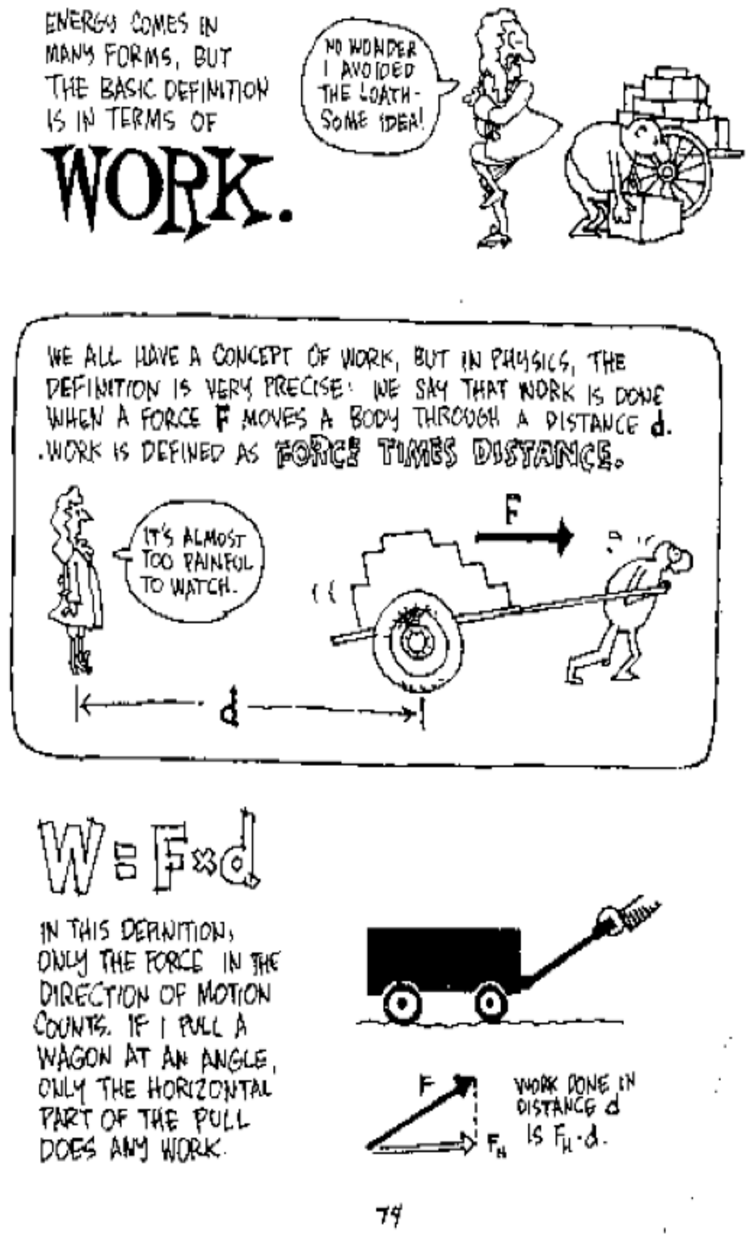
Lecture 4
Chapter 7
Work
and
Kinetic Energy

$$K = K_0 + W$$

$$K = \frac{1}{2}mv^2$$



Kinetic Energy

Energy is a quantity or characteristic of an object that is conserved or constant in a closed system. Kinetic energy is a particular form of energy that is related to an objects motion or speed.

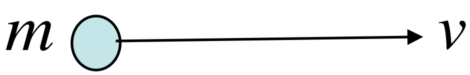
For a particle, the kinetic energy is defined as

$$K = \frac{1}{2}mv^2$$

The SI unit of energy is the joule.
1 joule = 1 kg m^2/s^2

Kinetic Energy

$$K = \frac{1}{2}mv^2$$



The 0.2 kg ball with speed v=10 m/s that hit Karyn was carrying

$$K = \frac{1}{2}(0.2kg)(10m/s)^2 = 10J$$

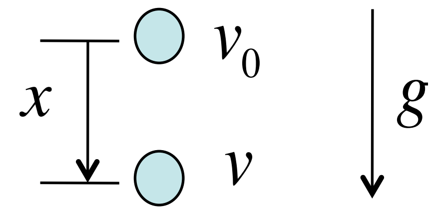
Kinetic energy is a scalar. It is not a vector. It has no components, even though you can write

$$v^2 = v_x^{\ 2} + v_y^{\ 2} + v_z^{\ 2} \qquad \vec{v} \bullet \vec{v} = \text{dot product}$$

Kinetic Energy -Work Theorem

$$K - K_0 = W$$

Recall we found the following relationship for a particle of mass m moving in a constant gravitational field.



$$v^2 = {v_0}^2 + 2gx$$

If we multiply both sides by $\frac{1}{2}m$ we get

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 + mgx \quad \text{and mg=F}$$

$$K = K_0 + Fx$$

where Fx is called W, the work done by the force acting on the particle.

$$K = K_0 + W$$

If the particle speed increases, the force does positive work.
If the particle speed decreases, the force does negative work.

WORK

$$W = Fx$$

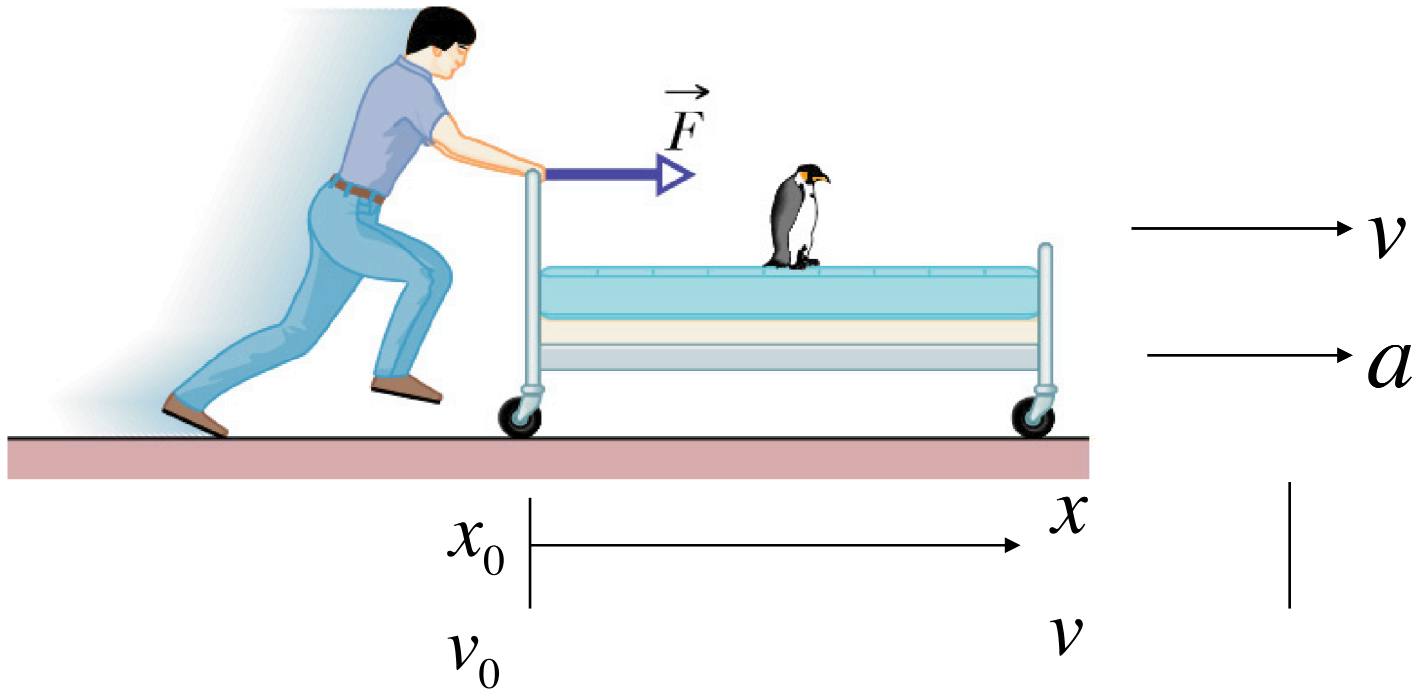
SI unit of work = Newton-meter = Joule

$$W = \Delta K = K - K_0$$

Net work done on the particle by the forces F in moving a distance x is the change in kinetic energy of the particle. This is true for any case with constant acceleration- not just gravity, which is force at a distance. It is also true for contact forces.

It is also true for a rigid body and not just a particle. By a rigid body we mean one that does not change shapes. Consider a man pushing a rigid cart along the floor.

Work - Kinetic Energy Theorem for any case with constant a



$v^2 = v_0^{\ 2} + 2ax$ is true for any constant acceleration. Also true for objects, if they are rigid and behave like particles.

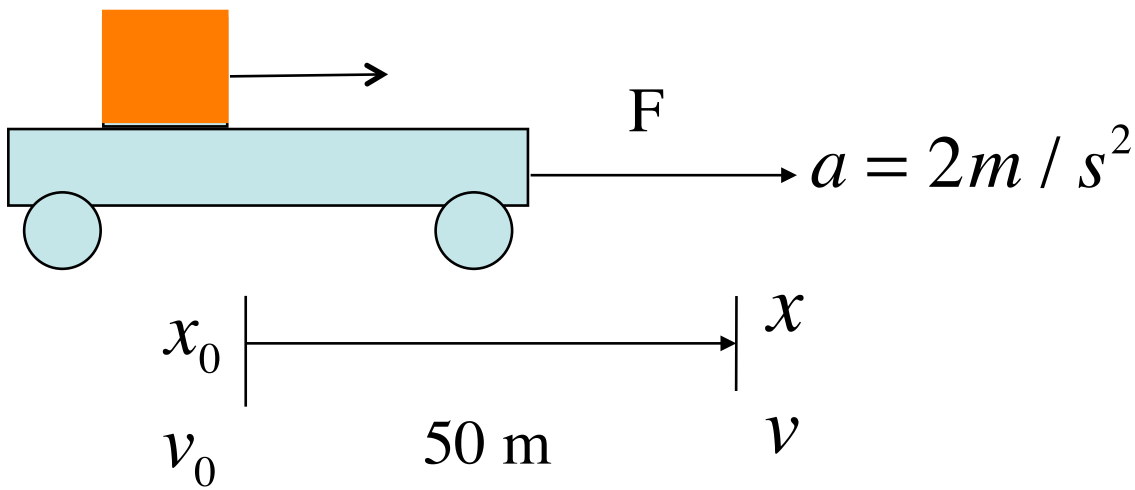
$$K - K_0 = Fx = W$$

Key: Point of application of the force is what is displaced

EXAMPLE

What is the work done on a 150 kg crate by a truck whose acceleration is 2 m/s^2 over a distance of 50 m. Draw the free body diagram of the crate first. Then find the force. Then find the work.

$$m = 150\,kg$$

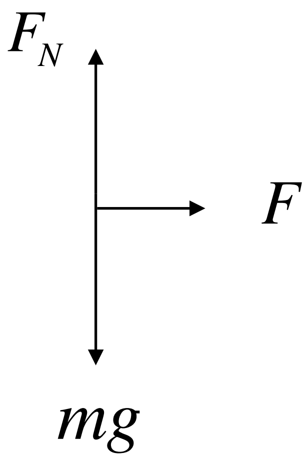


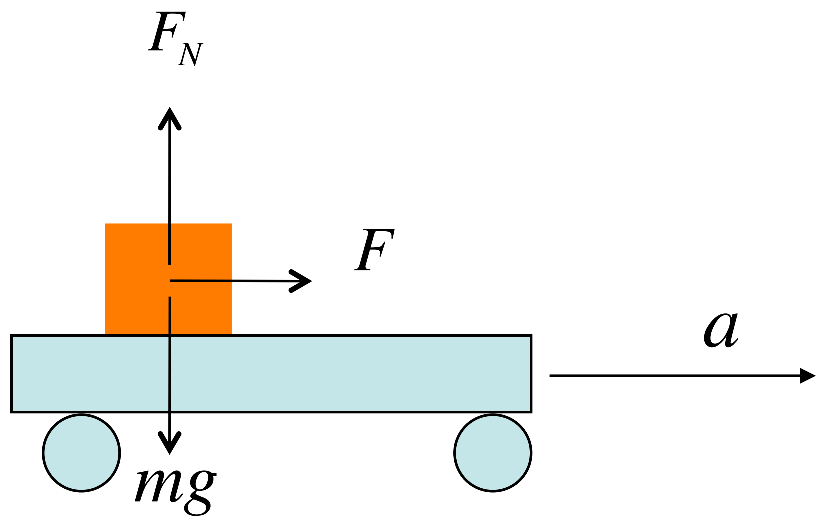
Work done by a TRUCK on a CRATE (Force comes from the truck and acts on the crate)

Key: Identify the two systems : Work done by a ….. on a ……

Key: Point of application of the force is what is displaced

Draw the free body diagram of the crate.





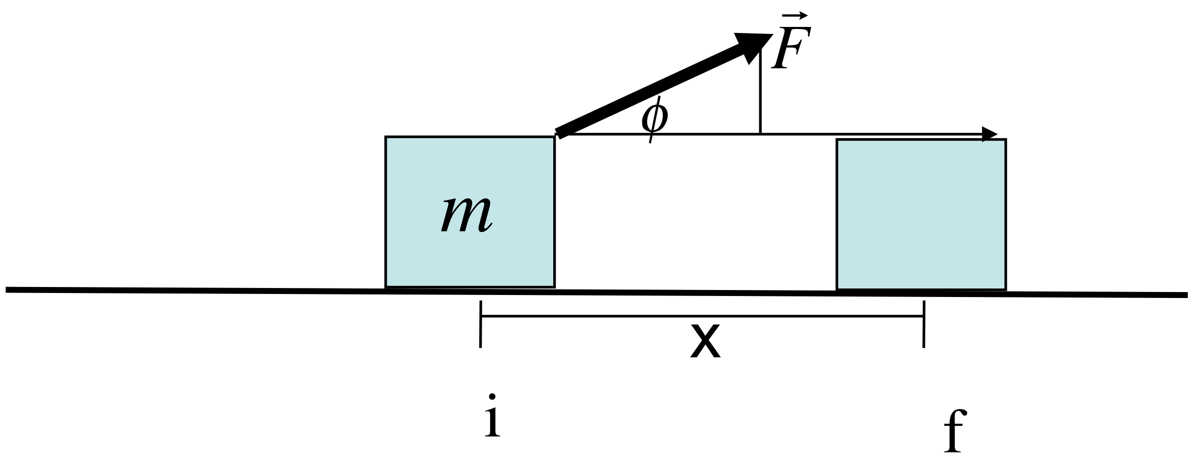
Find the force F

$$F = ma = (150kg)(2m/s^2) = 300N$$

Find the work.

$$W = Fx = (300N)(50m) = 15000J$$

How much work is done by a worker's force to pull a 50 kg crate a distance of 3 m across a frictionless horizontal floor with a force of 210 N 20 degrees above the horizontal?



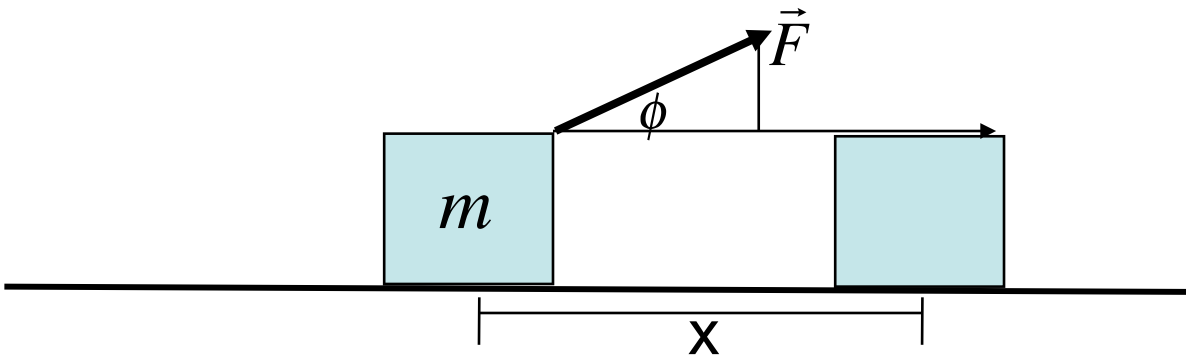
$$m = 50kg$$

$$x = 3m$$

$$F = 210N$$

$$\phi = 20^o$$

$$K_f - K_i = W$$



$m = 50kg$

$x = 3m$

$F = 210N$

$\phi = 20^o$

We want the component of the force along x

$$W = \vec{F} \bullet \vec{x} = F_x x$$

$$W = Fx\cos\phi = (210N)(3m)\cos(20^o) = 630(0.94) = 590J$$

$$K_f - K_i = W$$

What work is done by the gravitational force on the crate?

What work is done by the normal force on the crate?

Computing Work using unit vector notation

$$W = \vec{F} \bullet \vec{d} = F_x x + F_y y + F_z z$$

$$F = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

$$d = x\hat{i} + y\hat{j} + z\hat{k}$$

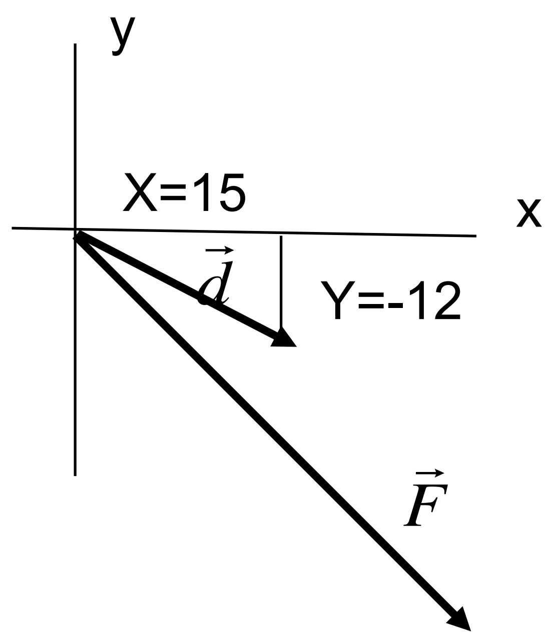
In 2 dimensions you would just have x and y components : Example 6E on pg 136 ed 6 or prob. 8 in ed 7

$$\vec{d} = 15\hat{i} - 12\hat{j}$$

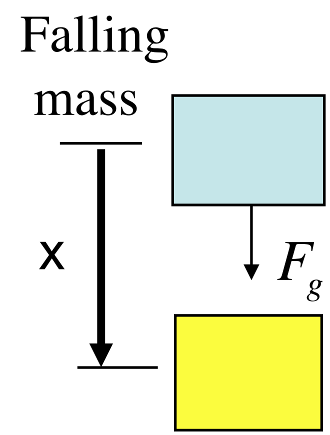
$$\vec{F} = 210\hat{i} - 150\hat{j}$$

$$W = (15)(210) + (-12)(-150)$$

$$= 3150 + 1800 = 4950J$$



What is the work done by gravity on a 1000 kg mass falling10 m starting from rest? What is the final kinetic energy?



$$W = \vec{F} \bullet \vec{x} = Mgx\cos\phi = Mgx\cos(0) = Mgx$$

$$W = (1000kg)(10m/s^2)(1m) = 10000J$$

f=0

We say the force of gravity did positive work on the falling block

$$K_f - K \ = W$$

$$K_i = 0$$

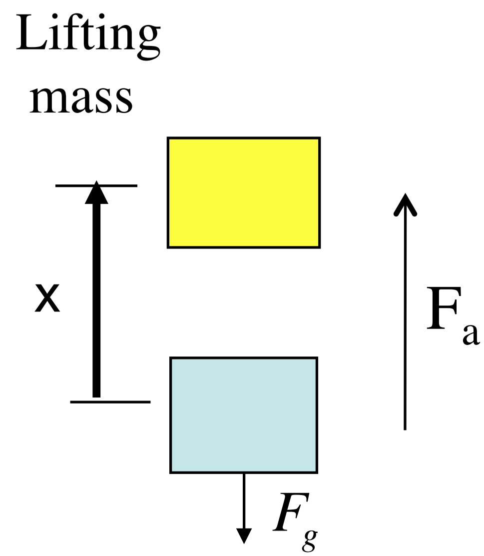
$$K_f = 10000J$$

There is also negative work.

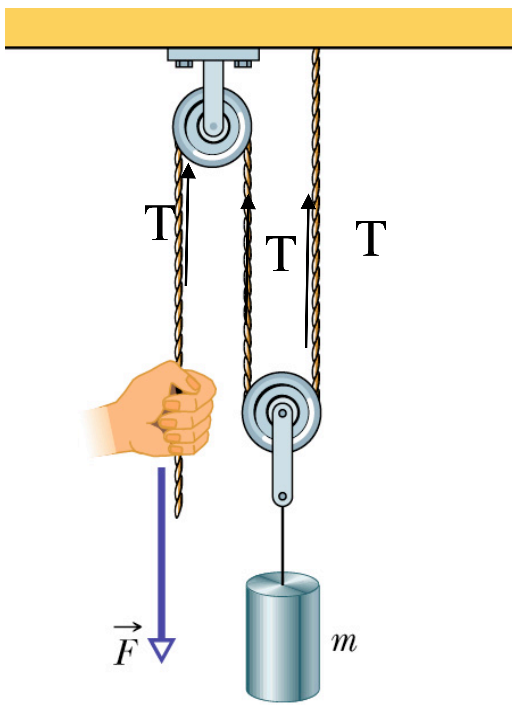
What is the work done by gravity when an applied force lifts a 1000 kg mass? (Be careful here)

$$W_g = \vec{F} \bullet \vec{x} = Mgx\cos\phi = Mgx\cos(180) = -Mgx$$

$$W_g = -(1000kg)(10m/s^2)(1m) = -10000J$$



The work done by the applied force in lifting the mass is W_a. If the mass is at rest initially and finally or the speed is the the same then, $W_a+W_g=\Delta K=0$ or $W_a=-W_g$



20 kg

a) What force F is needed to lift mass of 20 kg at constant speed? No acceleration

At hand

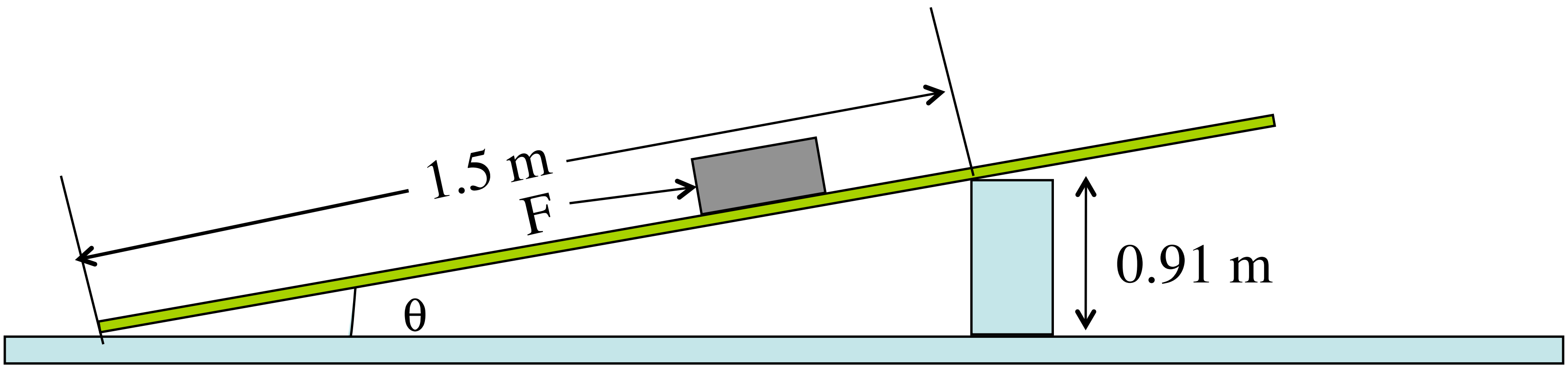
$$\sum F_y = 0 = T - F$$

At Pulley

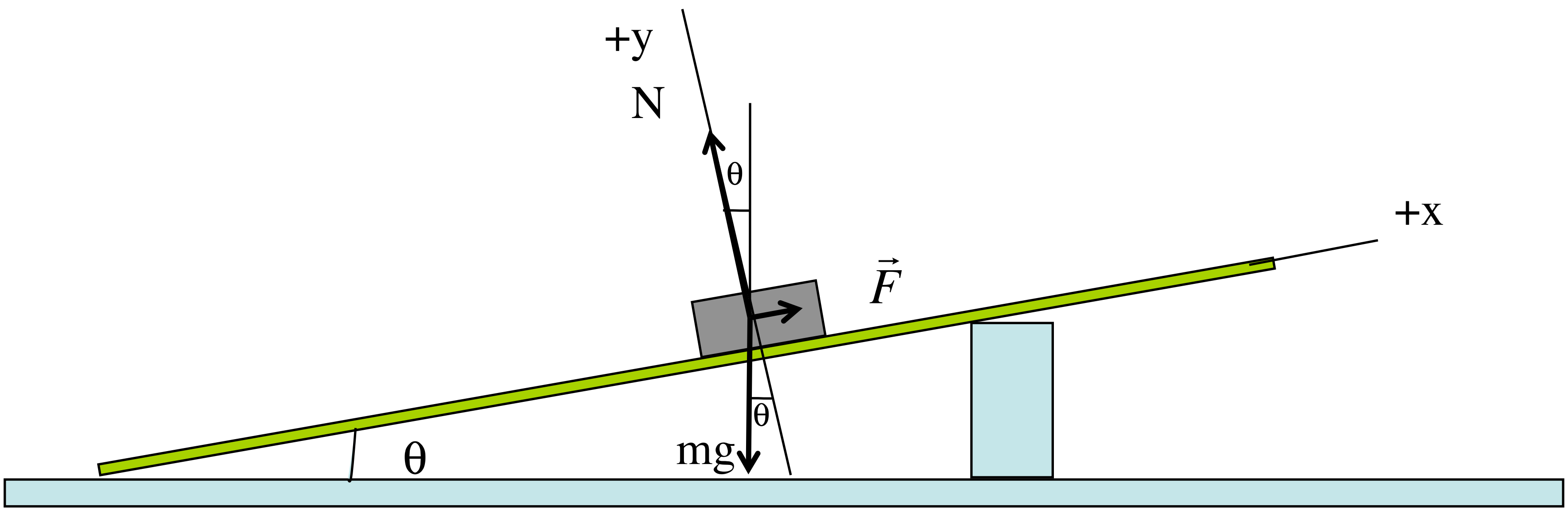
$$\sum F_y = 0 = 2T - mg$$

$$T = \frac{mg}{2} = \frac{(20kg)(9.8)}{2} = 98N$$

7-16E A worker pushes parallel to the incline plane on a block of ice just hard enough so that it slides down at constant speed. The mass of the block is m = 45 kg. The height of the support is 0.91 m



(a) Draw the free body diagram for the block

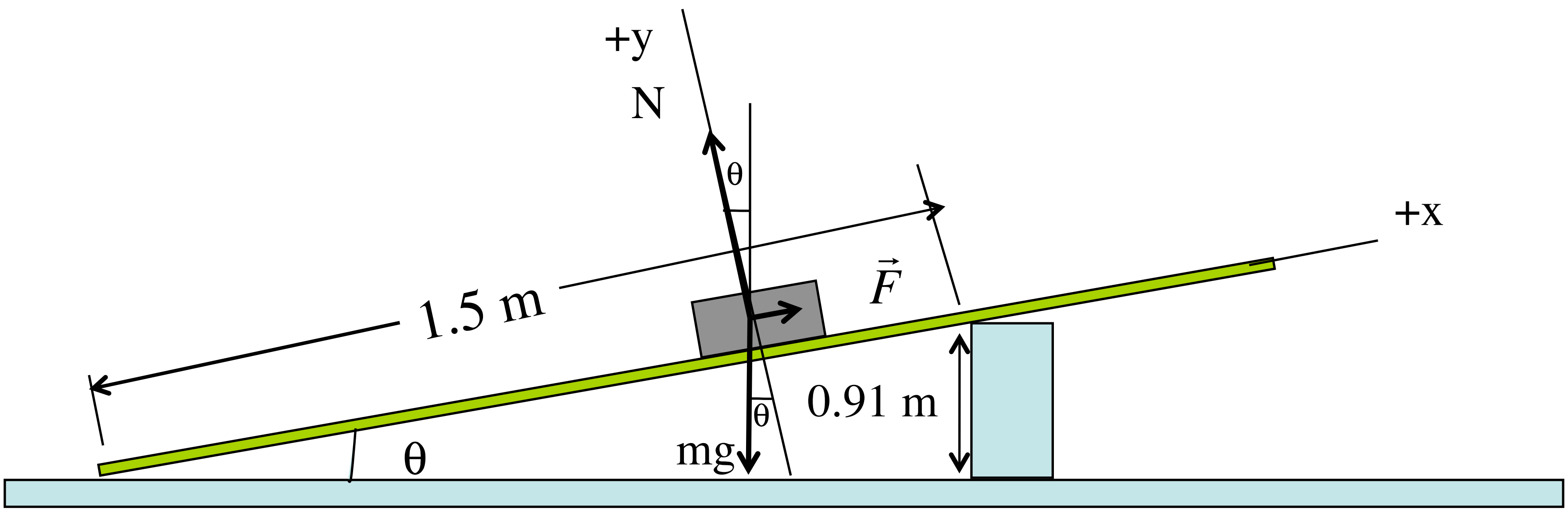


(b) How much work is done on the block by the worker's force F when it slides 1.5 m down the inclined plane?

Sum of the x-components of the forces = 0   Why?

$$\sum F_x = 0 = F - mg\sin\theta$$

$$W = F(-x) = -mgx\sin\theta = -(45)(9.8)(1.5)(.91/1.5) = -401J$$



Continuation

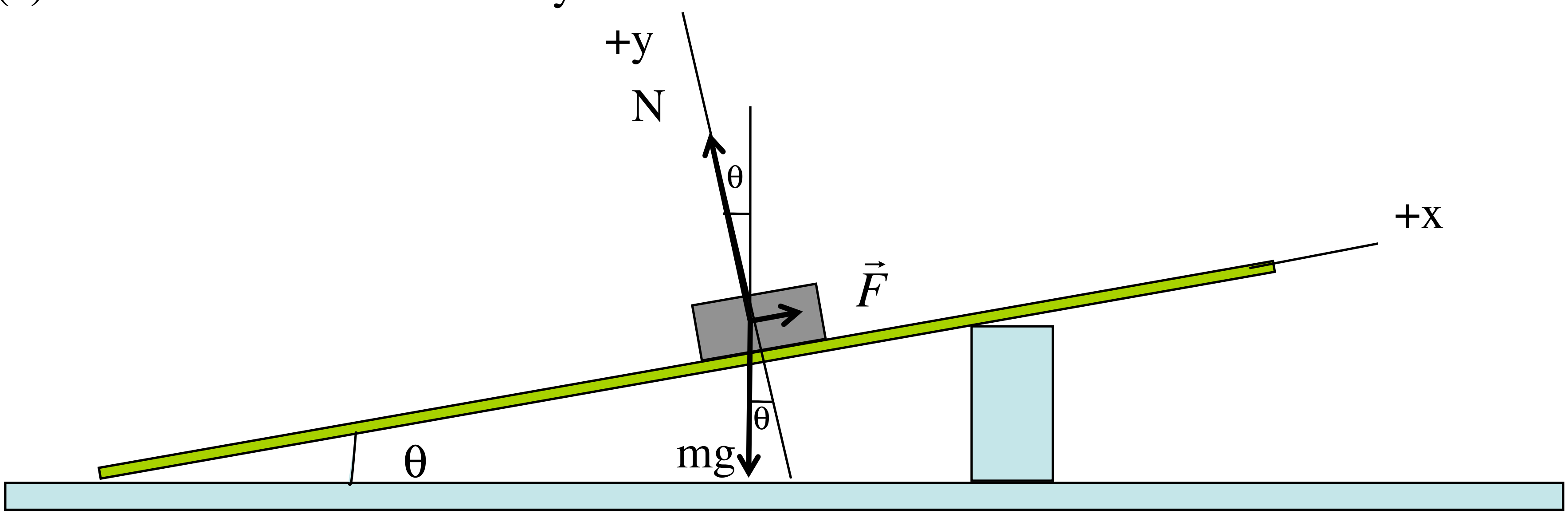
(c) How much work is done on the block by the gravitational force F when it slides down the inclined plane?

$$\sum F_x = 0 = F - mg\sin\theta$$

$$W = F_g(-x) = -mg(-x)\sin\theta = (45)(9.8)(1.5)(.91/1.5) = 401J$$

(d) by the normal force? 0 because cos(90) =0 $W = F \bullet d = 0$

(e) How much work is done by the net force on the block? 0 because a = 0

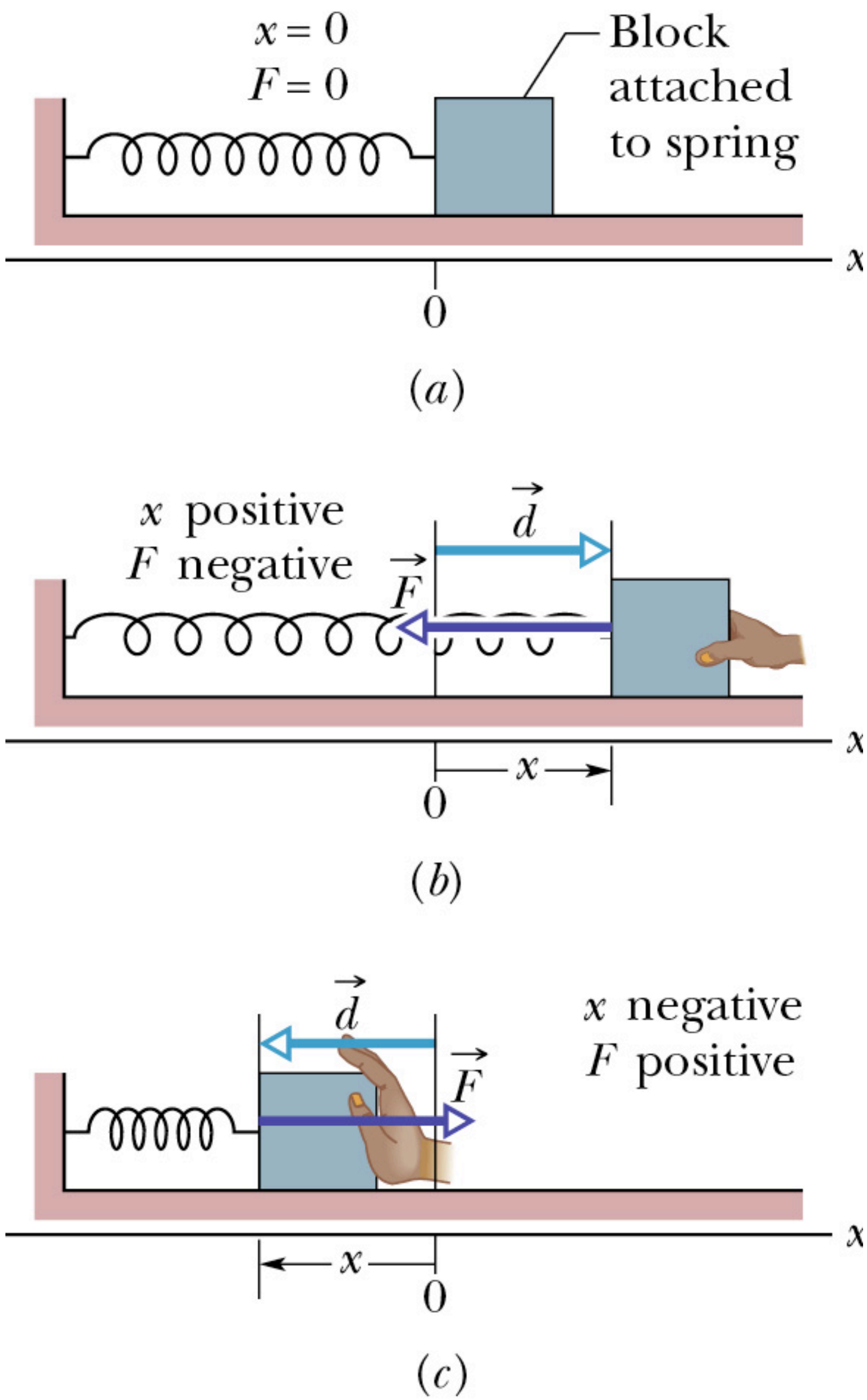


Does the force of friction do work?

$$K_f - K_i = W$$

Now look at a force that is not constant. Does a spring force do any work on a mass?

Consider the force $F = -kx$.
known as Hook's Law.



Find the Work when an applied force elongates or compresses the spring by acting on the mass

$$W = \int_0^d F\,dx = \int_0^d -kx\,dx$$

$$= -\tfrac{1}{2}kx^2\Big|_0^d = -\tfrac{1}{2}k(d^2 - 0)$$

$$W = -\tfrac{1}{2}kd^2$$

Or between any two arbitrary positions

$$W = \int_{x_i}^{x_f} F\,dx = \int_{x_i}^{x_f} -kx\,dx$$

$$= -\tfrac{1}{2}k\,x^2\Big|_{x_i}^{x_f} = -\tfrac{1}{2}k(x_f^2 - x_i^2) = \tfrac{1}{2}k(x_i^2 - x_f^2)$$

Where x is measured from equilibrium position

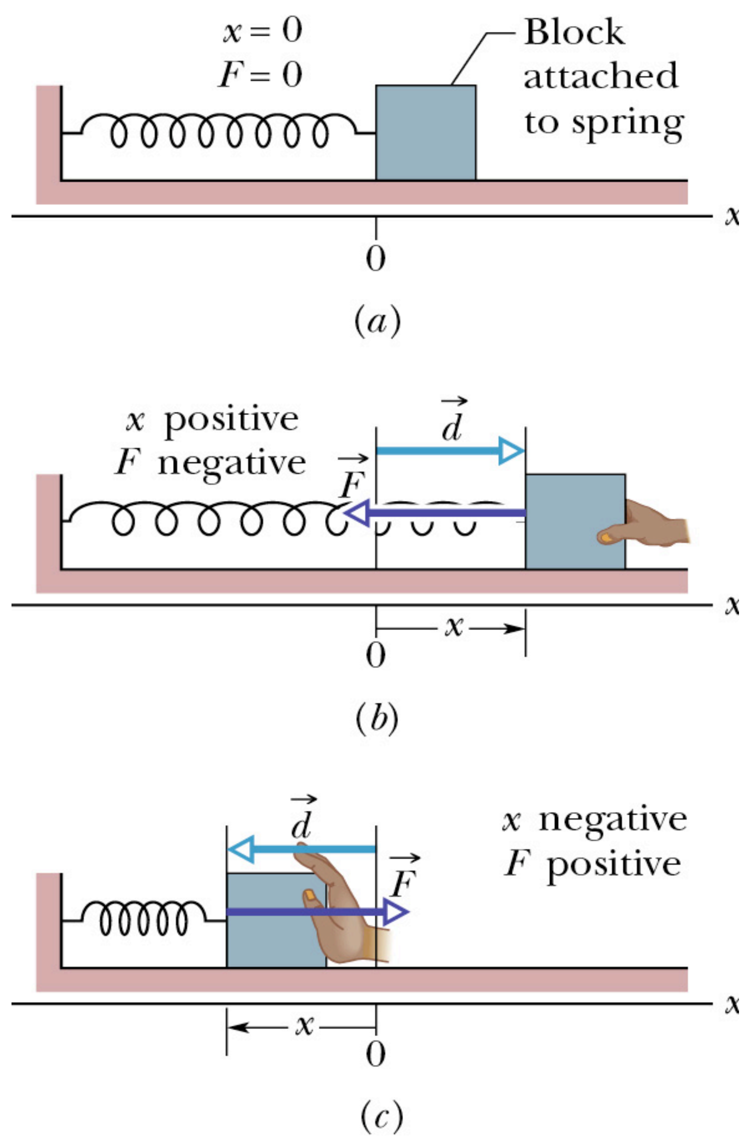
What is work done by me on the spring

W_a = work done by me pushing in the block

$$W_a = -W_s$$

$$\Delta K = K_f - K_i = W_s + W_a$$

$$\Delta K = 0$$



Measuring k for spring

Example of spring block system with friction

Work-Energy Theorem for a Variable Force

$$W = \sum_i F_i \Delta x_i$$

$$W = \int_{x_i}^{x_f} F(x)dx$$

$$W = \int_{\mathrm{x}_\mathrm{i}}^{\mathrm{x}_\mathrm{f}} ma\, dx$$

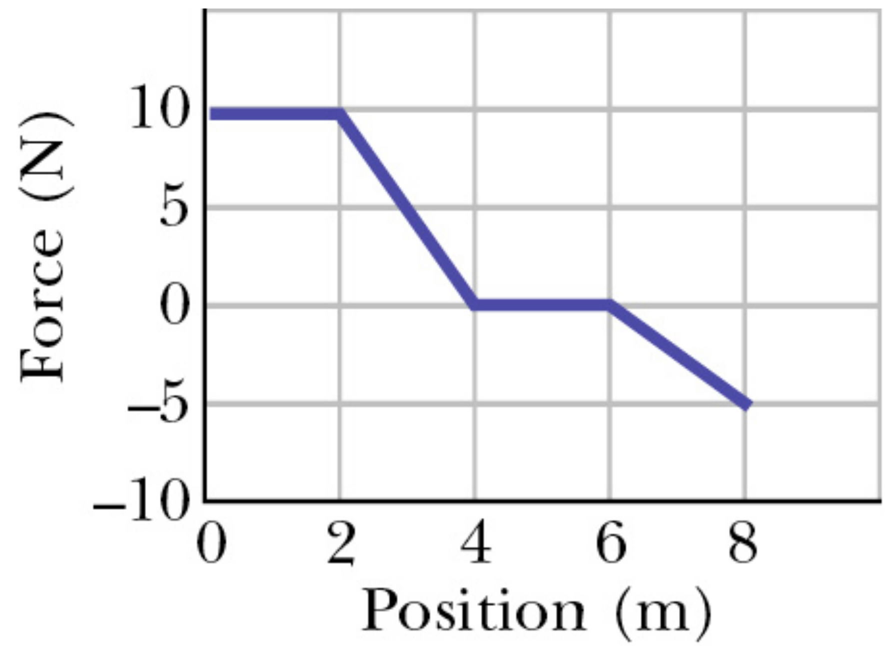
$$ma\, dx = m\frac{dv}{dt}dx = m\frac{dx}{dt}dv = mvdv$$

$$W = \int_{v_i}^{v_f} mvdv = 1/2mv^2\Big|_{v_i}^{v_f} = \tfrac{1}{2}mv_f{}^2 - \tfrac{1}{2}mv_i^2$$

$$W = K_f - K_i$$

Problem 24 ed 6 prob. 33 ed. 7

A 5.0 kg block moves in a straight line along a frictionless surface under the influence of the force shown in the figure. How much work is done by the force on the block in moving from the origin to x =8.0 m? If it starts from rest, what is its speed at x=8 m?



W = Force x Distance

W=(10) (2) + 1/2(2)(10) + 0 + 1/2(2)(-5) Nm

W=20+10-5=25 Nm=25 J

$$\Delta K = W = 25$$

$$K_f = 25$$

$$\frac{1}{2}mv^2 = 25 \rightarrow v = \sqrt{50 / m} \rightarrow \sqrt{50 / 5} \rightarrow \sqrt{10}$$

$$v = 3.16\ m / s$$

Power

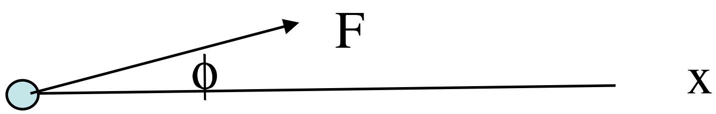
Power is the rate at which work is done.

Average Power = $\frac{W}{\Delta t}$

Instantaneous Power = $P = \frac{dW}{dt}$

The SI unit of power is joule/s or watt 1 kilowatt -hour = 3.60 MJ

Consider a particle moving along the x axis acted upon by a constant force at an angle ϕ with respect to the x axis



$$P = \frac{dW}{dt} = F\cos\theta\frac{dx}{dt} = Fv\cos\theta$$

$$P = F \bullet v$$

Sample Power Problem

ConcepTest 6.2a Friction and Work I

A box is being pulled across a rough floor at a constant speed. What can you say about the work done by friction?

1) **friction does no work at all**
2) **friction does negative work**
3) **friction does positive work**

ConcepTest 6.2d Tension and Work

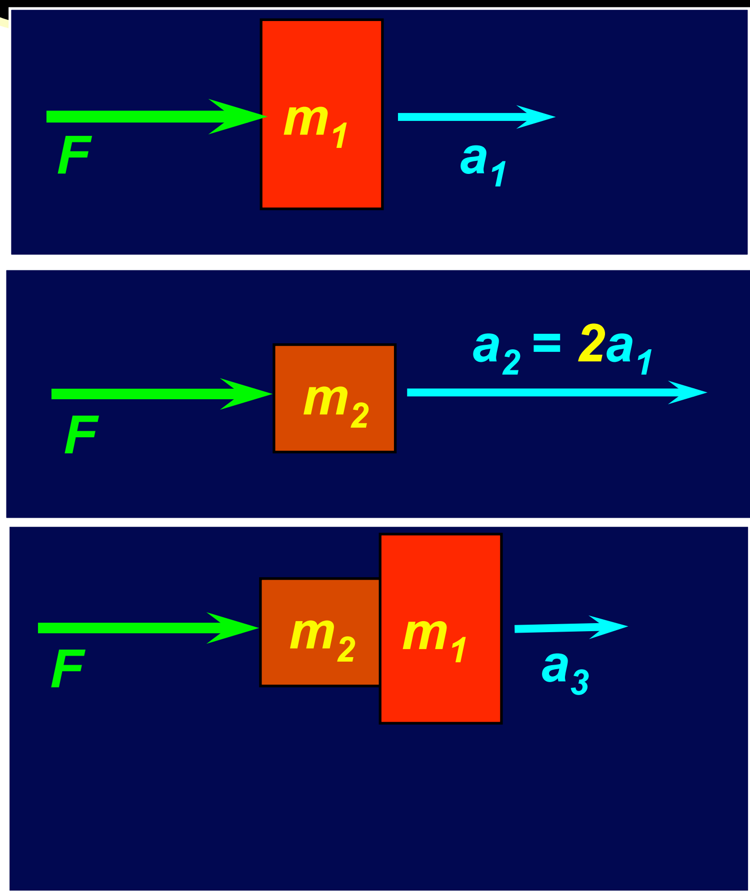
A ball tied to a string is being whirled around in a circle. What can you say about the work done by tension?

1) tension does no work at all
2) tension does negative work
3) tension does positive work

ConcepTest 4.6 Force and Two Masses

A force F acts on mass m_1 giving acceleration a_1. The same force acts on a different mass m_2 giving acceleration $a_2 = 2a_1$. If m_1 and m_2 are glued together and the same force F acts on this combination, what is the resulting acceleration?

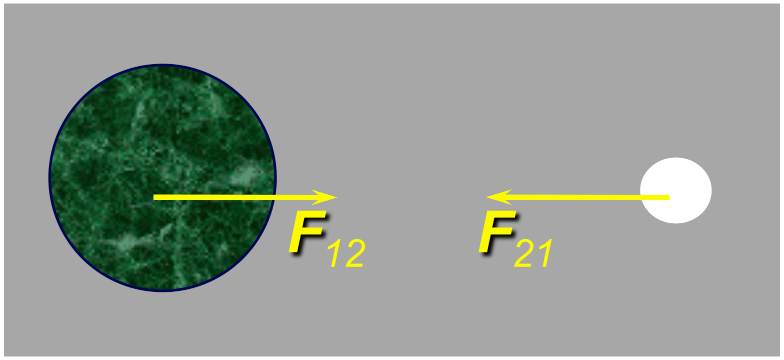
1) $3/4\ a_1$
2) $3/2\ a_1$
3) $1/2\ a_1$
4) $4/3\ a_1$
5) $2/3\ a_1$



ConcepTest 4.8a Bowling vs. Ping-Pong I

In outer space, a **bowling ball** and a **ping-pong ball** attract each other due to gravitational forces. How do the magnitudes of these attractive forces compare?

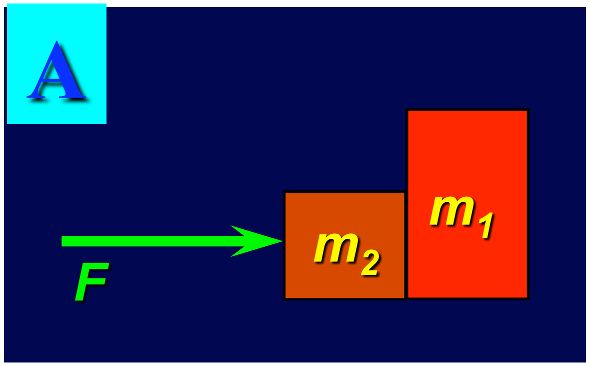
1) The bowling ball exerts a greater force on the ping-pong ball
2) The ping-pong ball exerts a greater force on the bowling ball
3) The forces are equal
4) The forces are zero because they cancel out
5) There are actually no forces at all

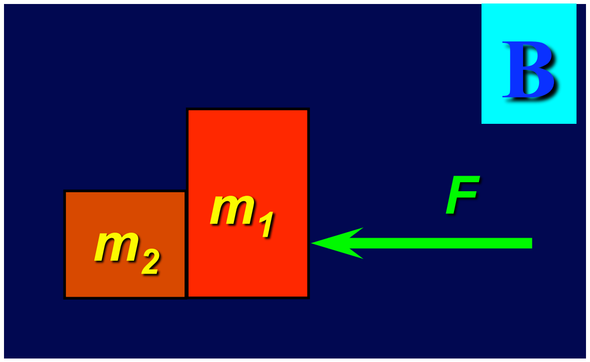


ConcepTest 4.10a Contact Force I

If you push with force F on either the heavy box (m_1) or the light box (m_2), in which of the two cases below is the contact force between the two boxes larger?

1) case A
2) case B
3) same in both cases





ConcepTest 5.3b Tension II

Two tug-of-war opponents each pull with a force of *100 N* on opposite ends of a rope. What is the tension in the rope?

1) ***0 N***
2) ***50 N***
3) ***100 N***
4) ***150 N***
5) ***200 N***

ConcepTest 5.3b Tension II

Two tug-of-war opponents each pull with a force of *100 N* on opposite ends of a rope. What is the tension in the rope?

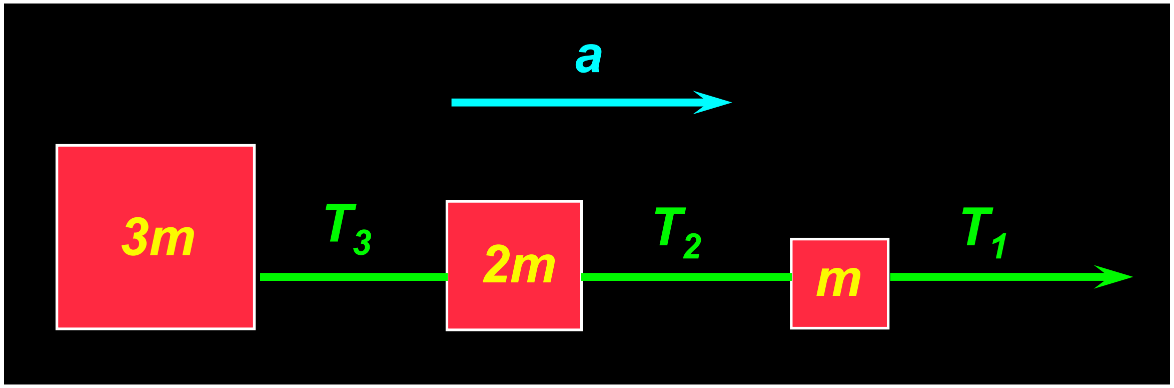
1) *0 N*
2) *50 N*
3) *100 N*
4) *150 N*
5) *200 N*

This is literally the identical situation to the previous question. The tension is not 200 N !! Whether the other end of the rope is pulled by a person, or pulled by a tree, the tension in the rope is still 100 N !!

ConcepTest 5.4 Three Blocks

Three blocks of mass *3m*, *2m*, and *m* are connected by strings and pulled with constant acceleration *a*. What is the relationship between the tension in each of the strings?

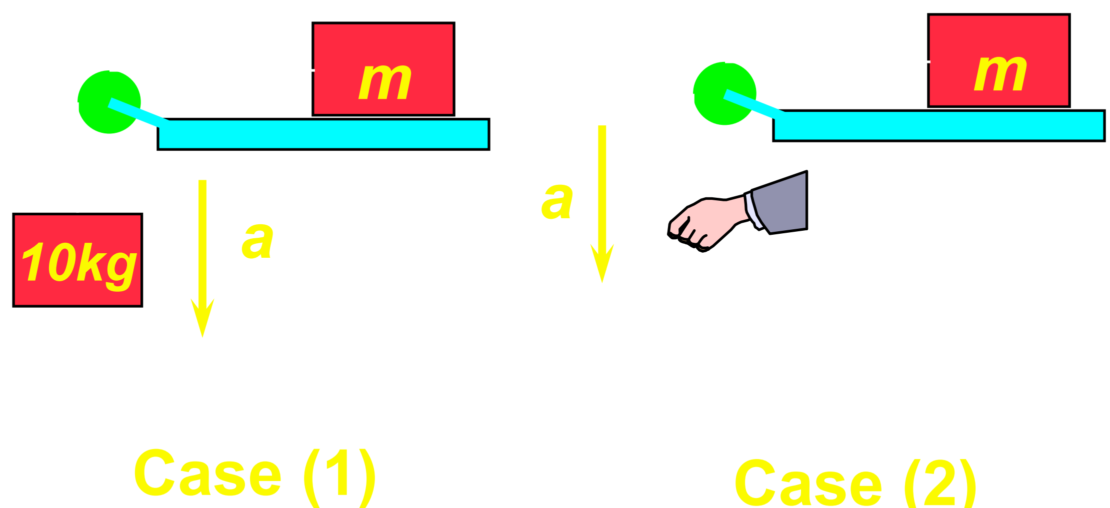
1) $T_1 > T_2 > T_3$
2) $T_1 < T_2 < T_3$
3) $T_1 = T_2 = T_3$
4) *all tensions are zero*
5) *tensions are random*



ConcepTest 5.5 Over the Edge

In which case does block *m* experience a larger acceleration? In (1) there is a *10 kg* mass hanging from a rope and falling. In (2) a hand is providing a constant downward force of *98 N*. Assume massless ropes.

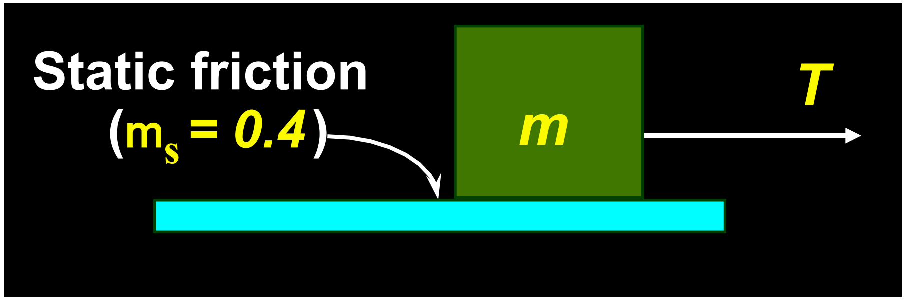
1) *case 1*
2) *acceleration is zero*
3) *both cases are the same*
4) *depends on value of m*
5) *case 2*



ConcepTest 5.12 Will it Budge?

A box of weight 100 N is at rest on a floor where m_s = *0.5.* A rope is attached to the box and pulled horizontally with tension *T = 30 N*. Which way does the box move?

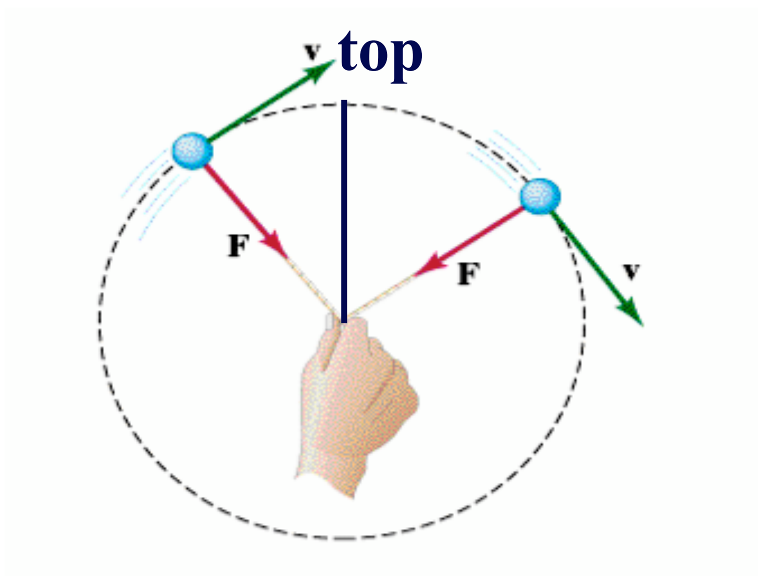
1) **moves to the left**
2) **moves to the right**
3) **moves up**
4) **moves down**
5) **the box does not move**



ConcepTest 5.19c Going in Circles III

You swing a ball at the end of string in a vertical circle. Since the ball is in circular motion there has to be a *centripetal force.* At the top of the ball's path, what is F_c equal to?

1) $F_c = T - mg$
2) $F_c = T + N - mg$
3) $F_c = T + mg$
4) $F_c = T$
5) $F_c = mg$

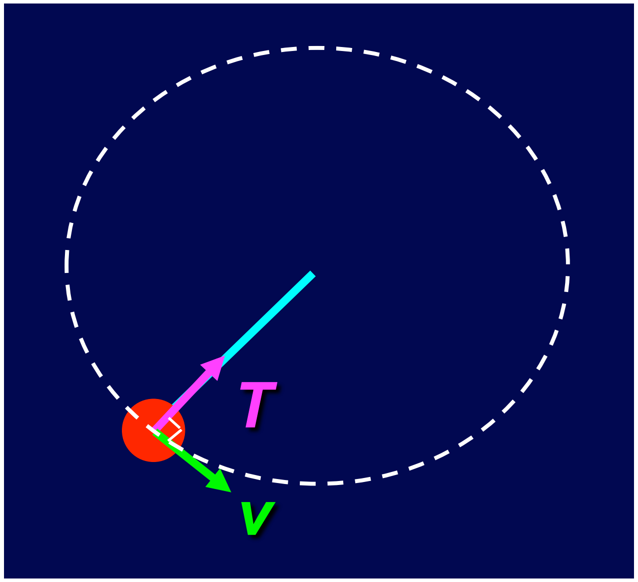


ConcepTest 6.2d Tension and Work

A ball tied to a string is being whirled around in a circle. What can you say about the work done by tension?

1) tension does no work at all
2) tension does negative work
3) tension does positive work

No work is done because the force acts in a **perpendicular** direction to the displacement. Or using the definition of work: $W = F\,d \cos q$ since $= 90^{\circ}$, then $W = 0$



Follow-up: Is there a force in the direction of the velocity?

ConcepTest 6.5b Kinetic Energy II

Car #1 has twice the mass of car #2, but they both have the same kinetic energy. How do their speeds compare?

1) $2\,v_1 = v_2$
2) $\sqrt{2}\,v_1 = v_2$
3) $4\,v_1 = v_2$
4) $v_1 = v_2$
5) $8\,v_1 = v_2$

ConcepTest 6.5b Kinetic Energy II

Car #1 has twice the mass of car #2, but they both have the same kinetic energy. How do their speeds compare?

1) $2 v_1 = v_2$
2) $\sqrt{2} v_1 = v_2$
3) $4 v_1 = v_2$
4) $v_1 = v_2$
5) $8 v_1 = v_2$

Since the kinetic energy is $\frac{1}{2} mv^2$, and the mass of car #1 is greater, then car #2 must be moving faster. If the ratio of m_1/m_2 is 2, then the ratio of v^2 values must also be 2. This means that the ratio of v_2/v_1 must be the square root of 2.

ConcepTest 6.7 Work and KE

A child on a skateboard is moving at a speed of 2 m/s. After a force acts on the child, her speed is 3 m/s. What can you say about the work done by the external force on the child?

1) positive work was done
2) negative work was done
3) zero work was done

ConcepTest 6.7 Work and KE

A child on a skateboard is moving at a speed of 2 m/s. After a force acts on the child, her speed is 3 m/s. What can you say about the work done by the external force on the child?

1) positive work was done
2) negative work was done
3) zero work was done

The kinetic energy of the child increased because her speed increased. This increase in KE was the result of positive work being done. Or, from the definition of work, since $W = \Delta KE = KE_f - KE_i$ and we know that $KE_f > KE_i$ in this case, then the work W must be positive.

Follow-up: What does it mean for negative work to be done on the child?

ConcepTest 6.11c Power

Engine #1 produces twice the power of engine #2. Can we conclude that engine #1 does twice as much work as engine #2?

1) yes
2) no

ConcepTest 6.11c Power

Engine #1 produces twice the power of engine #2. Can we conclude that engine #1 does twice as much work as engine #2?

1) yes

2) no

No!! We cannot conclude anything about how much work each engine does. Given the power output, the work will depend upon how much time is used. For example, engine #1 may do the same amount of work as engine #2, but in half the time.