the similar quantity on the right. This is also called torque.



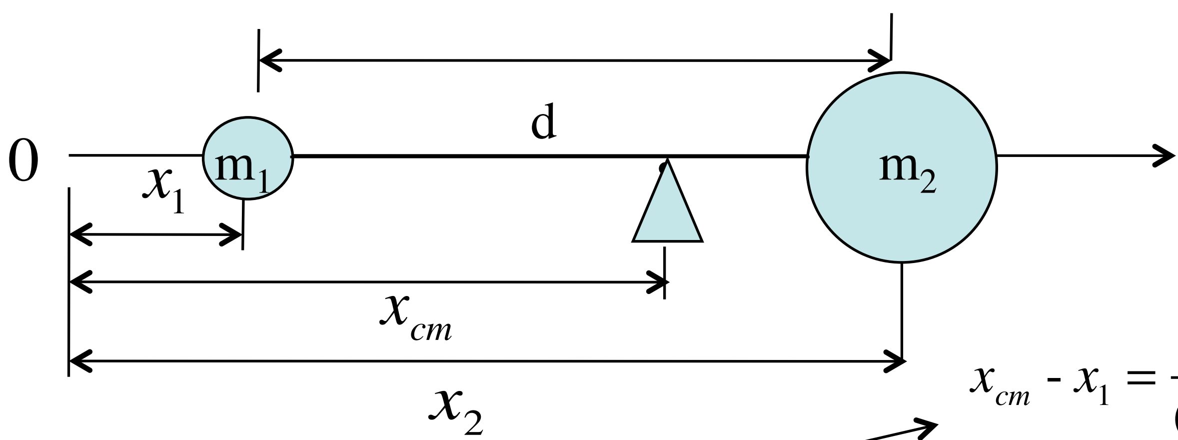
$$W_1 X_{cm} = W_2(d - X_{cm})$$ Equilibrium Condition

$$W_1 X_{cm} + W_2 X_{cm} = W_2 d$$

$$X_{cm} = \frac{W_2}{(W_1 + W_2)} d \qquad W_1 = m_1 g \qquad X_{cm} = \frac{m_2}{(m_1 + m_2)} d$$

Center of Mass

Change origin of coordinate system to some arbitrary location along x axis.



$$X_{cm} = \frac{m_2}{(m_1 + m_2)} d$$

$$x_{cm} - x_1 = \frac{m_2}{(m_1 + m_2)}(x_2 - x_1)$$

$$x_{cm}(m_1 + m_2) = m_2(x_2 - x_1) + x_1(m_1 + m_2)$$

$$x_{cm} M = x_1 m_1 + m_2 x_2$$

$$x_{cm} = \frac{1}{M}(x_1 m_1 + m_2 x_2)$$

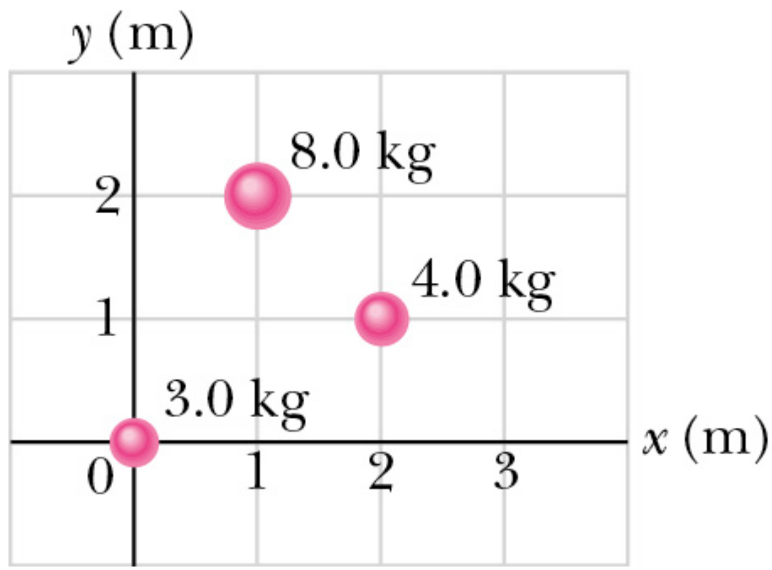
For a system of 3 particles along x axis

$$x_{cm} = \frac{(m_1x_1 + m_2x_2 + m_3x_3)}{M}$$

$$y_{cm} = \frac{(m_1y_1 + m_2y_2 + m_3y_3)}{M}$$

$$z_{cm} = \frac{(m_1z_1 + m_2z_2 + m_3z_3)}{M}$$

Problem 9.3



2 dimensions

Find x_{cm} and y_{cm}

$$x_{cm}M = m_1x_1 + m_2x_2 + m_3x_3$$

$$x_{cm}15 = (3*0)+(4*2)+(8*1)$$

$$x_{cm} = \frac{16}{15} = 1.1m$$

$$y_{cm}M = m_1y_1 + m_2y_2 + m_3y_3$$

$$y_{cm}15 = (3*0)+(4*1)+(8*2)$$

$$x_{cm} = \frac{20}{15} = 1.33m$$

Center of Mass for a system of n particles

$$x_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i$$

$$y_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i$$

$$z_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i$$

$$\vec{r}_{cm} = x_{cm}\hat{i} + y_{cm}\hat{j} + z_{cm}\hat{k}$$

$$\vec{r}_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i$$

Continuous Body (sum goes to integral)

$$x_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i$$

$$x_{cm} = \frac{1}{M}\int x dm$$

$$\rho = \frac{dm}{dV} = \frac{M}{V}$$

$$x_{cm} = \frac{1}{V}\int x dV$$

Show that the net force now acts on the center of mass

$\mathbf{F}_{net} = \mathbf{M}\mathbf{a}_{cm}$

Start here

$$\vec{r}_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i$$

take d/dt on both sides

$$\vec{v}_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{v}_i$$

take d/dt again

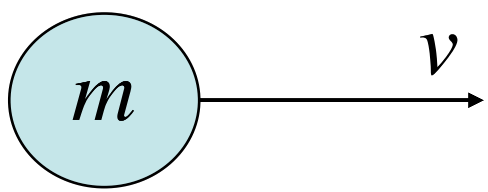
$$\vec{a}_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{a}_i$$

Identify ma as the force on each particle

$$\vec{a}_{cm} = \frac{1}{M}\sum_{i=1}^{n} \vec{F}_i \qquad \vec{a}_{cm} = \frac{1}{M}\vec{F}_{net} \qquad \vec{F}_{net} = M\vec{a}_{cm}$$

Momentum

What is momentum and why is it important? Momentum **p** is the product of mass and velocity for a particle or system of particles. The product of m and **v** is conserved in collisions and that is why it is important. It is also a vector which means each component of momentum is conserved. It has units of kg m/s or N-s.



$$\vec{p} = m\vec{v}$$

Large mass x low velocity can have high momentum because mass is so large

Prove total momentum is conserved stating with Newton's third law

Linear Momentum form of Newton's 2nd Law

$$\vec{\mathrm{F}}=\mathrm{m}\vec{\mathrm{a}} \;\; = md\vec{v}\,/\,dt = d(m\vec{v})\,/\,dt = d\vec{P}\,/\,dt$$

$$\vec{F} = d\vec{P}\,/\,dt$$

Force Law now generalized to include change in mass

$$\vec{\mathrm{F}}=d(m\vec{v})\,/\,dt = md\vec{v}\,/\,dt + \vec{v}dm\,/\,dt$$

Remember that F and v are both vectors

$$\vec{\mathrm{F}}_x=d(m\vec{v}_x)\,/\,dt = md\vec{v}_x\,/\,dt + \vec{v}_x dm\,/\,dt$$

$$\vec{\mathrm{F}}_y=d(m\vec{v}_y)\,/\,dt = md\vec{v}_y\,/\,dt + \vec{v}_y dm\,/\,dt$$

$$\vec{\mathrm{F}}_z=d(m\vec{v}_z)\,/\,dt = md\vec{v}_z\,/\,dt + \vec{v}_z dm\,/\,dt$$

$$\vec{F} = \hat{i}F_x + \hat{j}F_y + \hat{k}F_z$$

Linear Momentum form of Newton's 2nd Law for a system of particles

Important because it is a vector quantity that is conserved in interactions.

$$\vec{v}_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{v}_i$$

$\vec{p}_i = m_i \vec{v}_i$ is the definition of momentum of i'th particle

$\vec{P}_{cm} = M\vec{v}_{cm}$ is the momentum of the cm

$$\vec{P}_{cm} = \sum_{i=1}^{n} \vec{p}_i$$

Now take derivative d/dt of $\vec{P}_{cm} = M\vec{v}_{cm}$

$$\frac{d\vec{P}_{cm}}{dt} = M\frac{d\vec{v}_{cm}}{dt} = \mathrm{M}\vec{\mathrm{a}}_{\mathrm{cm}}$$

$$\vec{F}_{net} = \frac{d\vec{P}_{cm}}{dt}$$

Law of Conservation of Linear Momentum

If $\mathbf{F}_{net} = 0$ on a closed system where no mass enters or leaves the system, then $d\mathbf{P}/dt = 0$ or $\mathbf{P}$ = constant. Box of gas molecules.

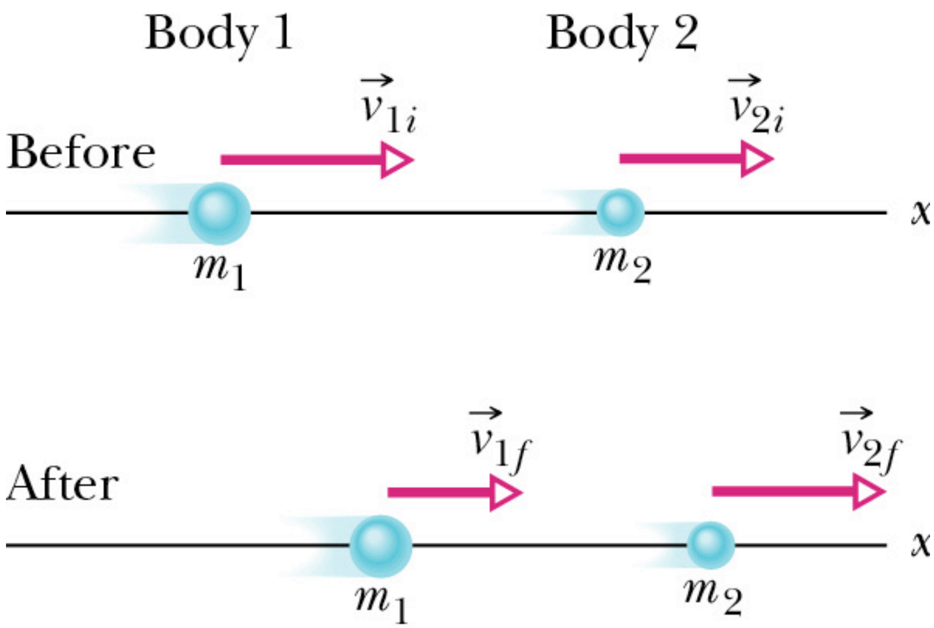
$\mathbf{P}_i = \mathbf{P}_f$ for a closed isolated system

Also each component of the momentum P_x, P_y, P_z is also constant since F_x, F_y, and F_z all = 0. Gives three equations.

P_x = constant
P_y = constant
P_z = constant

If one component of the net force is not 0, then that component of momentum is not a constant. For example, consider the motion of a horizontally fired projectile. The y component of P changes while the horizontal component is fixed after the bullet is fired.

One Dimension Elastic Collision



$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

Total momentum before = Total momentum after

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$$

$v_{2i} = 0$

Kinetic energy is conserved too.

$$\frac{1}{2} m_1 {v_{1i}}^2 = \frac{1}{2} m_1 {v_{1f}}^2 + \frac{1}{2} m_2 {v_{2f}}^2$$

Show how air track demonstrates the upper right hand results

Air Track (different cases)

Case			
1	• $m_2 = m_1$	$v_{1f} = 0$	$v_{2f} = v_{1i}$
2	• $m_2 > m_1$	$v_{1f} = -v_{1i}$	$v_{2f} = 0$
3	• $m_2 < m_1$	$v_{1f} = v_{1i}$	$v_{2f} = 2v_{1i}$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

$v_{2i} = 0$

Example of Collisions on Air track

Two carts of equal mass

one stopped one moving - demo momentum conservation

colliding head on

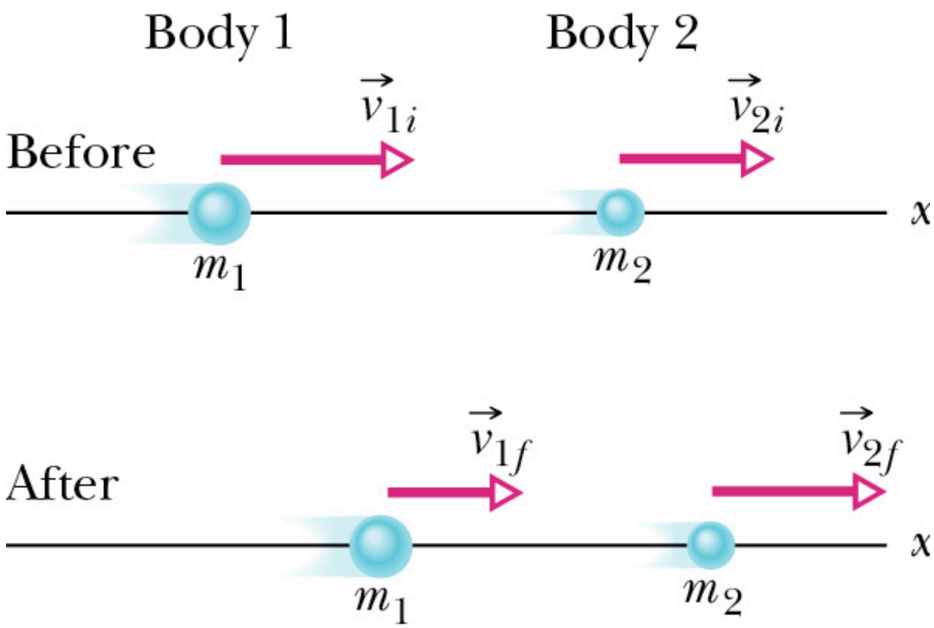
Two carts one large mass - one small mass

large mass moving small mass stopped.

small mass moving - large mass stopped

Two carts connected by a spring. Set them into oscillation by pulling them apart and releasing them from rest. Note cm does not move.

One Dimension Elastic Collision



$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

Total momentum before = Total momentum after

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$$

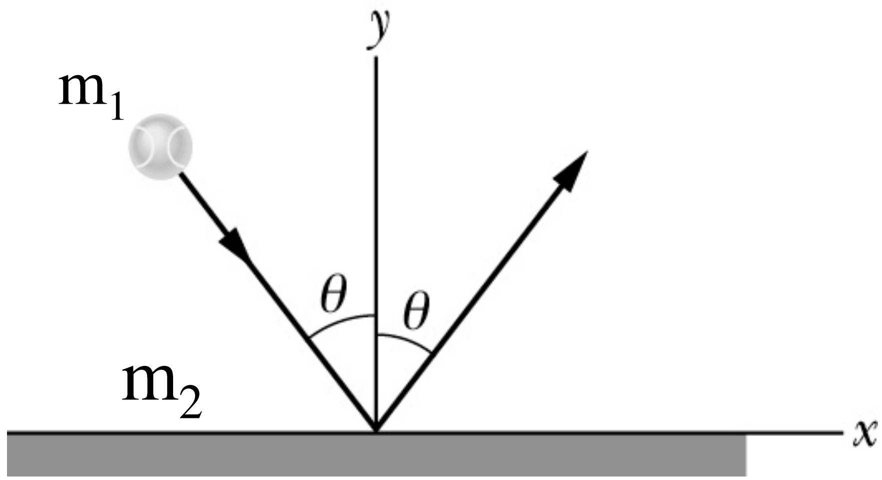
$v_{2i} = 0$

Kinetic energy is conserved too.

$$\frac{1}{2} m_1 {v_{1i}}^2 = \frac{1}{2} m_1 {v_{1f}}^2 + \frac{1}{2} m_2 {v_{2f}}^2$$

Show how air track demonstrates the upper right hand results

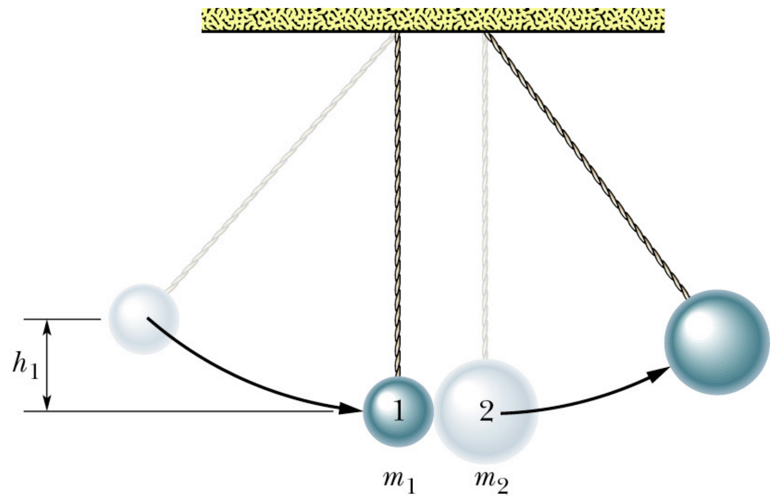
Balls bouncing off massive floors,
we have $m_2 >> m_1$



$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \cong \frac{-m_2}{m_2} v_{1i} \cong -v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} \cong \frac{2m_1}{m_2} v_{1i} \cong 0$$

Colliding pool balls
The executive toy
$m_2 = m_1$

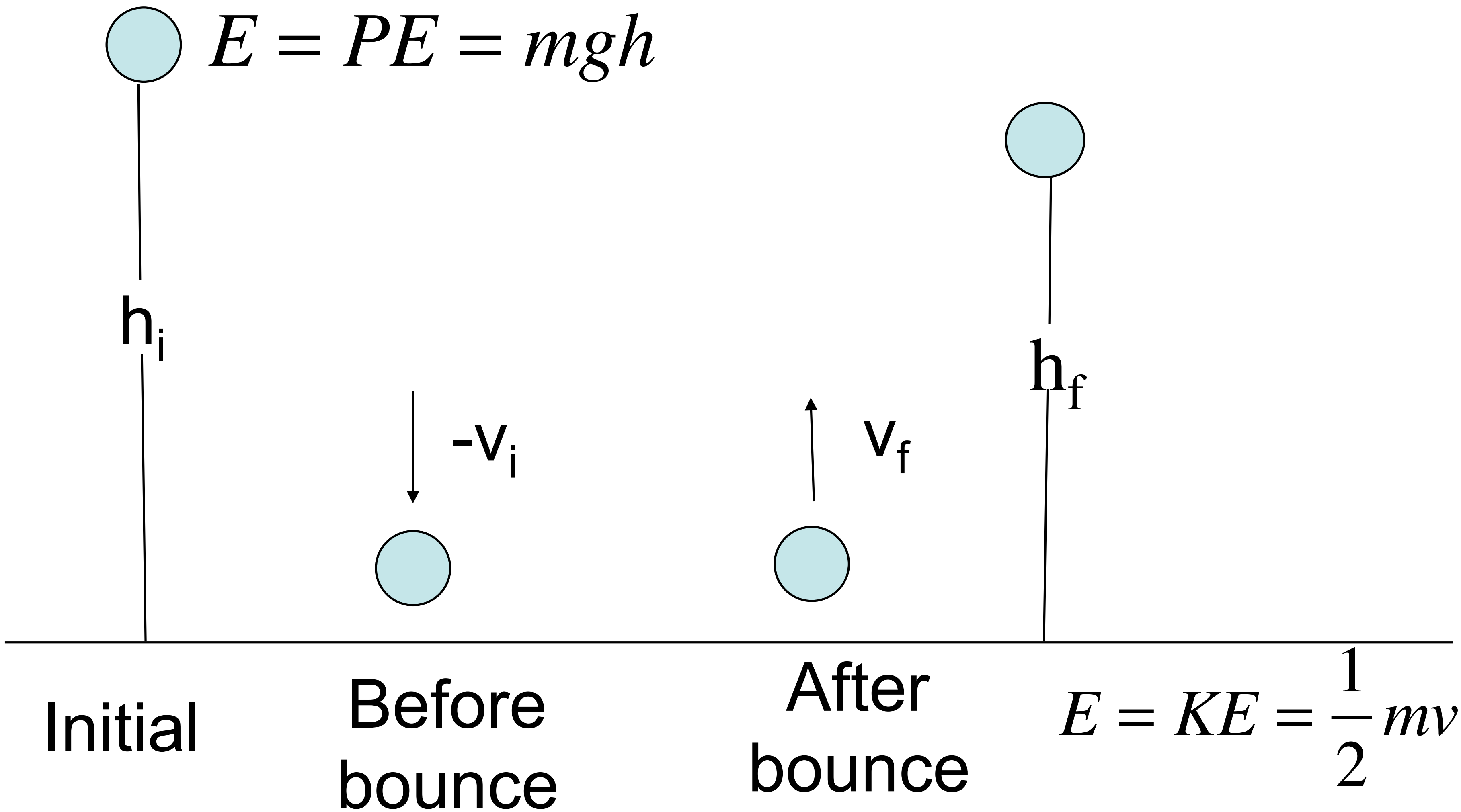


$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \cong 0 \qquad v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} \cong v_{1i}$$

Why don’t both balls go to the right each sharing the momentum and energy?

Almost elastic collision between wall and bouncing ball (Always lose a little energy)

$$E = PE = mgh$$



$$E = KE = \frac{1}{2}mv^2$$

Measuring velocities and heights of balls bouncing from a infinitely massive hard floor

Type of Ball	Coefficient of Restitution (C.O.R.)	Rebound Energy/ Collision Energy (R.E./C.E.)
Superball	0.90	0.81
Racquet ball	0.85	0.72
Golf ball	0.82	0.67
Tennis ball	0.75	0.56
Steel ball bearing	0.65	0.42
Baseball	0.55	0.30
Foam rubber ball	0.30	0.09
Unhappy ball	0.10	0.01
Beanbag	0.05	0.002

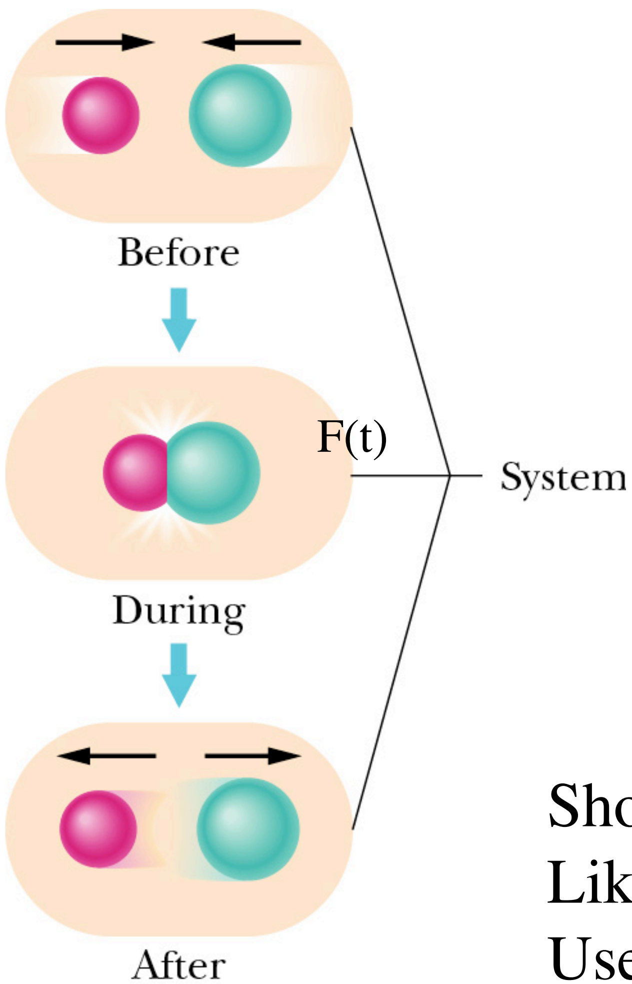
Almost elastic collision

$$C.O.R = \frac{v_f}{v_i}$$

$$\frac{R.E.}{C.E.} = \frac{H_i}{H_f}$$

Almost inelastic collision

What is a collision? A bullet striking a target.. Two balls colliding on a pool table. A billiard ball striking the cue stick. What about the air track?



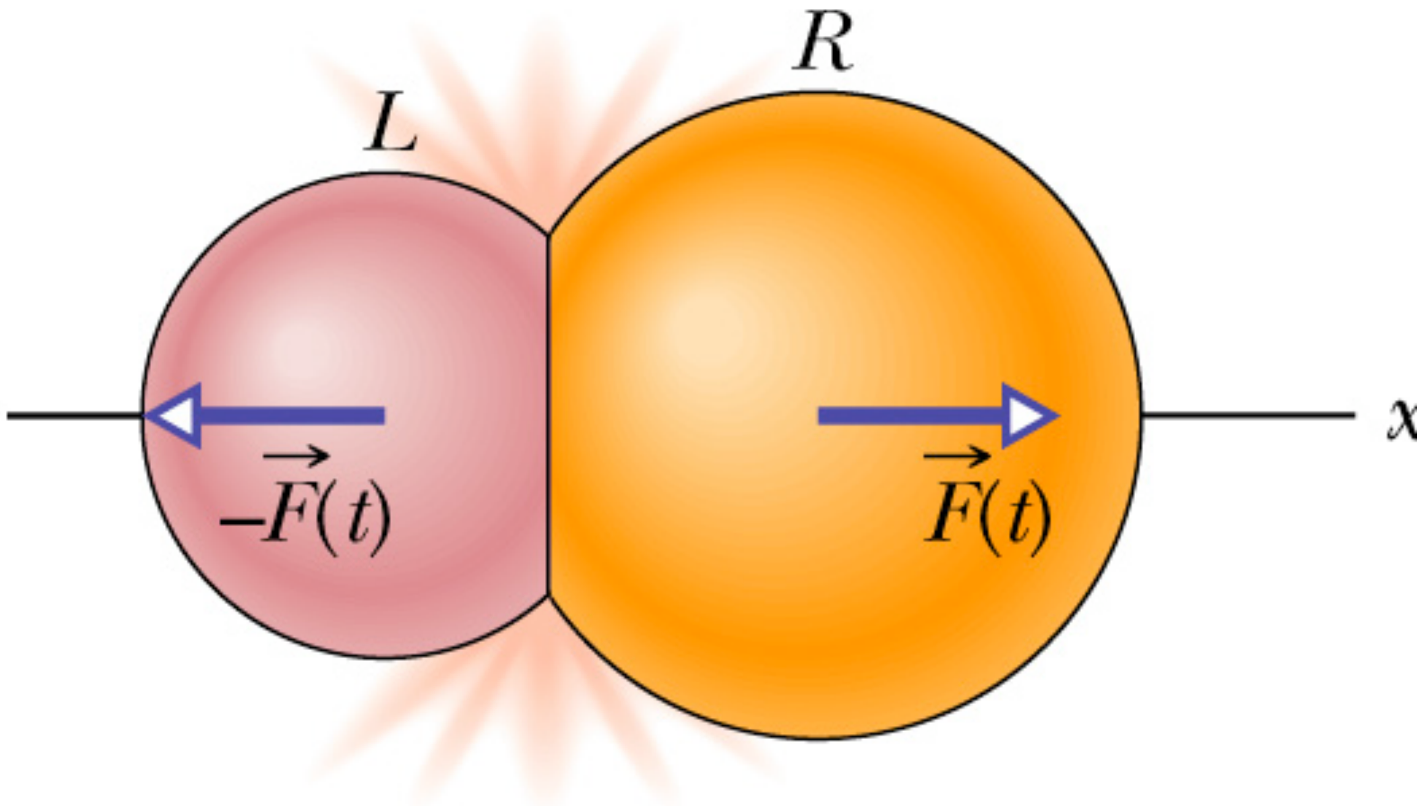
What happens during a collision on a short time scale?

Consider one object the projectile and the other the target.

Show 2 tennis balls colliding
Like two springs colliding on air track
Use as model.

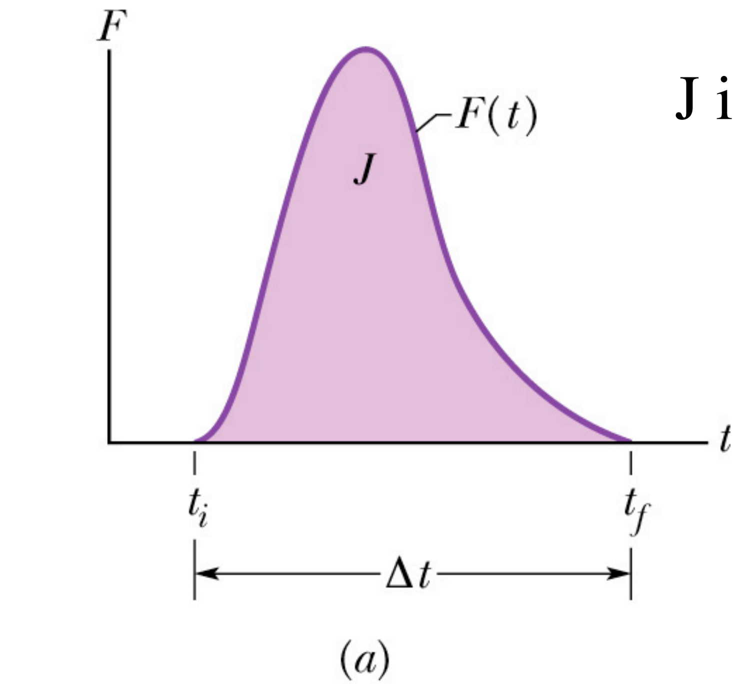
Shape of two objects while colliding with each other head-on.

Again two tennis like balls



Obeys Newtons third Law

$$J_L = -J_R$$

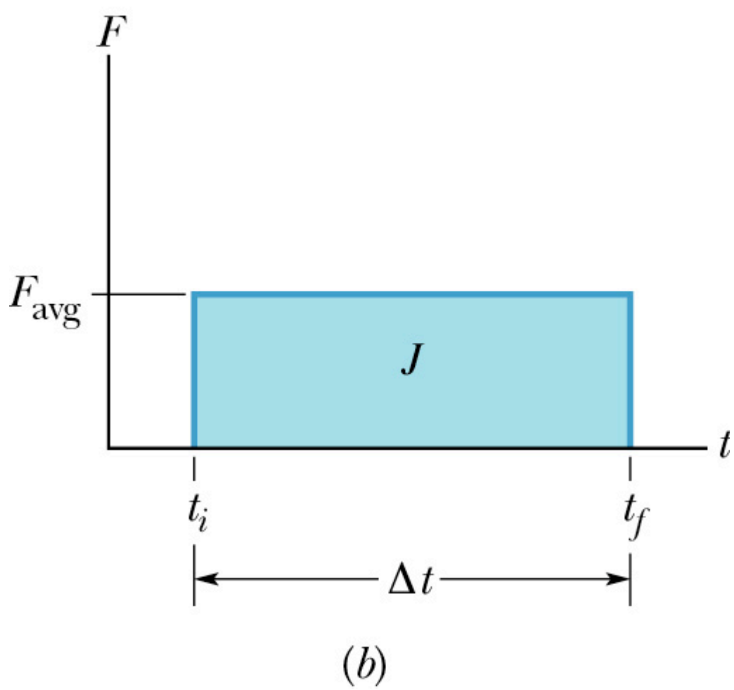


J is called the impulse = change in momentum

$$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)\,dt = \int_{t_i}^{t_f} \frac{d\vec{p}}{dt}\,dt = \int_{p_i}^{p_f} d\vec{p} = \vec{p}_f - \vec{p}_i$$

Change in momentum of the ball is to the left or right.

J is a vector



Also you can "rectangularize" the graph

$$J = F_{avg}\Delta t$$

Area is the same

Andy Rodick has been clocked at serving a tennis ball up to 149 mph(70 m/s). The time that the ball is in contact with the racquet is about 4 ms. The mass of a tennis ball is about 300 grams.

What is the average force exerted by the racquet on the ball?

$$F_{avg} = \frac{J}{\Delta t} = \frac{\Delta p}{\Delta t}$$

$$\Delta p = p_f - p_i = (70)(0.3) - 0 = 21 \text{ m/s kg}$$

$$F_{avg} = \frac{21 \text{ m/s kg}}{0.0004 \text{ s}} = 5250N$$

1lb = 4.45N

F_{avg}=5250/4.45=1180 lbs

What is the acceleration of the ball?

What distance does the racquet go through while the ball is still in contact?

So the average force exerted by the racquet on the ball is $F_{avg} = 5250N$

What is the acceleration of the ball?

$$a = \frac{F_{avg}}{m} = \frac{5250N}{0.3kg} = 17500m/s^2$$

What distance the racquet go through while the ball is still in contact?

$$v_f^{\ 2} - v_i^{\ 2} = 2ax$$

$$x = \frac{v_f^{\ 2}}{2a} = \frac{70^2}{2(17500)} = 0.14\ m$$

Machine Gun Force

$$F_{avg} = \frac{J}{\Delta t}$$

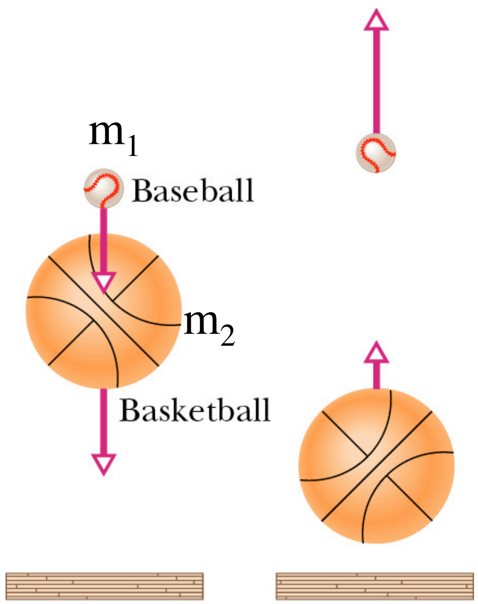
and J= -nΔp where n is number of bullets in time Δt

$$F_{avg} = -n\frac{\Delta \mathrm{p}}{\Delta t} = -nm\frac{\Delta v}{\Delta t} = -\frac{nm}{\Delta t}\Delta v = -\frac{\Delta m}{\Delta t}\Delta v$$

Example

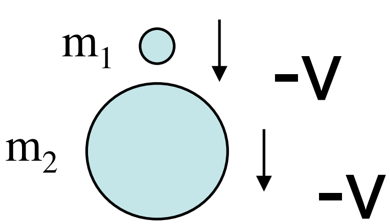
Two moving colliding objects

Show demo first

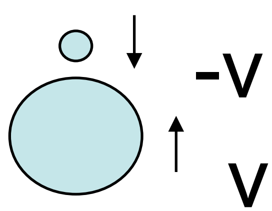


(a) Before (b) After

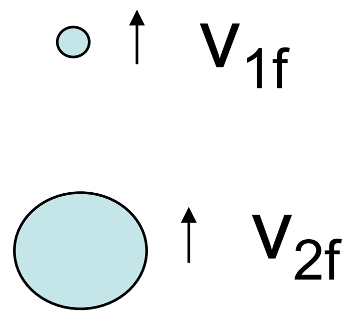
Just before each hits the floor



Just after the big ball bounced

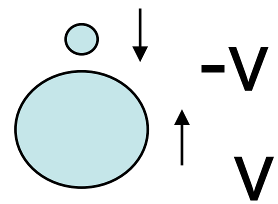


Just after the little ball bounced off the big ball



How high does superball go compared to dropping it off the floor? $h = \frac{V_{1f}^{2}}{2g}$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$



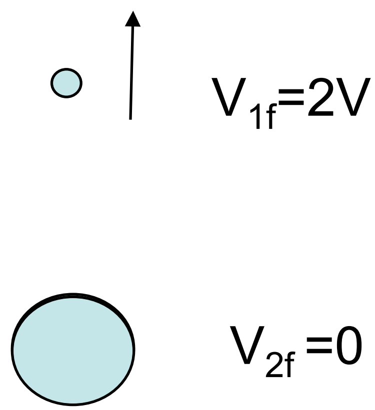
$$v_{1i} = -V$$

$$v_{2i} = V$$

$$v_{1f} = \frac{3m_2 - m_1}{m_1 + m_2}(V)$$

$$v_{2f} = \frac{m_2 - 3m_1}{m_1 + m_2}(V)$$

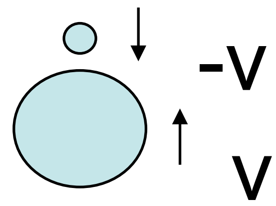
For
$m_2 = 3m_1$
$v_{1f} = 8/4V = 2V$
superball has twice as much speed.



4 times higher

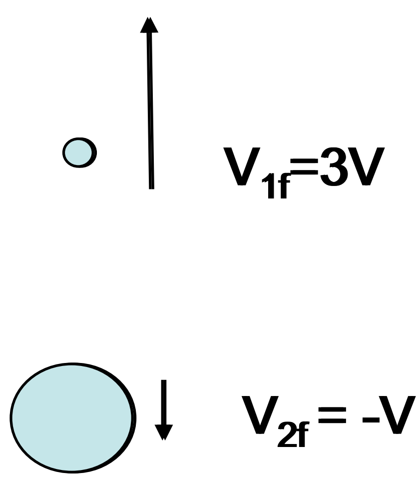
$$h = \frac{V_{1f}^{\ 2}}{2g} = \frac{(2V)^2}{2g}$$

$$v_{1f} = \frac{3m_2 - m_1}{m_1 + m_2}(V)$$



For maximum height consider $m_2 >> m_1$

How high does it go? $$h = \frac{(3V)^2}{2g}$$



9 times higher

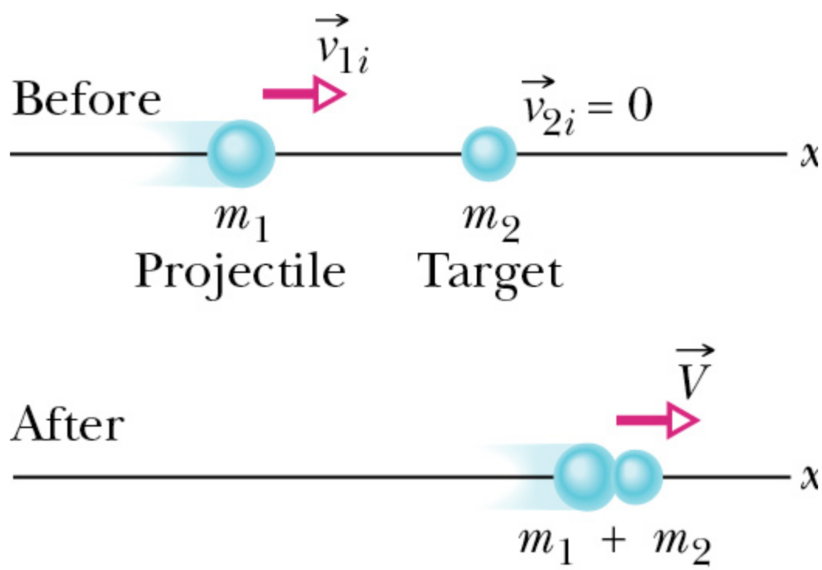
Types of Collisions

Elastic Collisions: Kinetic energy and momentum are conserved

Inelastic Collision: Only P is conserved. Kinetic energy is not conserved

Completely inelastic collision. Masses stick together

Illustrate with air track



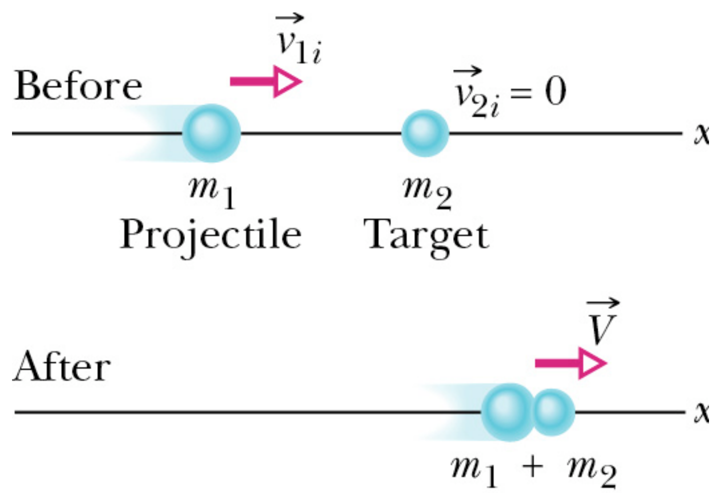
Completely Inelastic Collision (Kinetic energy is not conserved, but momentum is conserved)

Demo catching ball again

Conservation of Momentum

$$m_1 v_{1i} + 0 = (m_1 + m_2)V$$

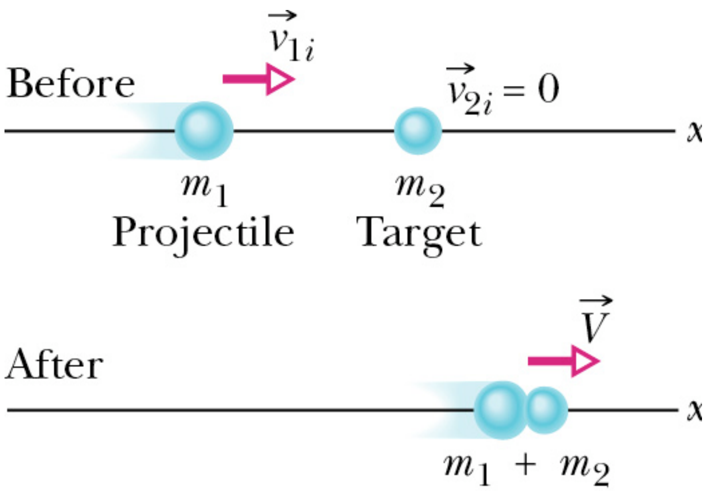
$$V = \frac{m_1 v_{1i}}{(m_1 + m_2)}$$



Now look at kinetic energy

$$V = \frac{m_1 v_{1i}}{(m_1 + m_2)}$$

Now look at kinetic energy



Before $\quad K_i = \frac{1}{2} m_1 v_{1i}^2$

After $\quad K_f = \frac{1}{2}(m_1 + m_2) V^2$

$$K_f = \tfrac{1}{2}(m_1 + m_2)\left(\frac{m_1 v_{1i}}{(m_1 + m_2)}\right)^2$$

$$K_f = \tfrac{1}{2}\frac{m_1}{m_1 + m_2} m_1 v_{1i}^2$$

Note K_i not equal to K_f

Completely Inelastic

$$K_f = \tfrac{1}{2}\frac{m_1}{m_1 + m_2} m_1 v_{1i}^2 \qquad \text{not equal to} \qquad K_i = \tfrac{1}{2} m_1 v_{1i}^2$$

For equal masses $K_f = 1/2\ K_i$, we lost 50% of K_i

Where did it go? It went into energy of binding the objects together, such as internal energy, rearrangement of the atoms, thermal, deformative, sound, etc.

Velocity of cm

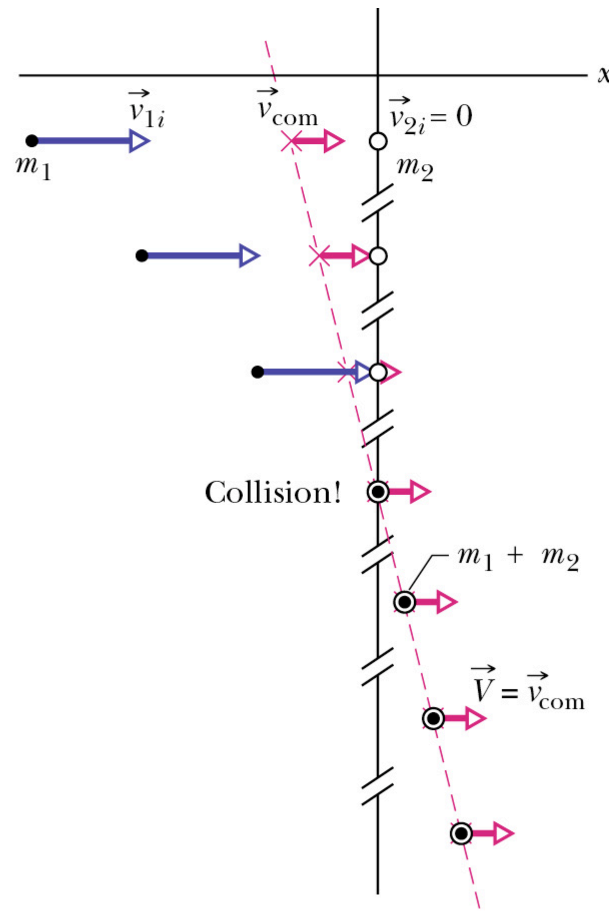
The velocity of the center of mass is a constant during the collision when there are no external forces. It is the same before and after the collision

The velocity of the cm is total momentum /total mass.

In general $v_{cm} = \dfrac{(p_{1i} + p_{2i})}{(m_1 + m_2)}$

Consider the total inelastic collision

In the initial state $V_{cm} = \dfrac{m_1 v_{1i}}{(m_1 + m_2)}$



Did the velocity of the center of mass stay constant for the inelastic collision?

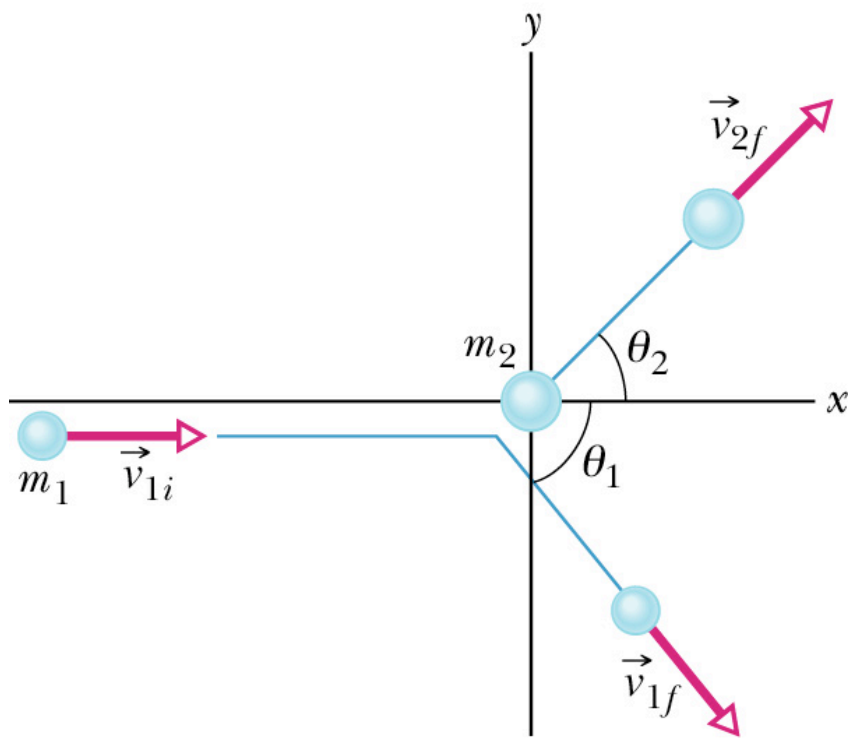
For equal mass objects $v_{cm} = \dfrac{v_{1i}}{2}$

After the inelastic collision what is V_{cm}. Since the particles are stuck together it must be the velocity of the stuck particles

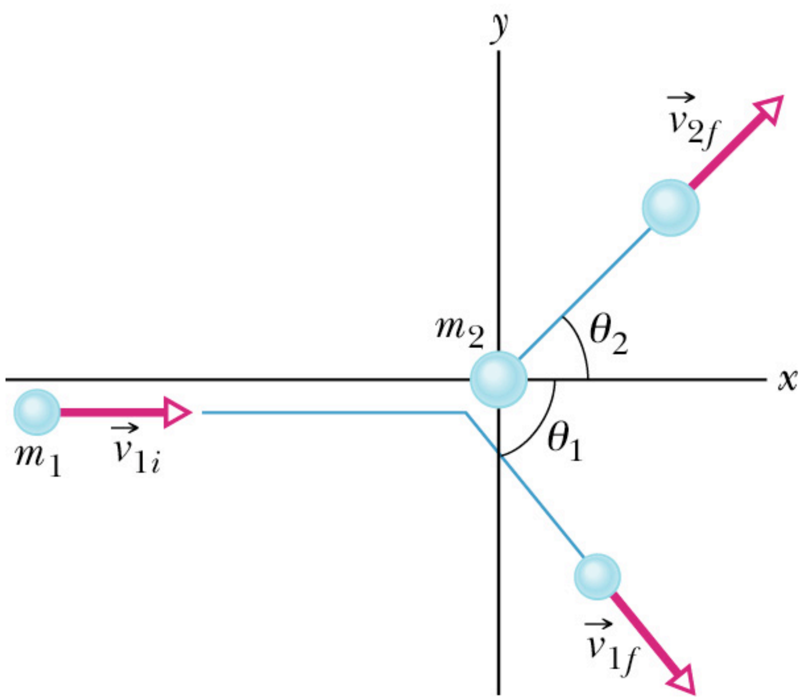
$$V = \frac{m_1 v_{1i}}{(m_1 + m_2)} = \frac{m v_{1i}}{m + m} = \frac{v_{1i}}{2}$$

which is $v_{1i}/2$. Hence, they agree.

Collisions in Two Dimensions



Write down conservation of momentum in x and y directions separately.
Two separate equations because momentum is a vector.
Eq 9-79 x axis
Eq 9-80 y axis
Write down conservation of kinetic energy - one equation 9-81

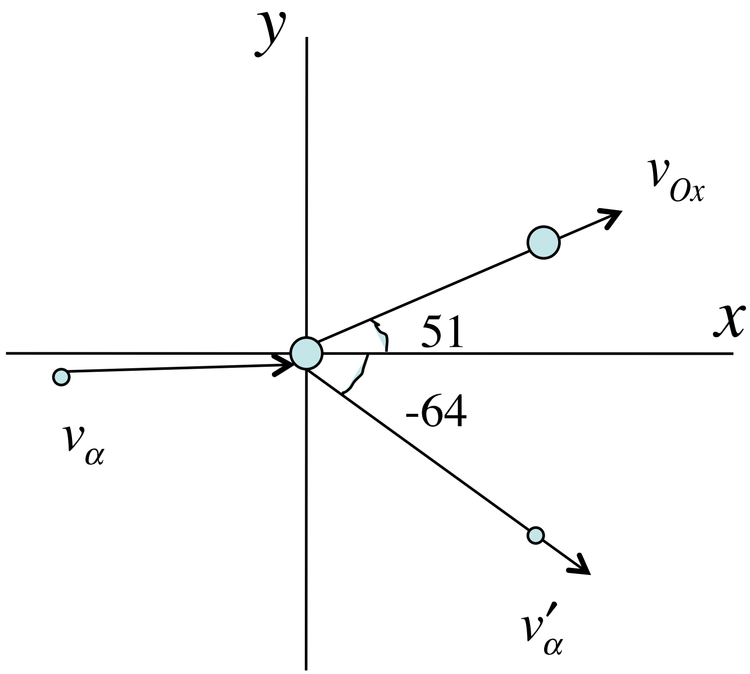


Conservation of momentum along x axis $m_1 v_{1i} = m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos\theta_2$

Conservation of momentum along y axis $0 = m_1 v_{1f} \sin\theta_1 + m_2 v_{2f} \sin\theta_2$

Conservation of Kinetic Energy $\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$

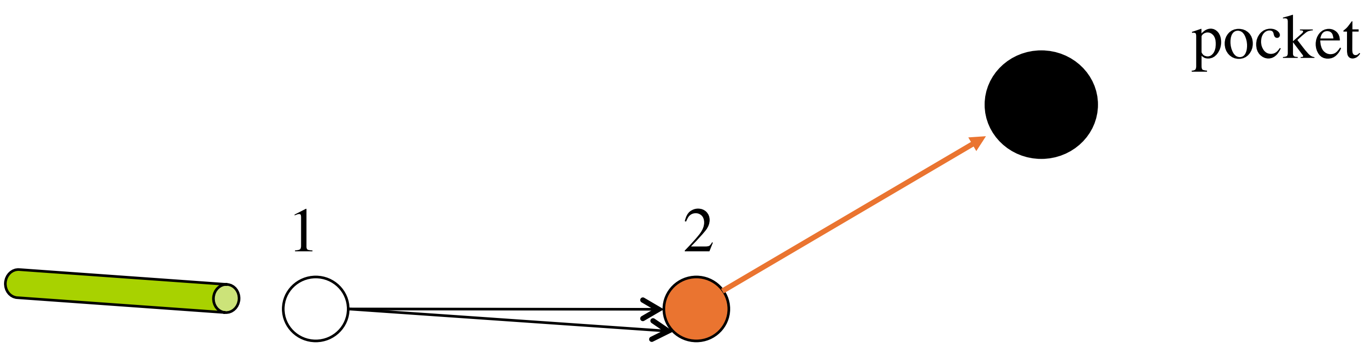
Problem 65 Chapter 9 Ed. 7

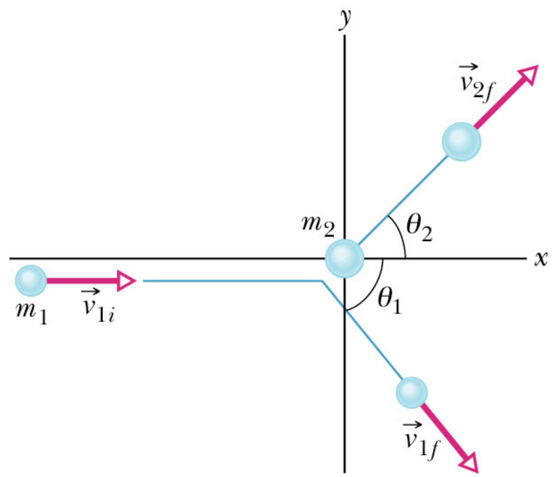


Let m_1 be an alpha particle and m_2 and oxygen nucleus. The alpha particle is scattered at 64 degrees and the oxygen nucleus recoils with speed 1.2 x 10^5 m/s at an angle of 51 degrees. In atomic mass units the alpha is 4 and oxygen is 16. What are the (a) final and (b) initial speeds of the alpha?

$$v_{Ox} = 1.2 \times 10^5 \frac{m}{s}$$

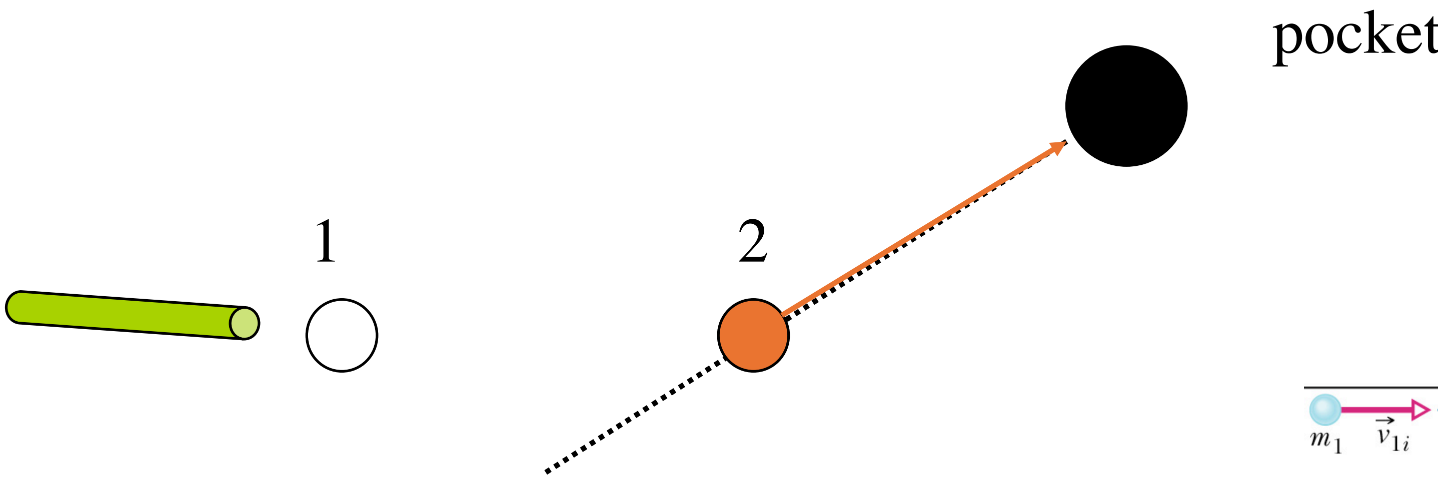
Example in Billiards- Line of Action: How do you know where ball 1 should strike ball 2 to make ball 2 go into the pocket?

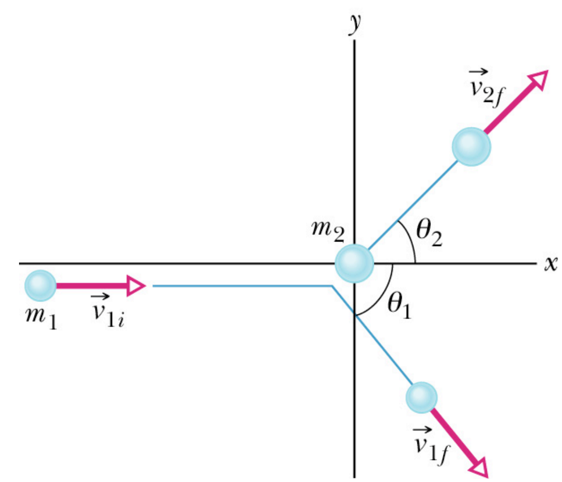




Assuming no spin
Assuming elastic collision

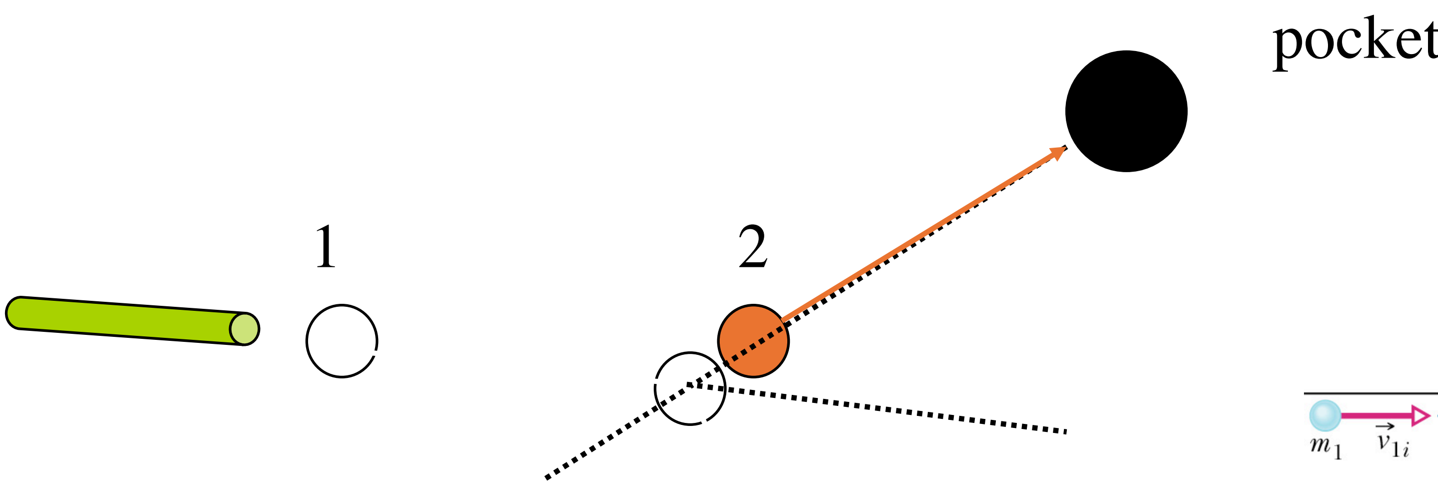
Example in Billiards- Line of Action: How do you now where ball 1 should strike ball 2 to make ball 2 go into the pocket?

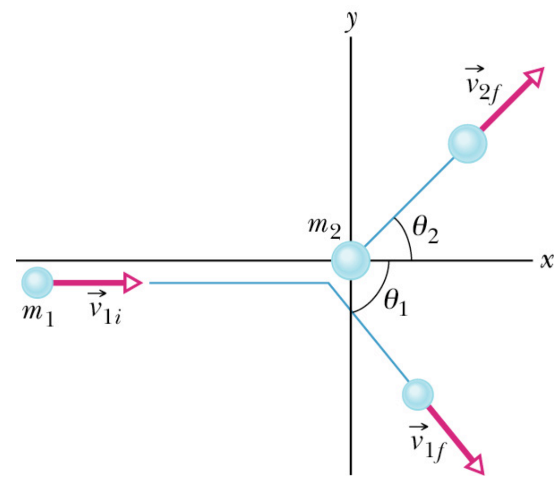




Assuming no spin
Assuming elastic collision

Example in Billiards- Line of Action: How do you know where to aim ball ball 1 to make ball 2 go into the pocket?

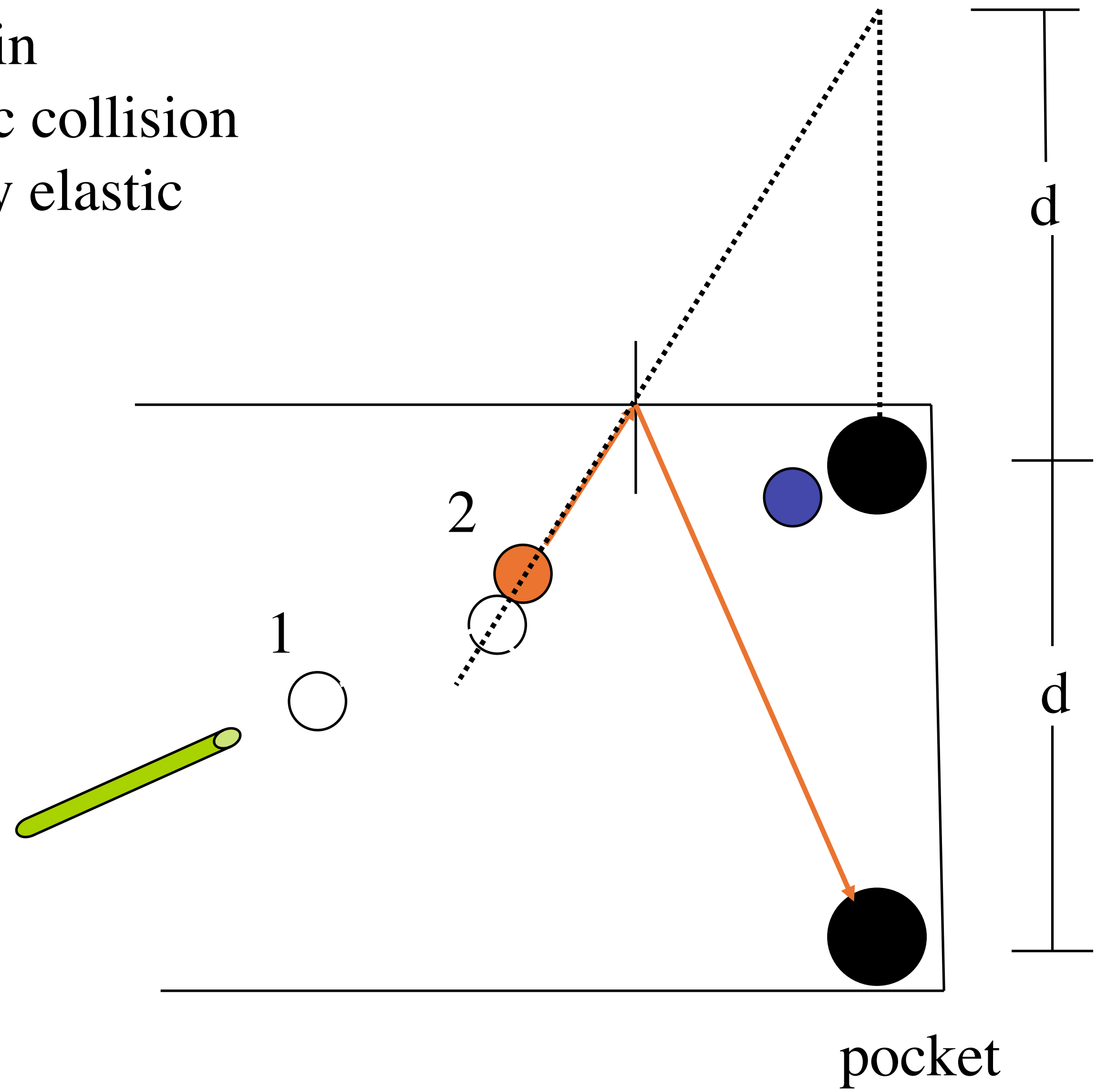




Assuming no spin
Assuming elastic collision

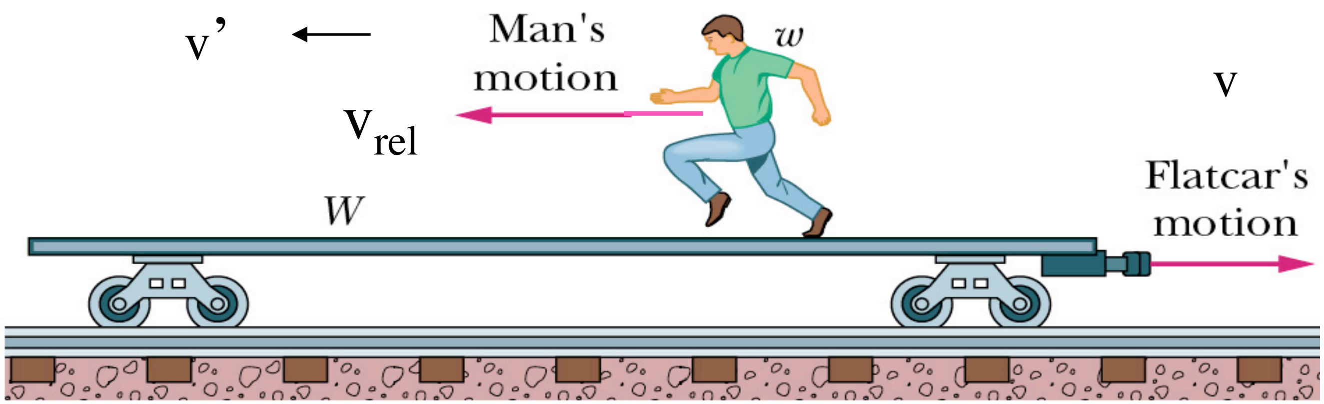
Bank shots: How do you know where to aim ball ball 1 to make ball 2 hit the bank and go into the lower corner pocket?

Assuming no spin
Assuming elastic collision
Bank is perfectly elastic



Problem 9-32 ed 6

A man of weight w is at rest on a flat car of weight W moving to the right with speed $v_{0.}$ The man starts running to the left with speed v_{rel} relative to the flatcar. What is the change in the velocity of the flatcar $\Delta v = v - v_0$?

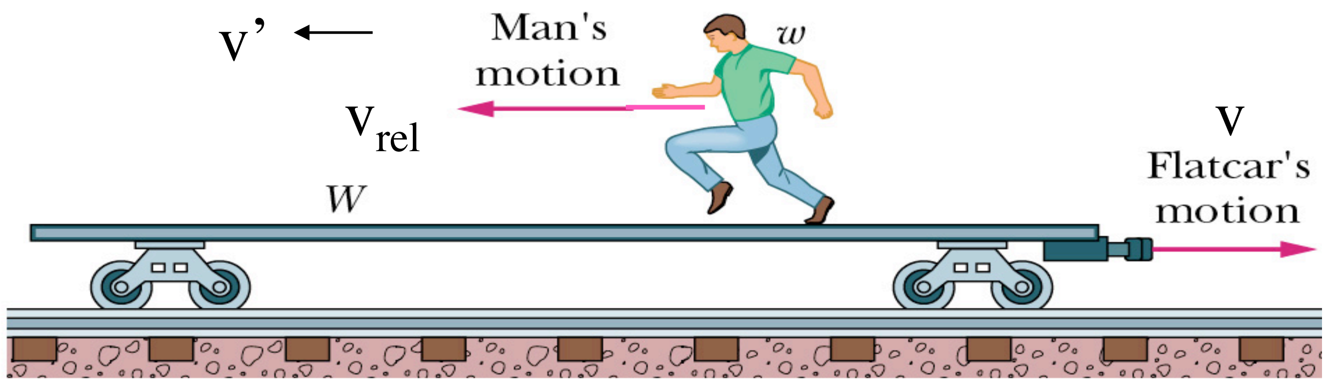


v_0 = initial velocity of flat car (before man runs) relative to tracks.
v = final velocity of flat car relative to tracks.
v_{rel}= velocity of man relative to flat car.
v' = velocity of man relative to the tracks.

Initial momentum = Final momentum

$$\frac{(W+w)v_0}{g} = \frac{(W)v}{g} - \frac{wv'}{g}$$

$$v' = v_{rel} - v$$



What is the change in the velocity of the car $\Delta v = v - v_0$?

$$\frac{(W+w)v_0}{g} = \frac{(W)v}{g} - \frac{w(v_{rel}-v)}{g}$$

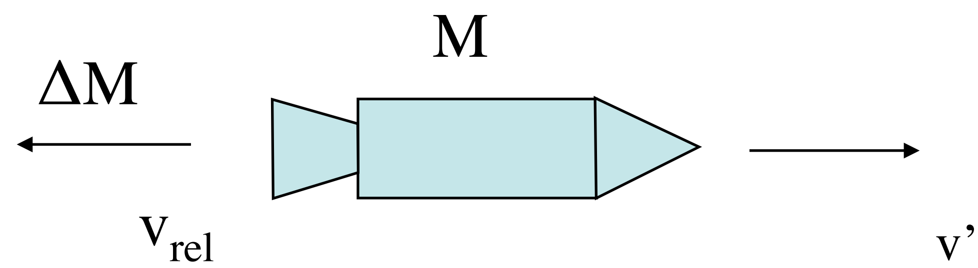
$$(M+m)v_0 = (M)v - m(v_{rel}-v)$$

$$0 = -(M+m)v_0 + (M+m)v - m(v_{rel})$$

$$0 = (M+m)(v-v_0) - mv_{rel}$$

$$(v-v_0) = \frac{mv_{rel}}{(M+m)}$$

Derivation of rocket equation



$$v - v_0 = \frac{mv_{rel}}{(M + m)}$$

$$mv_{rel} = (M + m)(v - v_0)$$

m = mass of fuel

M=mass of Rocket without fuel

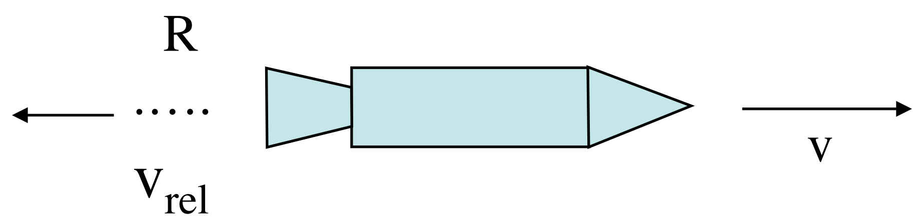
$m + M$ = mass of fuel + mass of Rocket $\rightarrow$ M

$v - v_0 = \Delta v$

let m = - ΔM

- ΔMv_{rel} = MΔv

Derivation of rocket equation



$$\frac{\Delta M v_{rel}}{\Delta t} = \frac{M\,\Delta v}{\Delta t}$$ divide by Δt

$$R = \frac{\Delta M}{\Delta t}$$ defined as mass rate of fuel consumption

$$R v_{rel} = \frac{M\,\Delta v}{\Delta t}$$

$$R v_{rel} = Ma$$ first rocket equation

$$\frac{-dM}{dt} v_{rel} = Ma$$

$$a = \frac{dv}{dt}$$

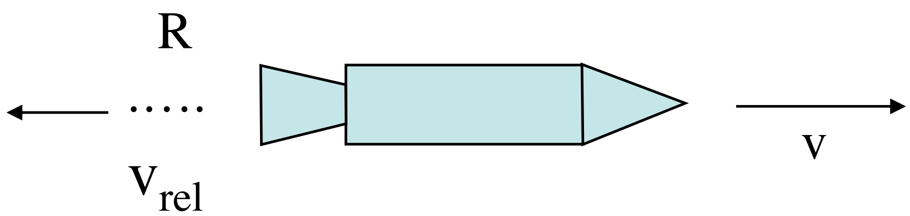
$$\frac{-dM}{dt} v_{rel} = M \frac{dv}{dt}$$

$$dv = -v_{rel} \frac{dM}{M}$$

$$\int_{v_i}^{v_f} dv = -v_{rel} \int_{M_i}^{M_f} \frac{dM}{M} = -v_{rel} \ln M \Big|_{M_i}^{M_f} = -v_{rel} (\ln M_f - \ln M_i)$$

$$v_f - v_i = v_{rel} \ln \frac{M_i}{M_f} \qquad (2)$$

Problem 9-71 ed 7



$$v_i = 0$$

$$M_{\text{rocket+fuel}} = 2.55 \times 10^5 \, kg$$

$$\text{M}_{\text{fuel}} = 1.81 \times 10^5 \, kg$$

$$R = 480 \, kg/s$$

$$v_{rel} = 3.27 \, km/s$$

(a) What is the thrust?

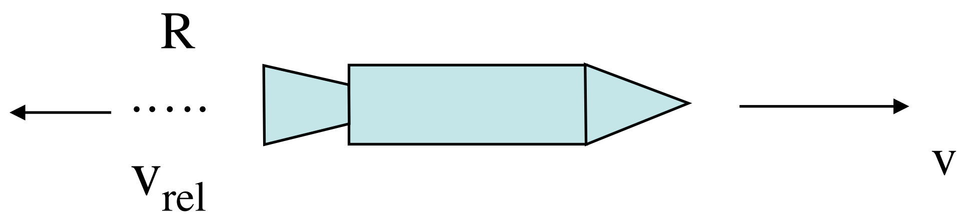
$$T = v_{rel} R = (3270)(480) = 1.57 \times 10^6 \, N$$

(b) What is the rockets mass +fuel after 250 s of firing?

$$\Delta M = (250)(480) = 1.2 \times 10^5 \, kg$$

$$M_{\text{rocket+fuel}} = 2.55 - 1.20 = 1.35 \times 10^5 \, kg$$

Problem 9-71 ed 7



$$v_i = 0$$

$$M_{\text{rocket+fuel}} = 2.55 \times 10^5\, kg$$

$$M_{\text{fuel}} = 1.81 \times 10^5\, kg$$

$$R = 480\, kg / s$$

$$v_{rel} = 3.27\, km / s$$

(c) What is the speed after 250 s?

$$v_f - v_i = v_{rel} \ln \frac{M_i}{M_f} = 3270 \ln \frac{2.55}{1.35} = 2079 m / s$$

$$v_f = 2079 m / s \text{ since } v_i = 0$$

ConcepTest 8.19 Motion of CM

Two equal-mass particles (A and B) are located at some distance from each other. Particle A is held stationary while B is moved away at speed *v*. What happens to the center of mass of the two-particle system?

1) it does not move

2) it moves away from A with speed *v*

3) it moves toward A with speed *v*

4) it moves away from A with speed 1/2 *v*

5) it moves toward A with speed 1/2 *v*

ConcepTest 8.2b Momentum and KE II

A system of particles is known to have a total momentum of zero. Does it necessarily follow that the total kinetic energy of the system is also zero?

1) yes
2) no

ConcepTest 8.3a Momentum and Force

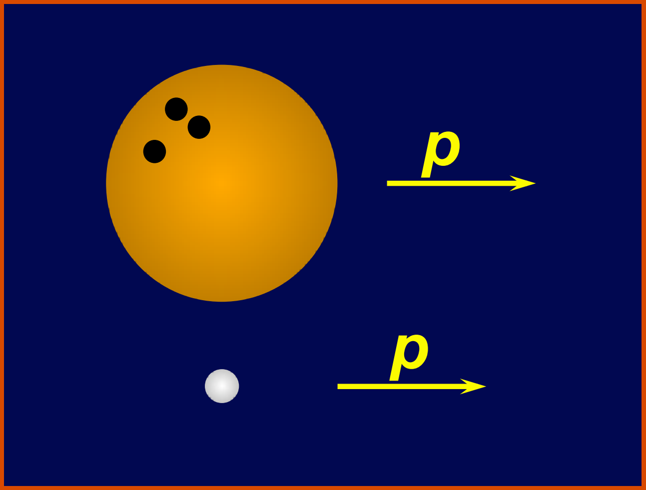
A net force of 200 N acts on a 100-kg boulder, and a force of the same magnitude acts on a 130-g pebble. How does the rate of change of the boulder's momentum compare to the rate of change of the pebble's momentum?

1) greater than
2) less than
3) equal to

ConcepTest 8.9a Going Bowling I

A bowling ball and a ping-pong ball are rolling toward you with the **same momentum**. If you exert the **same force** to stop each one, which takes a **longer time** to bring to rest?

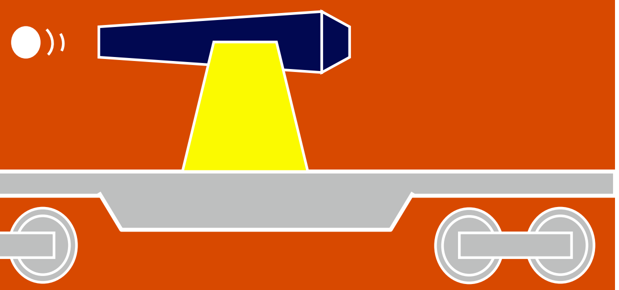
1) the bowling ball
2) same time for both
3) the ping-pong ball
4) impossible to say



ConcepTest 8.14b Recoil Speed II

A cannon sits on a stationary railroad flatcar with a total mass of **1000 kg**. When a **10-kg** cannon ball is fired to the left at a speed of **50 m/s**, what is the recoil speed of the flatcar?

1) 0 m/s
2) 0.5 m/s to the right
3) 1 m/s to the right
4) 20 m/s to the right
5) 50 m/s to the right



ConcepTest 8.19 Motion of CM

Two equal-mass particles (A and B) are located at some distance from each other. Particle A is held stationary while B is moved away at speed *v*. What happens to the center of mass of the two-particle system?

1) **it does not move**
2) **it moves away from A with speed *v***
3) **it moves toward A with speed *v***
4) **it moves away from A with speed 1/2 *v***
5) **it moves toward A with speed 1/2 *v***

Let's say that A is at the origin (x = 0) and B is at some position x. Then the center of mass is at $x/2$ because A and B have the same mass. If $v = Dx/Dt$ tells us how fast the position of B is changing, then the position of the center of mass must be changing like $D(x/2)/Dt$, which is simply 1/2 v.

ConcepTest 8.2b Momentum and KE II

A system of particles is known to have a total momentum of zero. Does it necessarily follow that the total kinetic energy of the system is also zero?

1) yes
2) no

Momentum is a vector, so the fact that $p_{tot} = 0$ does not mean that the particles are at rest! They could be moving such that their momenta cancel out when you add up all of the vectors. In that case, since they are moving, the particles would have non-zero KE.

ConcepTest 8.3a Momentum and Force

A net force of 200 N acts on a 100-kg boulder, and a force of the same magnitude acts on a 130-g pebble. How does the rate of change of the boulder's momentum compare to the rate of change of the pebble's momentum?

1) greater than
2) less than
3) equal to

The rate of change of momentum is, in fact, the force. Remember that $F = \Delta p/\Delta t$. Since the force exerted on the boulder and the pebble is the same, then the rate of change of momentum is the same.

ConcepTest 8.14b Recoil Speed II

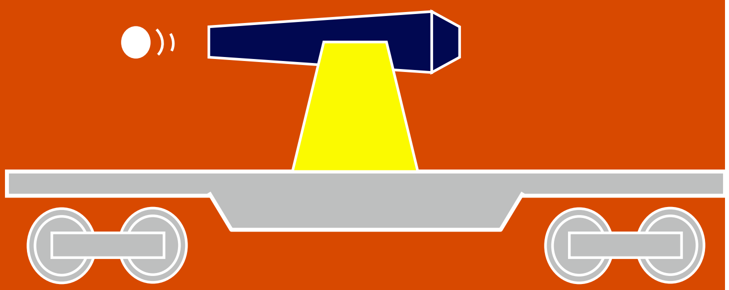
A cannon sits on a stationary railroad flatcar with a total mass of **1000 kg**. When a **10-kg** cannon ball is fired to the left at a speed of **50 m/s**, what is the recoil speed of the flatcar?

1) 0 m/s
2) 0.5 m/s to the right
3) 1 m/s to the right
4) 20 m/s to the right
5) 50 m/s to the right

Since the initial momentum of the system was zero, the final total momentum must also be zero. **Thus, the final momenta of the cannon ball and the flatcar must be equal and opposite.**

$$p_{\text{cannonball}} = (10\text{ kg})(50\text{ m/s}) = 500\text{ kg-m/s}$$

$$p_{\text{flatcar}} = 500\text{ kg-m/s} = (1000\text{ kg})(0.5\text{ m/s})$$



ConcepTest 8.9a Going Bowling I

A bowling ball and a ping-pong ball are rolling toward you with the same momentum. If you exert the same force to stop each one, which takes a longer time to bring to rest?

1) the bowling ball
2) same time for both
3) the ping-pong ball
4) impossible to say

We know: $F_{av} = \dfrac{\Delta p}{\Delta t}$ so $\Delta p = F_{av}\Delta t$

Here, *F* and D*p* are the *same* for both balls!

It will take the *same amount of time* to stop them.