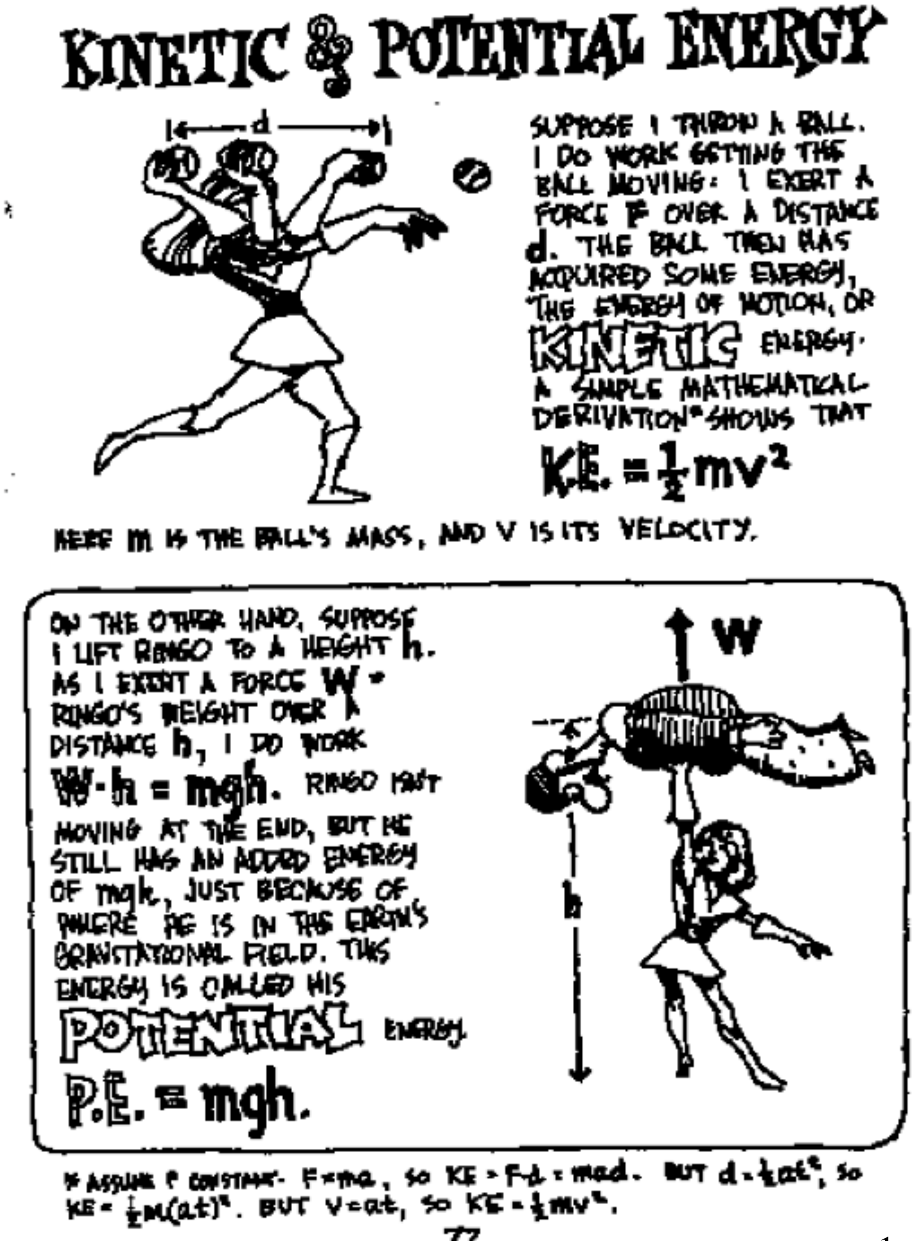
Lecture 5
Chapter 8
Potential Energy and Conservation of Energy



We want to define potential energy and change in potential energy. It is a useful concept for Conservative forces. When combined with kinetic energy it becomes an invariant or constant of the motion.

- Conservative Forces that are very common are:
  - gravitational forces
  - spring-like or elastic forces
  - electric force

- Non-conservative forces
  - friction

- Path independence of conservative forces.

The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle. It only depends on the initial and final positions. Will prove this later.

Show example raising and lowering a book.

Straight up and down, zig zag path, curved path.

Gravitational Force - Potential Energy - U

- **The change in potential energy, ΔU, of an object is defined as the negative of the work done on the object by the force. Consider a ball, the earth and the gravitational force between them.**

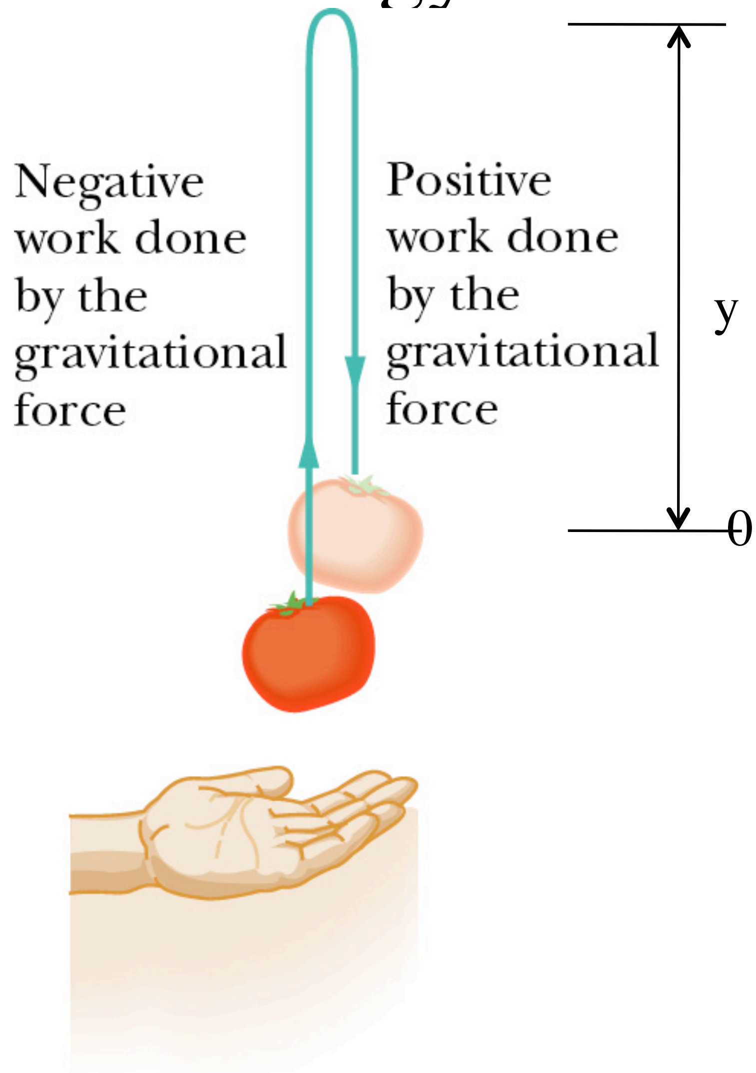
$$U_f - U_i = \Delta U$$

$$\Delta U = -W$$

$$W = \vec{F} \cdot \vec{y}$$

$$F = -mg$$

$$\Delta U = mgy$$



More exact relationship between conservative force and potential energy

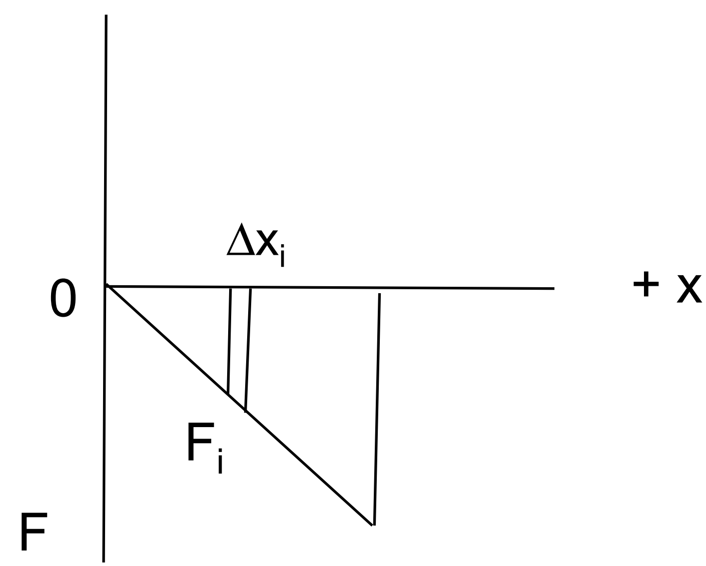
$$\Delta U = -W$$

$$W = F(x)\Delta x$$

$$\Delta U = -F(x)\Delta x$$

$$F(x) = \frac{-\Delta U(x)}{\Delta x}$$

$$F(x) = \frac{-dU(x)}{dx}$$



Find the value of potential energy for a gravitational system

- From $F(x) = \frac{-dU(x)}{dx}$ or $dU(x) = -F(x)dx$ In one dimension

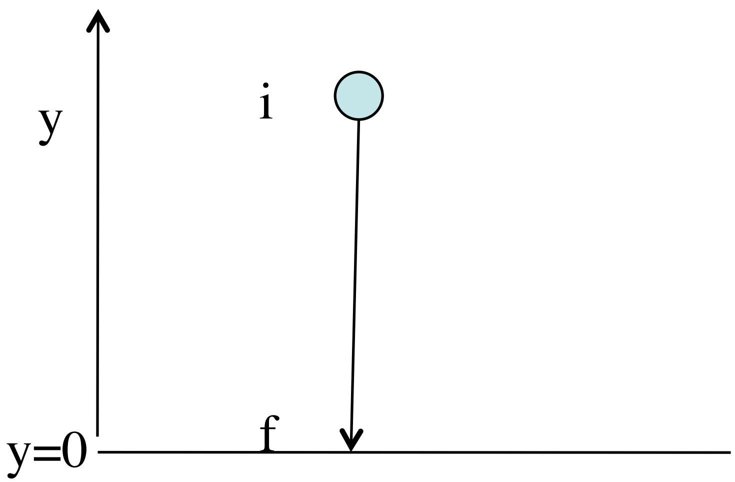
$$\int_{U_i}^{U_f} dU = -\int_{x_i}^{x_f} F(x)dx$$

$$U_f - U_i = -\int_{x_i}^{x_f} F(x)dx$$

For gravity acting in the downward y-direction F= - mg

$$U_f - U_i = -\int_{y_i}^{y_f} (-mg)dy = mg\Big|_{y_i}^{y_f} = mg(y_f - y_i)$$

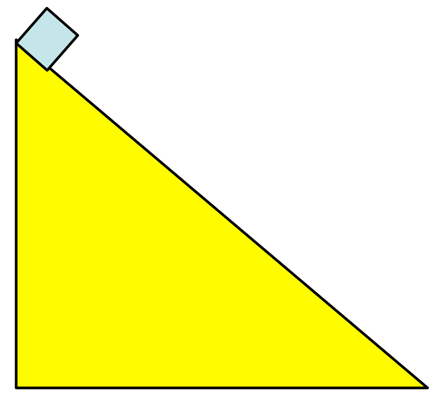
Reference point $U_i = 0$, $y_i = 0$ Hence $U = mgy$



$$U = mgy$$

$$U = 0$$

W is the same here

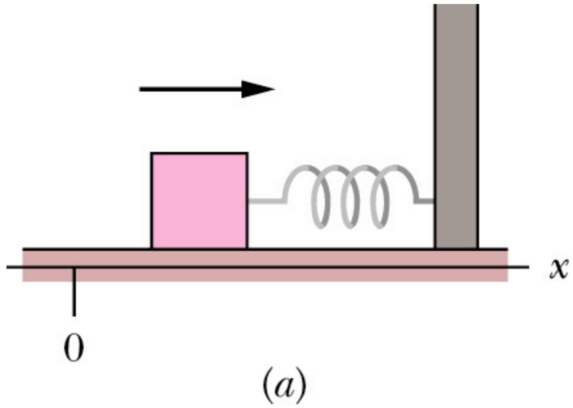


$$W = -\Delta U$$

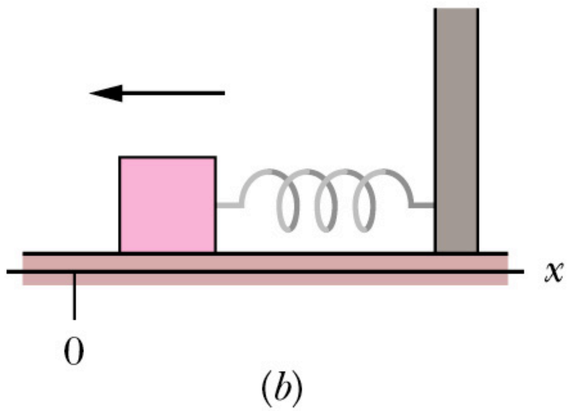
$$W = -(U_f - U_i) = -(0 - mgy) = mgy$$

W is positive so gravity does work on the ball

Potential energy-U can also be defined for elastic spring



Work done on the spring



Work done by the spring

$$\Delta U = -W$$

$$W = Fx$$

$$F = -kx$$

$$\Delta U = \frac{1}{2}kx^2$$

Find the value of potential energy for an ideal spring

$$F = -kx$$

$$U_f - U_i = -\int_{x_i}^{x_f} F(x)dx$$

$$U_f - U_i = -\int_{x_i}^{x_f} (-kx)dx = \tfrac{1}{2}kx^2\Big|_{x_i}^{x_f} = \tfrac{1}{2}k(x^2{}_f - x^2{}_i)$$

Reference point: $U_i = 0$, $x_i = 0$

Again the potential energy
and work only depend on the end points
That is the initial and final positions.

Hence $U = \dfrac{1}{2}kx^2$

Conservation of Mechanical Energy (defined as sum of kinetic and potential)

Assumptions: No friction forces
No drag forces
No velocity dependent forces
No external forces acting outside the system
Only conservative forces acting

$$\Delta K = W$$ From the Work-Energy Theorem

$$\Delta U = -W$$ Definition of potential energy

$$\Delta U = -\Delta K \qquad (U_2 - U_1) = -(K_2 - K_1)$$

$$K_2 + U_2 = K_1 + U_1 \qquad E_{mec} = K + U$$

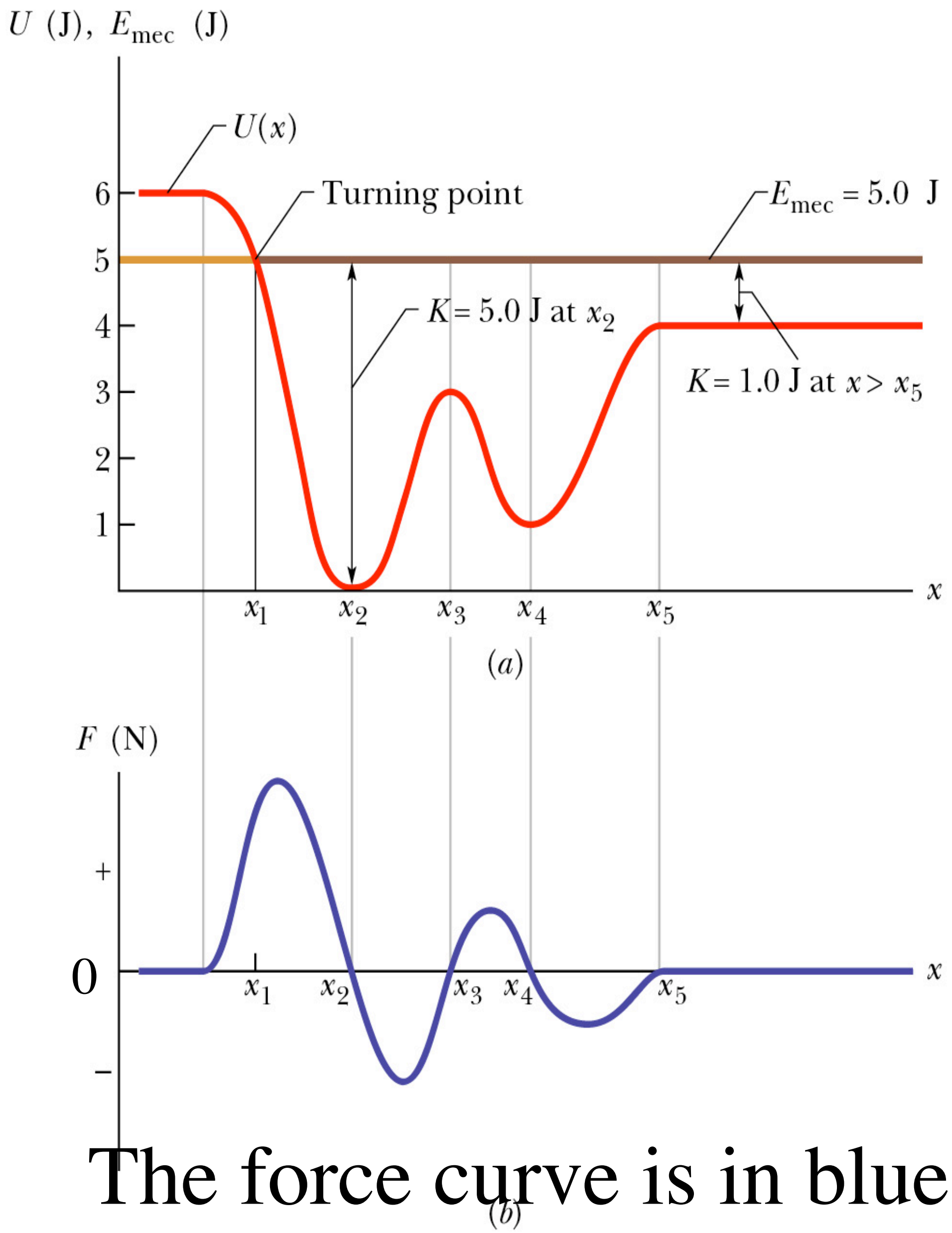
All the information
is in the potential energy

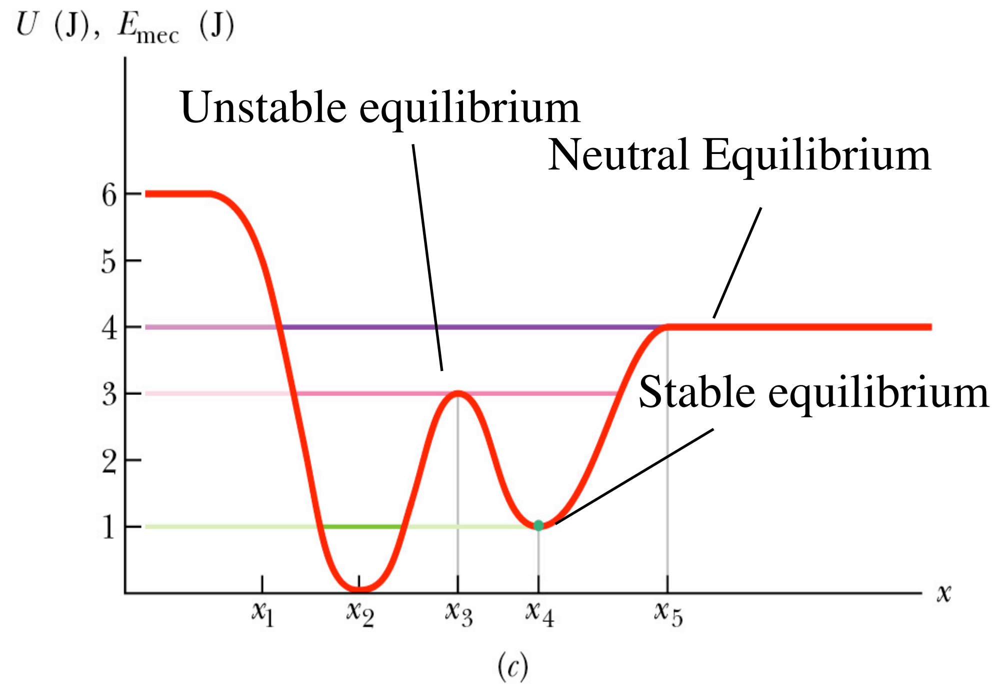
Potential Energy Curve

Turning points

Equilibrium Points

The Potential Energy Curve is in red





$$F(x) = \frac{-dU(x)}{dx}$$

The force curve is in blue

Question 6 pg 159

Rank regions according to the magnitude of the force. AB, GH, EF, CD,

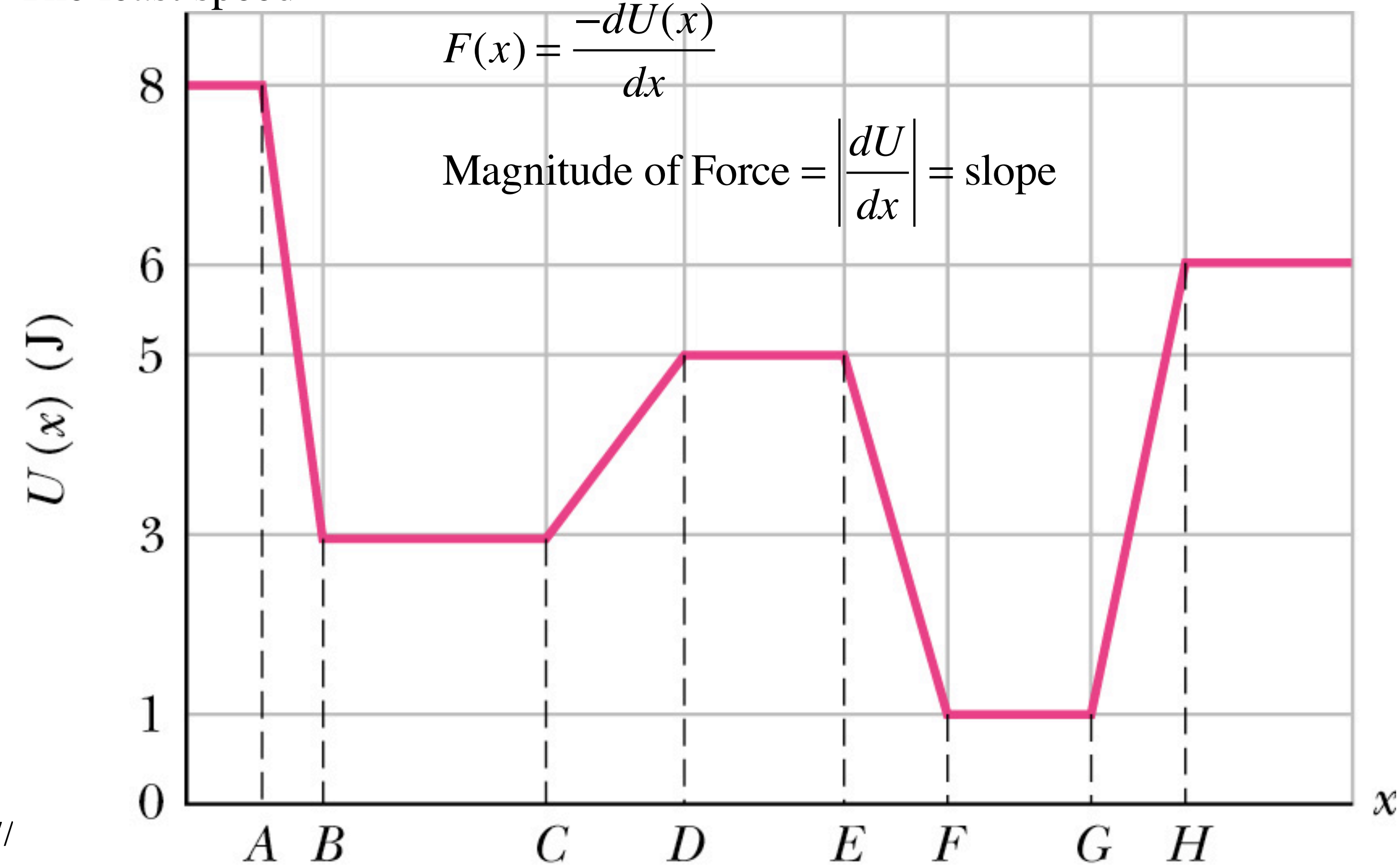
What is the max E for the particle to be trapped in BC? 5 J

What is the max E for the particle to be trapped in FG? 5 J

Move between BC and FG but not to the right of H? Between 5-6 J

For the above case, where will the particle have the max K? FG

The least speed DE



$$F(x) = \frac{-dU(x)}{dx}$$

$$\text{Magnitude of Force} = \left|\frac{dU}{dx}\right| = \text{slope}$$

Principle of Conservation of Mechanical Energy

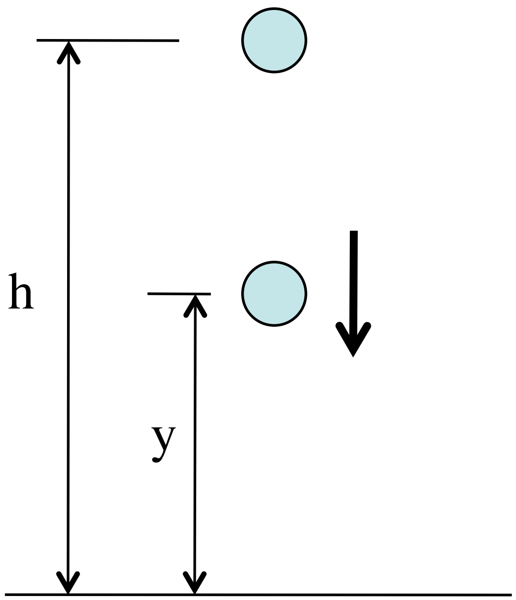
- E remains constant as an object moves provided that no work is done on it by external friction forces.

Using the Conservation of Mechanical Energy

- Identify important forces. Friction forces must be absent or small.
- Choose height where gravitational PE is zero.
- Set initial KE + PE equal to final KE + PE

Ball in Free Fall

A Ball is dropped from rest at a height h above the ground. What is its speed at a height y above the ground. Neglect air resistance. Use conservation of mechanical energy in the ball-earth system.



$$K_f + U_f = K_i + U_i$$

$$\frac{1}{2}mv_f^2 + mgy = 0 + mgh$$

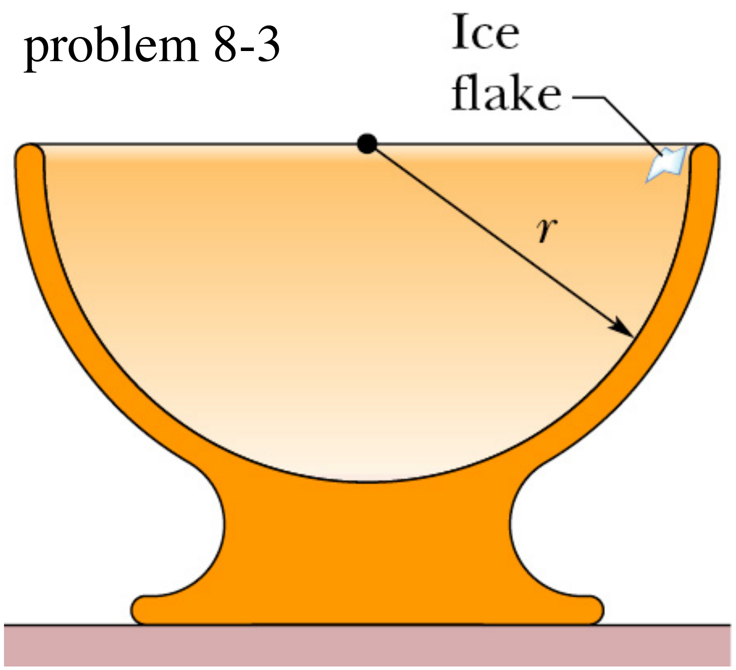
$$v_f = \sqrt{2g(h-y)}$$

Take h=1.0 m
y=0.5 m
v = 3.1 m/s

Suppose the ball had some initial speed at h, say v_i.
What is the speed at y?

$$\frac{1}{2}mv_f^2 + mgy = \frac{1}{2}mv_i^2 + mgh$$

problem 8-3



a) How much work is done on the ice flake by the gravitational force during its descent to the bottom of the bowl ?

$$W = F \bullet d \qquad F = mg \qquad d = r \qquad W = mgr$$

(Why isn't there an angle involved?)

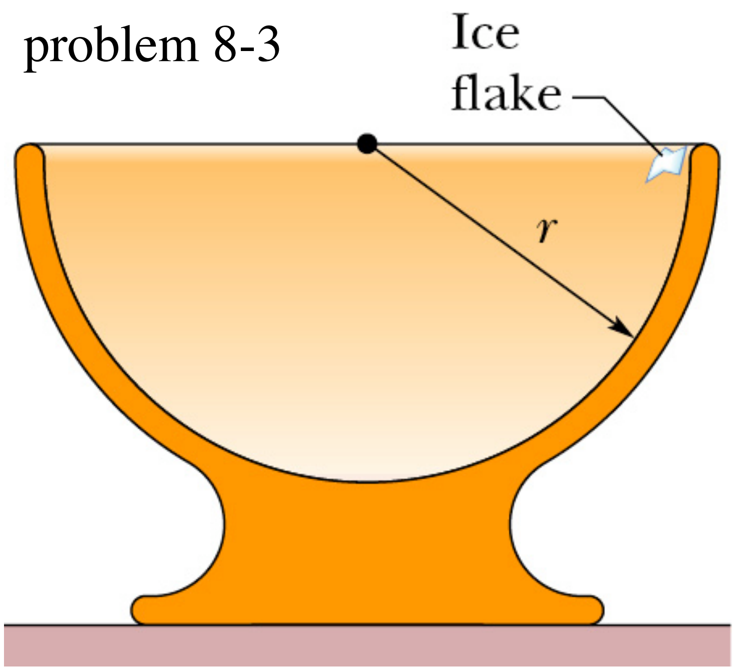
(gravity is conservative and no friction)

b) What is the change in the potential energy of the flake-earth system during descent?

$$\Delta U = -W \qquad W = mgr \qquad \Delta U = -mgr$$

Potential energy is lower at the bottom of the bowl

problem 8-3



$$\Delta U = -mgr$$

c. If U=0 at the bottom, what is U at the top?

U (Bottom) – U (Top) = - mgr

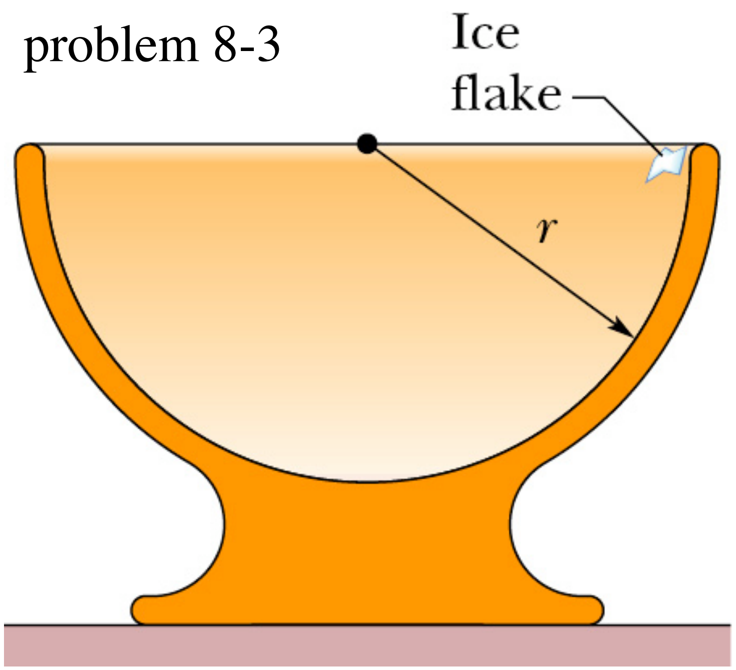
U(top)= U(Bottom) +mgr

U(top)= +mgr

d. If U=0 at the release point, then what is U at the bottom?
U(Bottom) = - mgr

The same

problem 8-3



e. If the mass of the flake is doubled, what are the answers to a-d?

They are doubled because they all involve the mass

f. What is the speed at the bottom of the bowl?

$$K_b + U_b = K_t + U_t$$

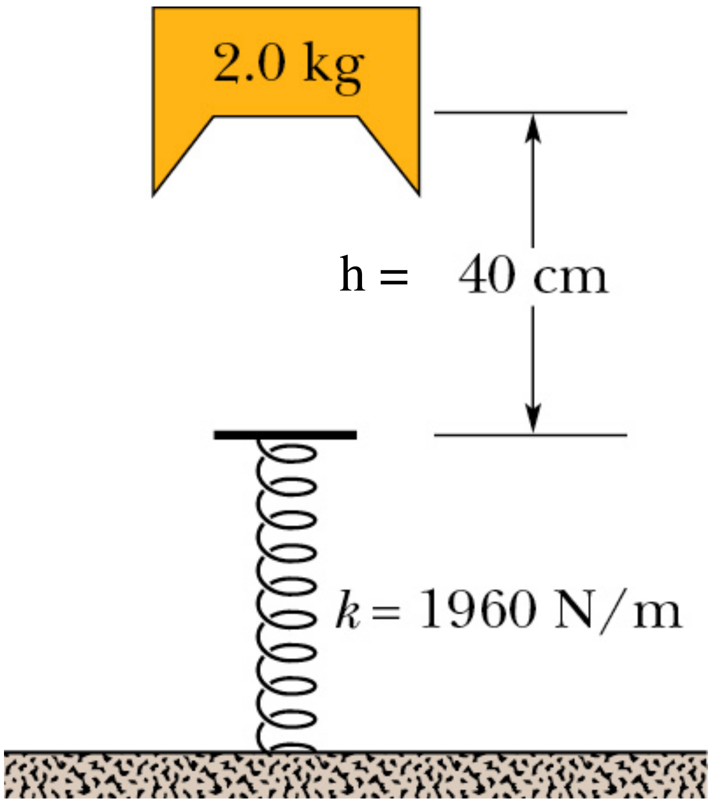
$$K_b - K_t = U_t - U_b$$

$$\frac{1}{2}mv^2 - 0 = -\Delta U = -(-mgr - 0)$$

$$v = \sqrt{2gr}$$

g. If the flake had twice the mass, what is the speed? The same

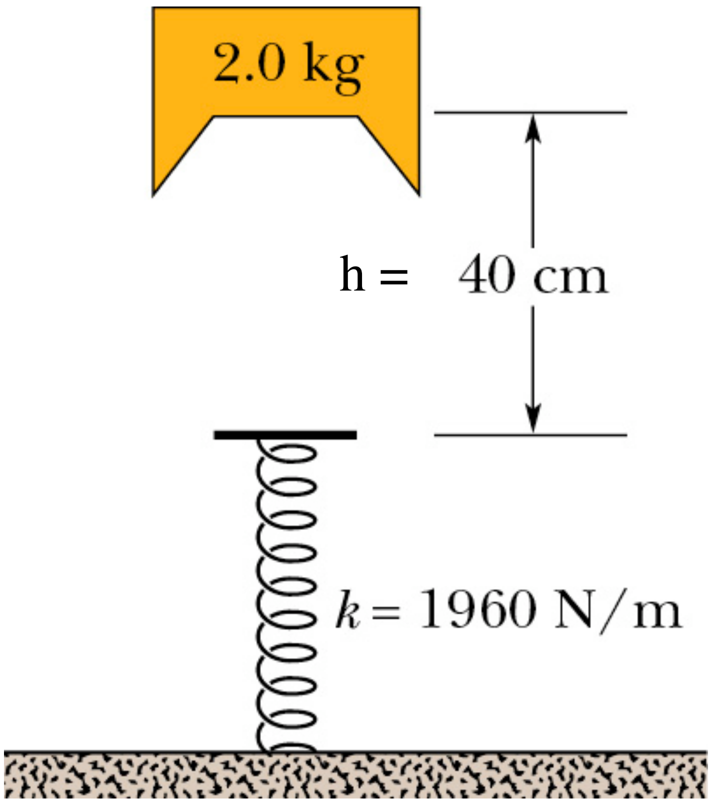
Problem 8-18

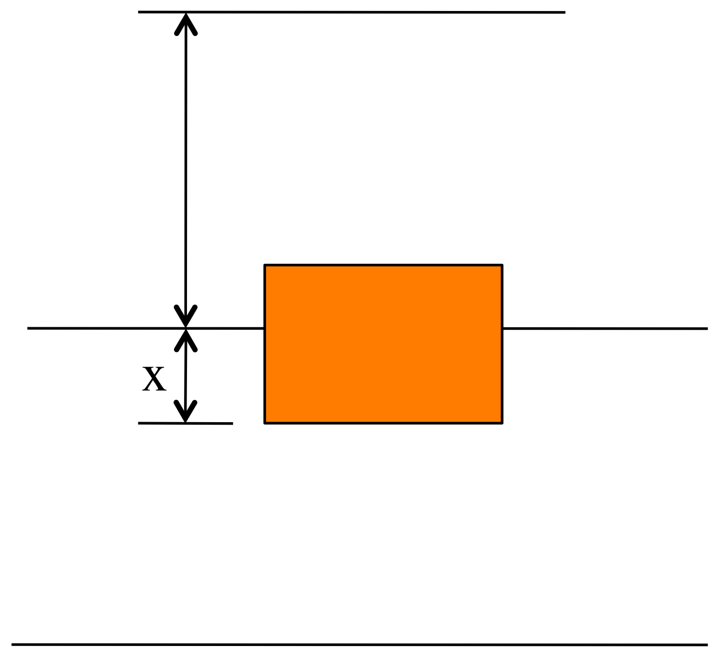


Find the maximum amount the spring is compressed after the block is dropped on it.

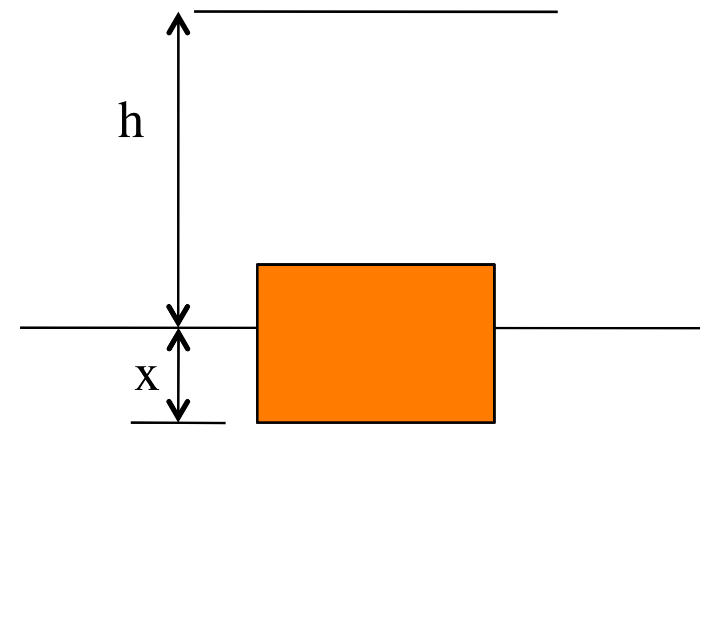
Problem 8-18

Find the maximum amount the spring is compressed after the block is dropped on it.





Problem 8-18



Find the maximum amount the spring is compressed after the block is dropped on it.

$$K_i + U_i = K_f + U_f$$

$$U_i = U_i^b + U_i^s = mg(x+h) + 0,$$

$$U_f = U_f^s + U_f^b = \frac{1}{2}kx^2 + 0$$

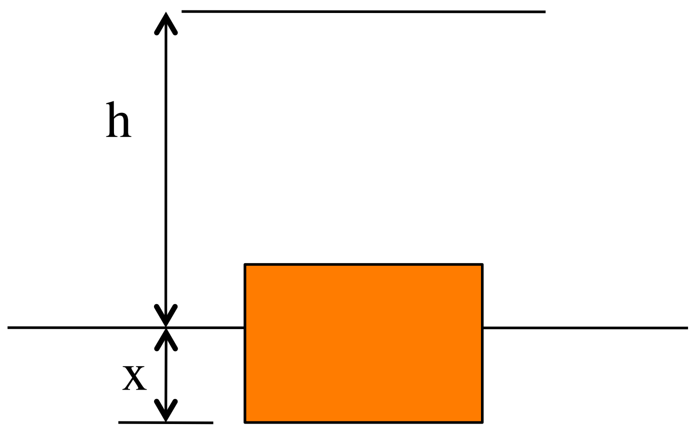
$$K_i = K_f = 0$$

$$mg(h+x) = \frac{1}{2}kx^2$$

$$\frac{1}{2}kx^2 - mgx - mgh = 0$$

$$x = \frac{mg \pm \sqrt{(mg)^2 + 2mghk}}{k}$$

Problem 8-18



$$x = \frac{mg \pm \sqrt{(mg)^2 + 2mghk}}{k}$$

$$x = \frac{19.6 \pm \sqrt{(19.6)^2 + (4)(9.8)0.4)(1960)}}{1960}$$

$$x = \frac{19.6 \pm \sqrt{384.2 + 30732.8}}{1960}$$

$$x = \frac{19.6 \pm 176.4}{1960}$$

$$x_+ = \frac{19.6 + 176.4}{1960} = \frac{196}{1960} = 0.1m$$

Spring is compressed= 0.1 m

Does the second solution have any meaning?

What is the other solution?

$x_{-} = -0.08m$

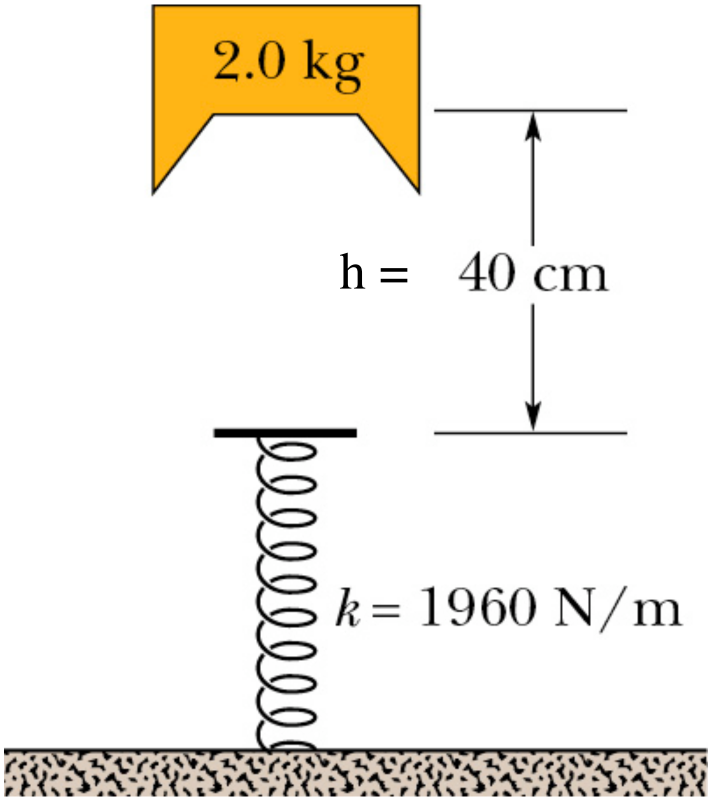
Equilibrium position

$kx = mg$

$x = mg / k = 19.6 / 1960 = 0.01m$

Assuming the block stuck to the spring, the block would oscillate up and down about the equilibrium position with amplitude equal to 0.1 – 0.01= 0.09 m. The amplitude on the other side is also 0.09 but when measured from the equilibrium position, the absolute location on the x axis would be at - 0.08-the second solution.

Problem 8-18



Find the velocity of the block just before it hits the spring

Potential Energy in 2-3 Dimensions

Conservation of Energy leads to

$$\frac{1}{2}mv^2 + U(x,y,z) = cons\tan t$$

where $v^2 = v_x{}^2 + v_y{}^2 + v_z{}^2$

$$U_f - U_i = -\int_{x_i}^{x_f} F(x)dx - \int_{y_i}^{y_f} F(y)dy - \int_{z_i}^{z_f} F(z)dz$$

Example: Simple Pendulum

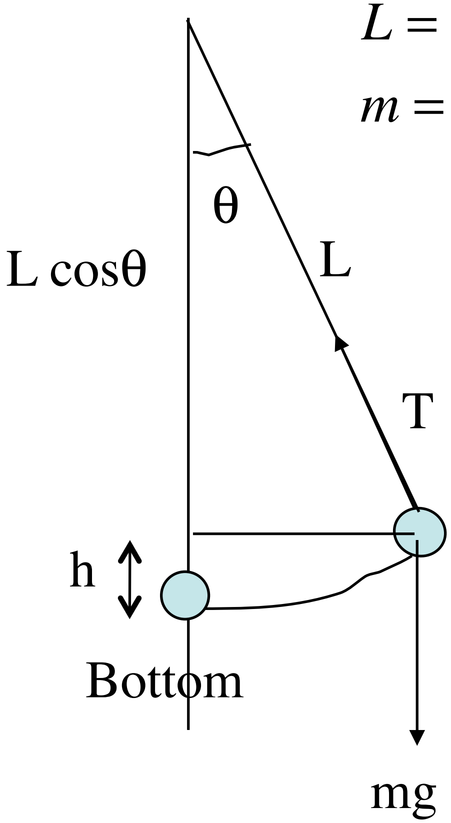
Pendulum

a) The bob is pulled out with force F so that the angle θ=30 .
What is the speed when θ=0. Use conservation of mechanical energy.

$$\theta = 30$$

$$L = 1.4m$$

$$m = 300gm$$



Measure the gravitational potential energy
Relative to the bottom of the swing.

$$K_f + U_f = K_i + U_i$$

$$\frac{1}{2}mv_1^2 + 0 = 0 + mgh$$

$$h = L - L\cos\theta = L(1-\cos\theta)$$

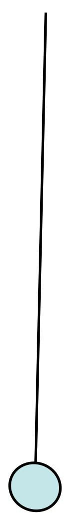
$$\frac{1}{2}mv_1^2 + 0 = 0 + mgL(1-\cos\theta)$$

$$v_1 = \sqrt{2gL(1-\cos\theta})$$

$$= \sqrt{2*9.8*1.4(1-\cos 30)} = 1.9m/s$$

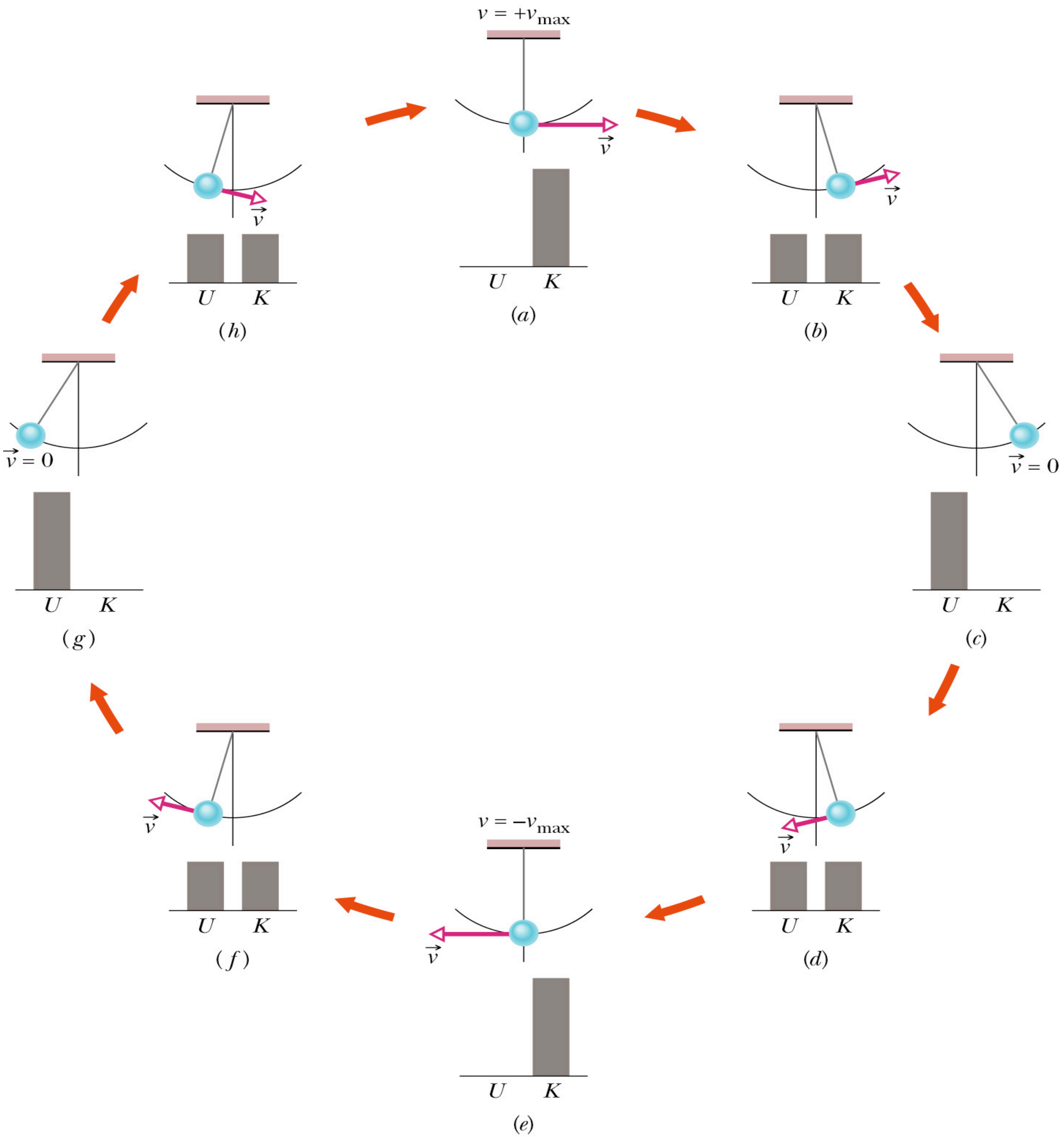
Pendulum

What is the tension in the string at the bottom ?



$$mg - T = -mv^2 / L$$

$$T = mg + mv^2 / L$$



v = +v max
v
U K
(a)
v
U K
(b)
v = 0
U K
(c)
v
U K
(d)
v = −v max
v
U K
(e)
v
U K
(f)
v = 0
U K
(g)
v
U K
(h)

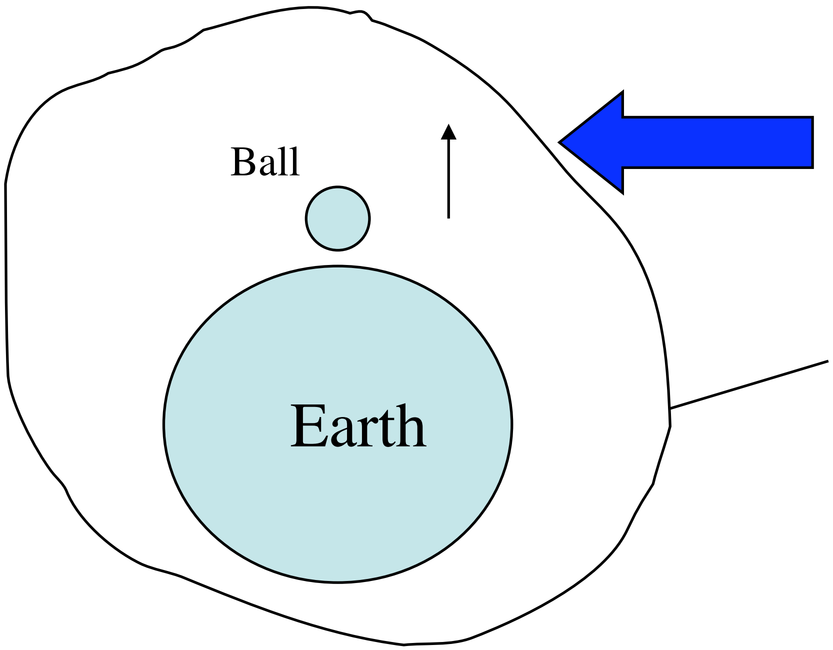
Friction and other internal forces

$$W = \Delta K + \Delta U + \Delta E_{th} + \Delta E_{\mathrm{int}}$$

For a system isolated from the environment where no energy transfers can take place between the system and the environment, then

$$\Delta K + \Delta U + \Delta E_{th} + \Delta E_{\mathrm{int}} = 0$$

External Work (No Friction)



W (eg. I could lift the ball)

Ball-Earth system

$$E = K_B + K_E + U_{BE}$$

$$\Delta E = \Delta K + \Delta U = 0$$

With no external forces

If an external force does work, W, then we must include it.

$$W = \Delta K + \Delta U$$

Friction Present

$$W = \Delta K + \Delta U + \Delta E_{th}$$

$$\Delta E_{th} = f_k d$$

Isolated system

For a system isolated from the environment where no energy transfers can take place between the system and the environment, then

$$\Delta K + \Delta U + \Delta E_{th} + \Delta E_{\text{int}} = 0$$

Here we have included an internal energy change to account for Other processes than friction.

Summary

$$E = \frac{1}{2}mv^2 + mgh$$ Constant for gravity

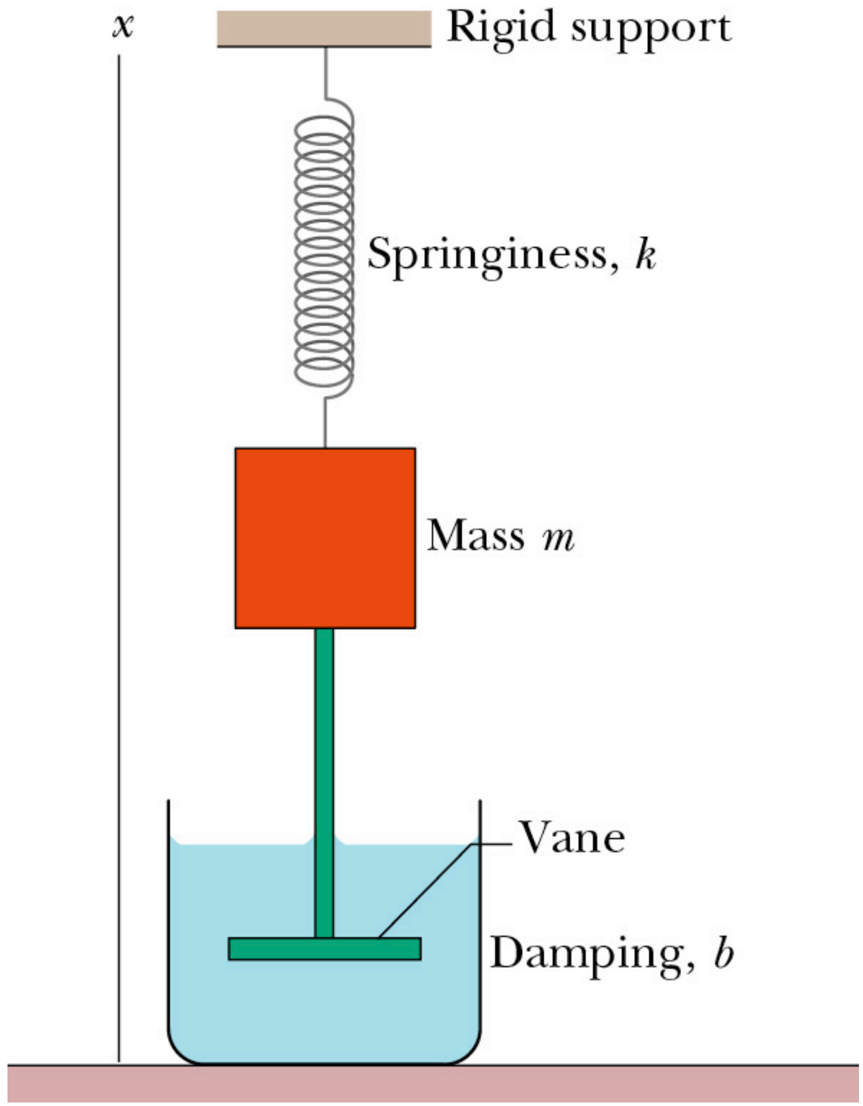
$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$ Constant for spring

$$E = \frac{1}{2}mv^2 + mgh + \frac{1}{2}kx^2$$ Constant for combined system

Assume all other forces are zero

Damped Harmonic Motion Spread Sheet Problem
Due Friday July 17 9:00 AM

Figure 15-15 HRW



Variables and constants:

t = time

x = displacement of mass

v = velocity of mass

a = acceleration of the mass

x_0 = initial displacement =0.12 m

k = spring constant=8.0 N/m

b = damping constant= 0.230 kg/s

m = mass of block=1.50 kg

From Newtons 2nd Law, the following equation describes the motion of the mass on a spring with spring constant k and a damping mechanism with a damping constant b.

$$F = -kx - bv$$

$$ma = -kx - bv$$

$$mdv / dt = -kx - bv$$

$$dv / dt + kx / m + bv / m = 0$$

This equation is equivalent to eq. 15-41 in HRW, where v=dx/dt.

Using the method of numerical integration in an excel spreadsheet, solve the following differential equation by answering the questions below.

$$dv / dt + kx / m + bv / m = 0$$

Follow the example I gave in class, which can be found in class materials on the website for Lecture 3. Also see section 15-8 in HRW.

1. Create an excel spread sheet as I did in class. Make three columns: one for time, one for velocity, and one for displacement. Include enough rows to reach 15 seconds. Turn in your spread sheet as well as the results below.
2. Plot the velocity of the mass as a function of time for 15 seconds
3. Plot the displacement x as a function of time for 15 seconds.
4. Check your result in 2 to Fig. 15-16 in HRW. It should be similar.
5 Find the number of oscillations of the mass per second or frequency in units of inverse sec.
6. Find the time it takes for the amplitude of motion to fall to 1/3 of its initial value.
7. Set the damping constant b=0 and repeat answers to questions 1,2 and 3.
8. The answer to 5 should be $f = \frac{1}{2\pi}\sqrt{k/m}$

ConcepTest 7.2 KE and PE

You and your friend both solve a problem involving a skier going down a slope, starting from rest. The two of you have chosen different levels for *y* = 0 in this problem. Which of the following quantities will you and your friend agree on?

1) only B

2) only C

3) A, B and C

4) only A and C

5) only B and C

A) skier's PE **B) skier's change in PE** **C) skier's final KE**

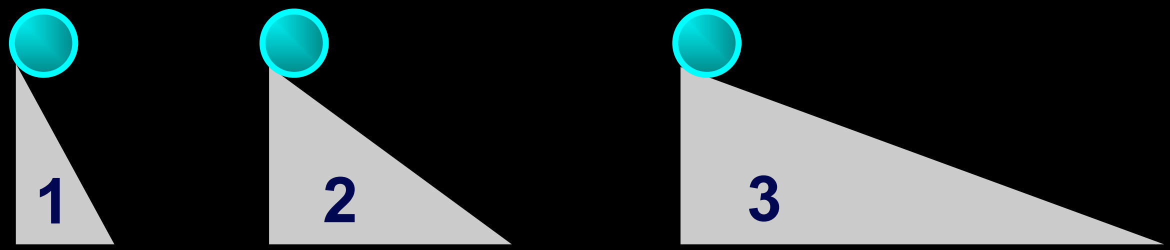
ConcepTest 7.4 Elastic Potential Energy

How does the work required to stretch a spring 2 cm compare with the work required to stretch it 1 cm?

1) same amount of work
2) twice the work
3) 4 times the work
4) 8 times the work

ConcepTest 7.6 Down the Hill

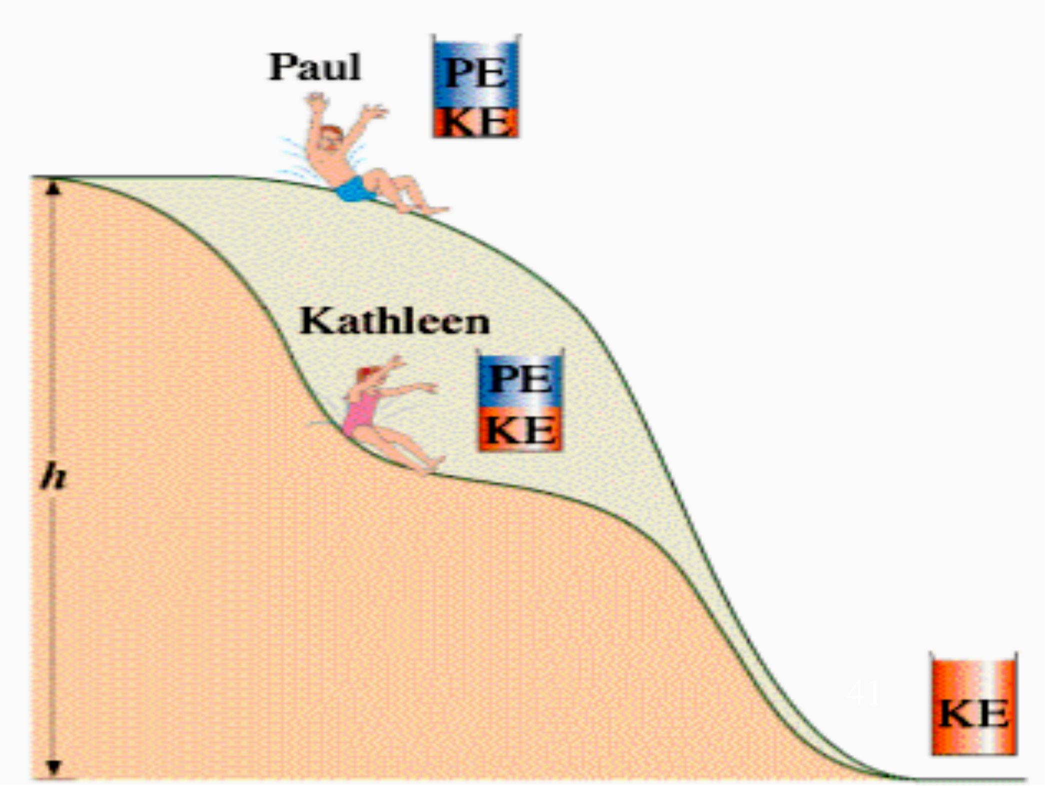
Three balls of equal mass start from rest and roll down different ramps. All ramps have the same height. Which ball has the greater speed at the bottom of its ramp?



ConcepTest 7.8b Water Slide II

Paul and Kathleen start from rest at the same time on frictionless water slides with different shapes. Who makes it to the bottom first?

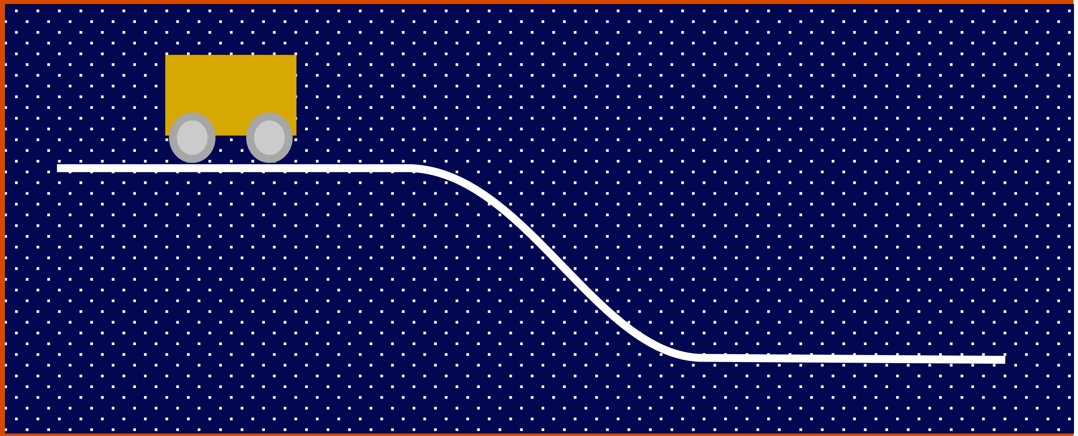
1) Paul
2) Kathleen
3) both the same



ConcepTest 7.9 Cart on a Hill

A cart starting from rest rolls down a hill and at the bottom has a speed of 4 m/s. If the cart were given an initial push, so its initial speed at the top of the hill was 3 m/s, what would be its speed at the bottom?

1) 4 m/s
2) 5 m/s
3) 6 m/s
4) 7 m/s
5) 25 m/s



ConcepTest 7.2 KE and PE

You and your friend both solve a problem involving a skier going down a slope, starting from rest. The two of you have chosen different levels for $y = 0$ in this problem. Which of the following quantities will you and your friend agree on?

1) only B
2) only C
3) A, B and C
4) only A and C
5) only B and C

A) skier's PE
B) skier's change in PE
C) skier's final KE

The gravitational PE depends upon the reference level, but the *difference* DPE does not! The work done by gravity must be the same in the two solutions, so DPE and DKE should be the same.

Follow-up: Does anything change *physically* by the choice of $y = 0$?

ConcepTest 7.4 Elastic Potential Energy

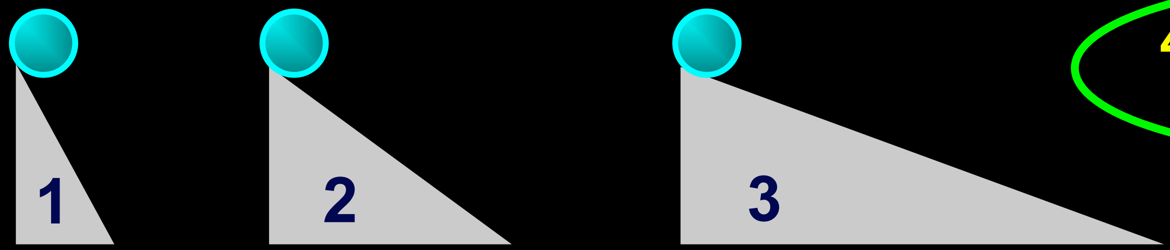
How does the work required to stretch a spring 2 cm compare with the work required to stretch it 1 cm?

1) **same amount of work**
2) **twice the work**
3) **4 times the work**
4) **8 times the work**

The elastic potential energy is $\frac{1}{2}kx^2$. So in the second case, the elastic PE is 4 times greater than in the first case. Thus, the work required to stretch the spring is also 4 times greater.

ConcepTest 7.6 Down the Hill

Three balls of equal mass start from rest and roll down different ramps. All ramps have the same height. Which ball has the greater speed at the bottom of its ramp?



4) same speed for all balls

All of the balls have the same initial gravitational PE, since they are all at the same height (PE = *mgh*). Thus, when they get to the bottom, they all have the same final KE, and hence the same speed (KE = 1/2 mv^2).

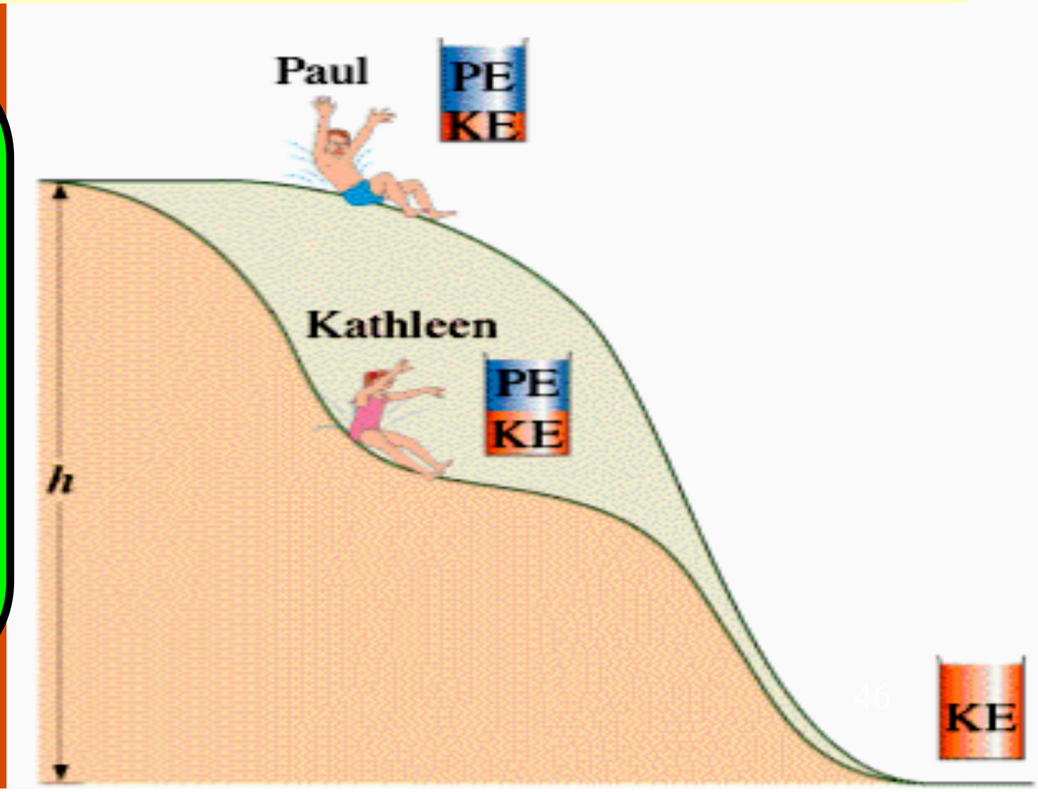
Follow-up: Which ball takes longer to get down the ramp?

ConcepTest 7.8b Water Slide II

Paul and Kathleen start from rest at the same time on frictionless water slides with different shapes. Who makes it to the bottom first?

1) Paul
2) Kathleen
3) both the same

Even though they both have the same *final velocity*, **Kathleen is at a lower height than Paul for most of her ride.** Thus she always has a ***larger velocity*** during her ride and therefore arrives earlier!



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A cart starting from rest rolls down a hill and at the bottom has a speed of 4 m/s. If the cart were given an initial push, so its initial speed at the top of the hill was 3 m/s, what would be its speed at the bottom?

1) 4 m/s
2) 5 m/s
3) 6 m/s
4) 7 m/s
5) 25 m/s

- When starting from rest, the cart's PE is changed into KE:
- DPE = DKE = $1/2\, m(4)^2$

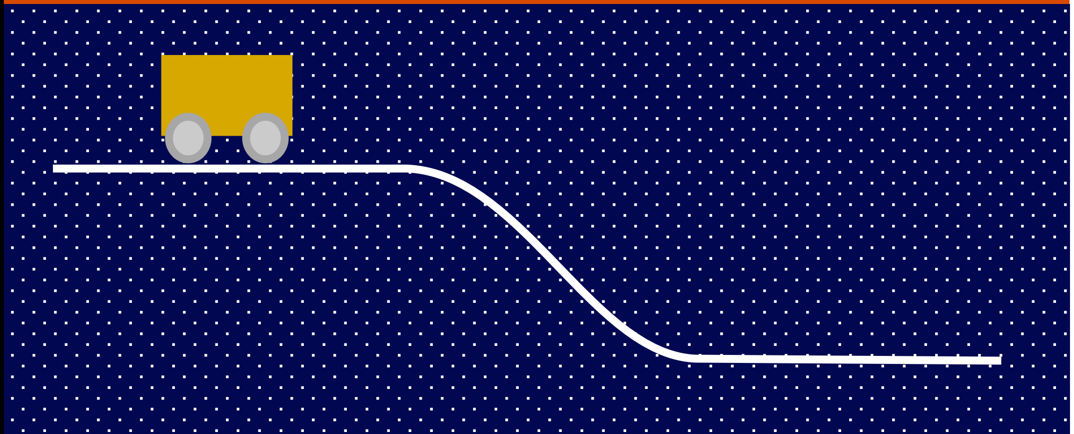
When starting from 3 m/s, the final KE is:

KE_f = KE_i + DKE

= $1/2\, m(3)^2 + 1/2\, m(4)^2$

= $1/2\, m(25)$

= $1/2\, m(5)^2$



Speed is not the same as kinetic energy