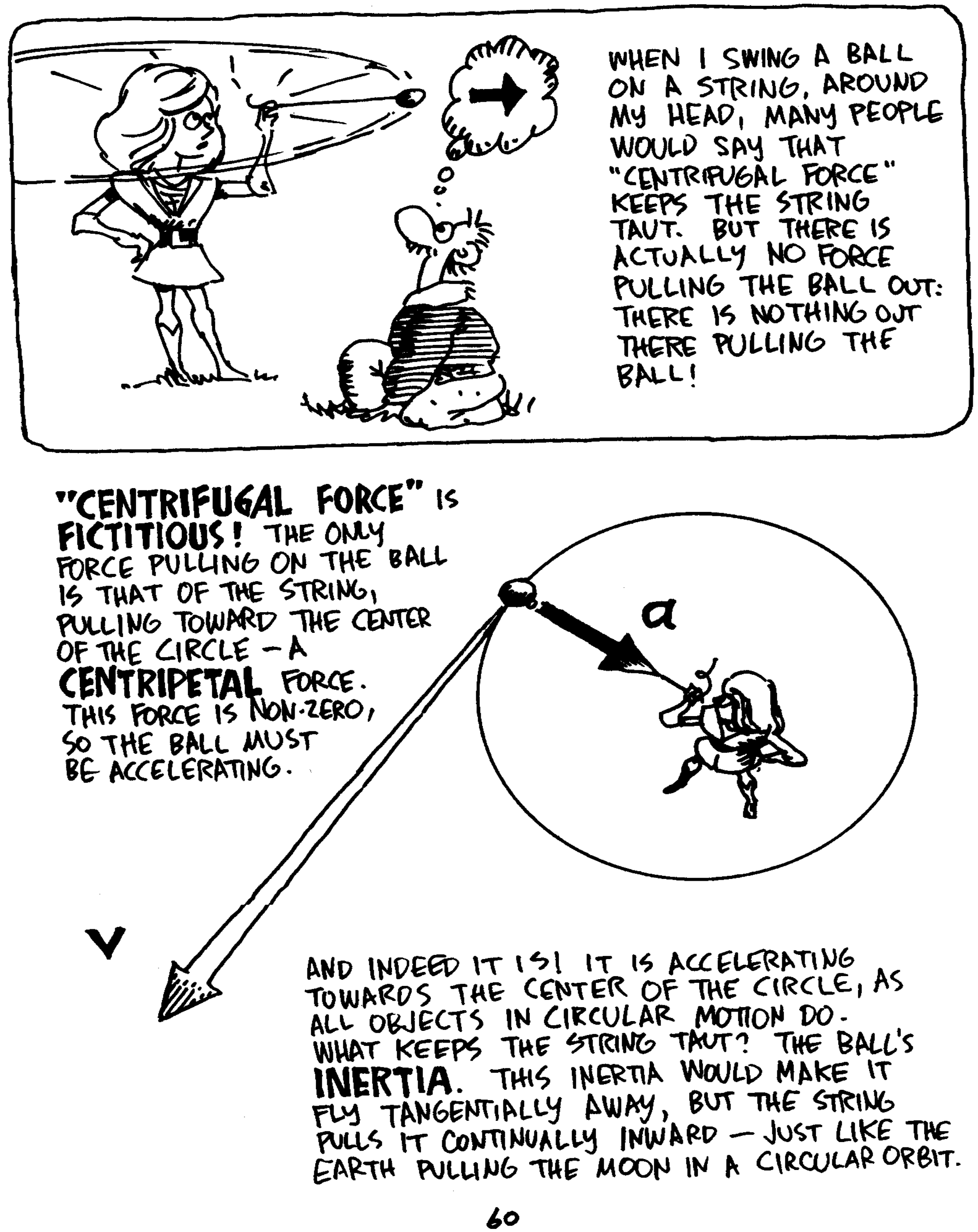
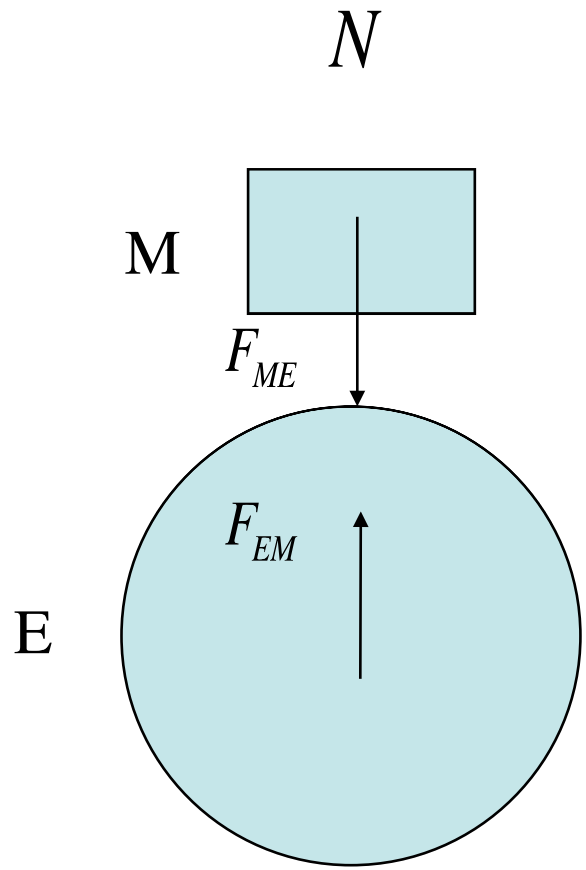
Lecture 3
Chapter 6
Force and Motion II

End of Lecture 2
Friction
More Circular motion
Apparent Weight
Drag Forces
Numerical integration
Misconceptions



NEWTON'S THIRD LAW

When two bodies interact, the forces on the bodies due to each other are always equal in magnitude and opposite in direction.



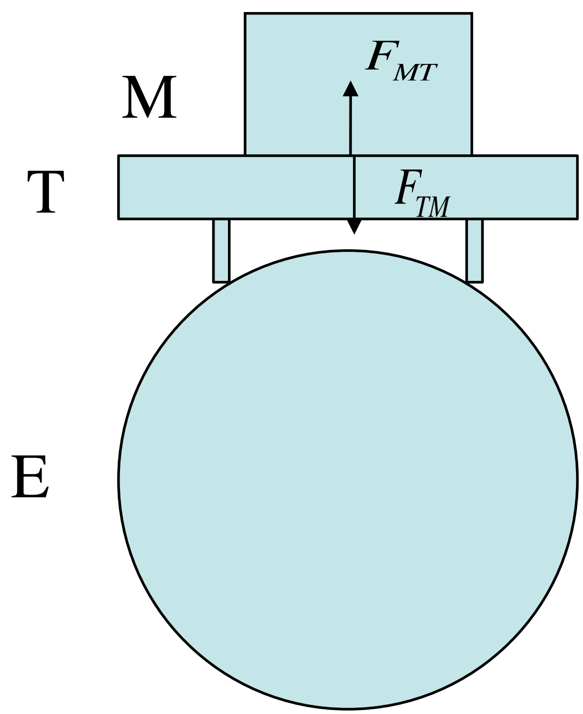
$$\vec{F}_{ME} = -\vec{F}_{EM}$$

Gravitational Interaction

NEWTON'S THIRD LAW

When two bodies interact, the forces on the bodies due to each other are always equal in magnitude and opposite in direction.

N



$$\vec{F}_{MT} = -\vec{F}_{TM}$$

Contact Interaction

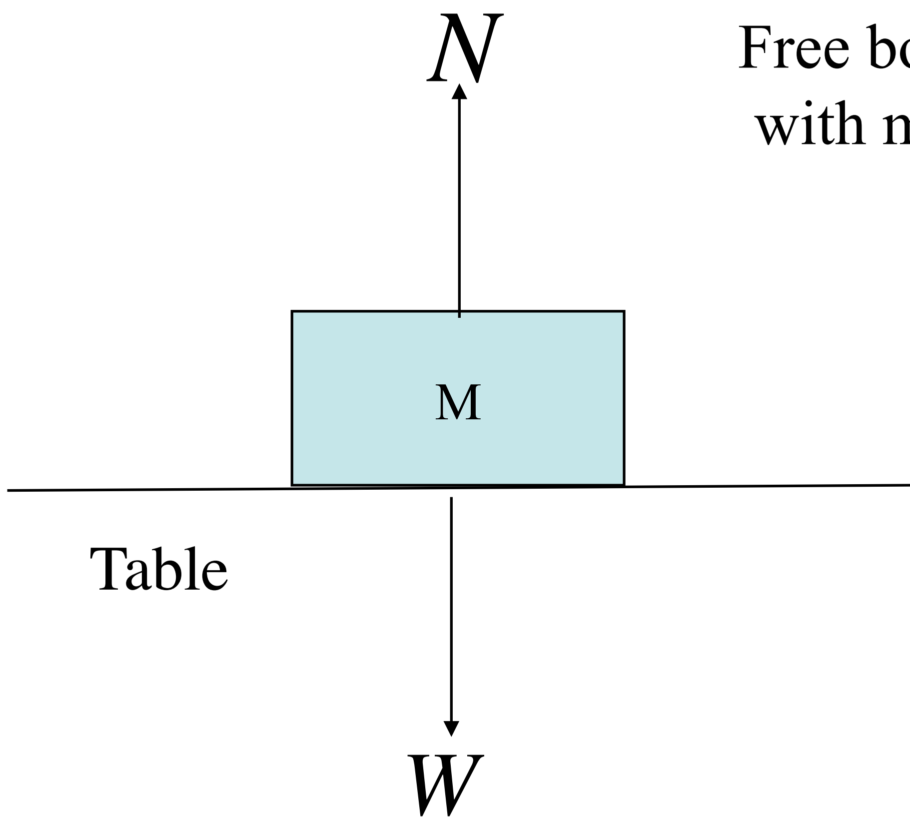
Rules for drawing free body diagrams. Isolates the forces acting on one body

1) Represent the body by a point.
2) Each force acting on the body is represented by a vector with tail at the point and the length of vector indicating the approximate magnitude of the force.
3) A coordinate system is optional.
4) If the situation consist of several bodies which are rigidly connected, you can still represent all the bodies by a point and use the total mass. Internal forces are not included.

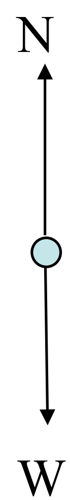
What is the free body diagram of the block at rest on the table?

$$F_{MT} = N$$

$$F_{ME} = W$$



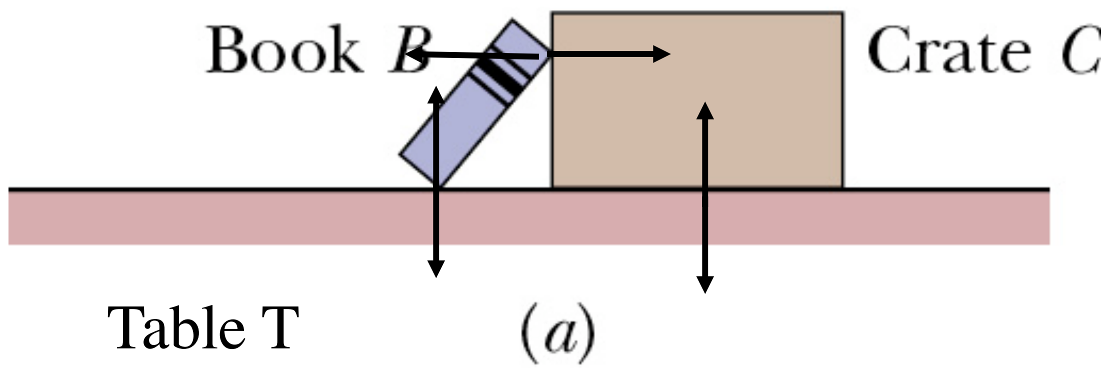
Free body diagram of object with mass M



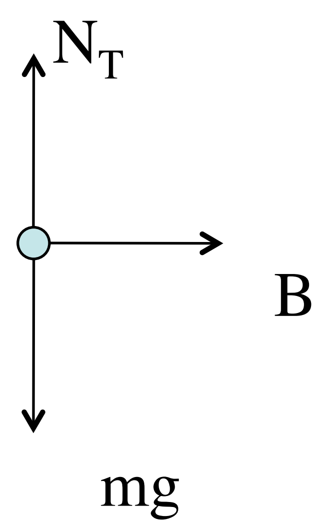
$$\vec{N} = -\vec{W}$$

$$W = Mg$$

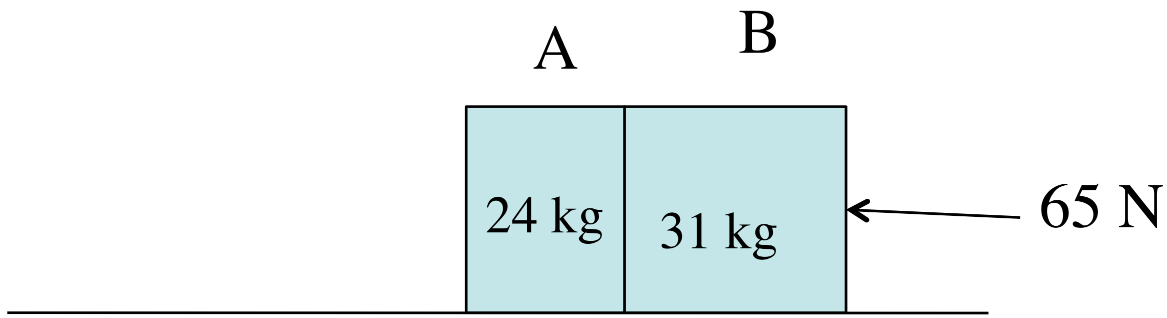
Book leaning against a crate on a table at rest. What are the action –reaction pairs?

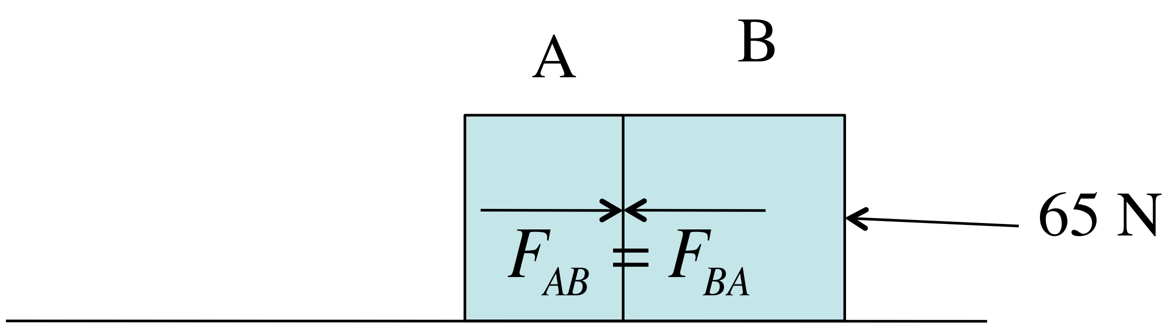


1) Draw a free body diagram of the forces acting on the crate

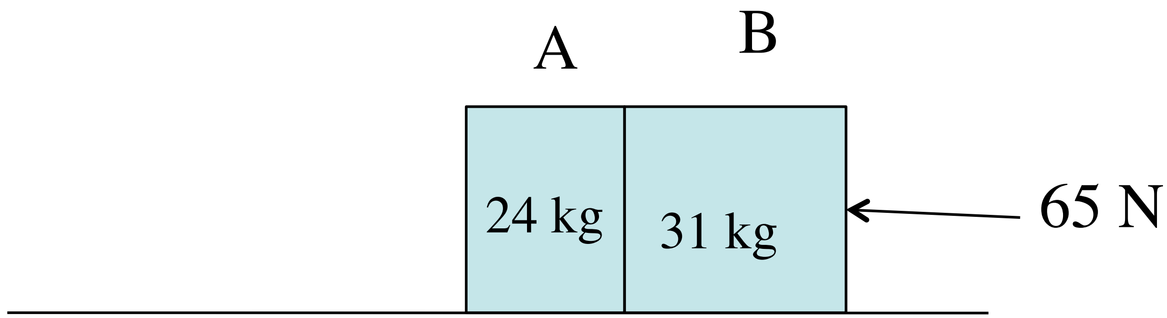


Problem: What is the acceleration of the system of the two blocks and the contact force between the blocks? What is the net force on Block B?



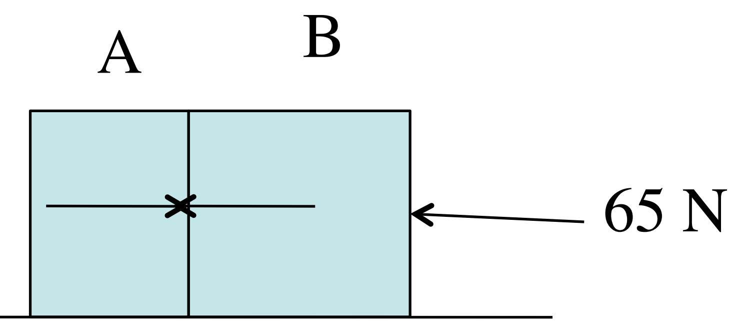


Problem: What is the acceleration of the system of the two blocks and the contact force between the blocks? What is the net force on Block B?



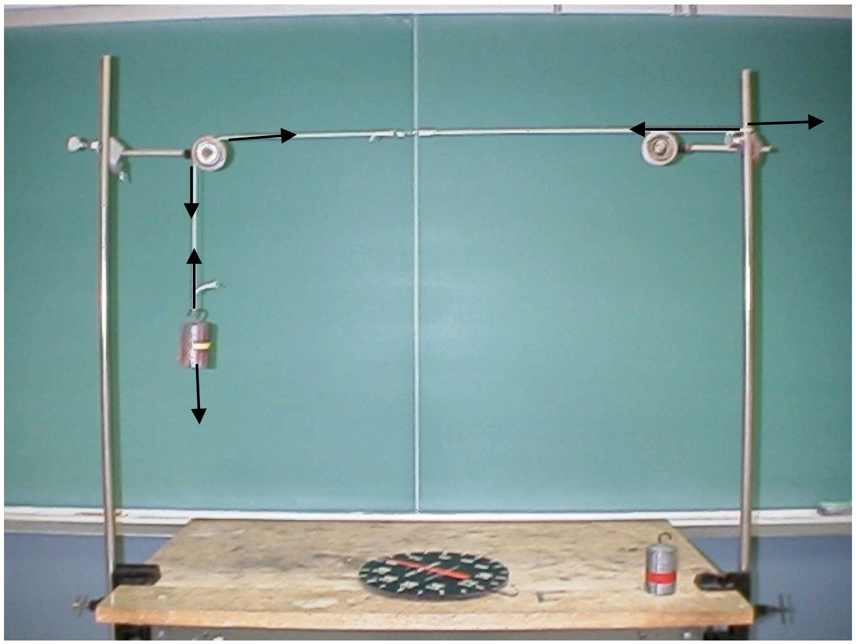
Problem: What is the acceleration of the system of the two blocks and the contact force between the blocks? What is the net force on Block B?

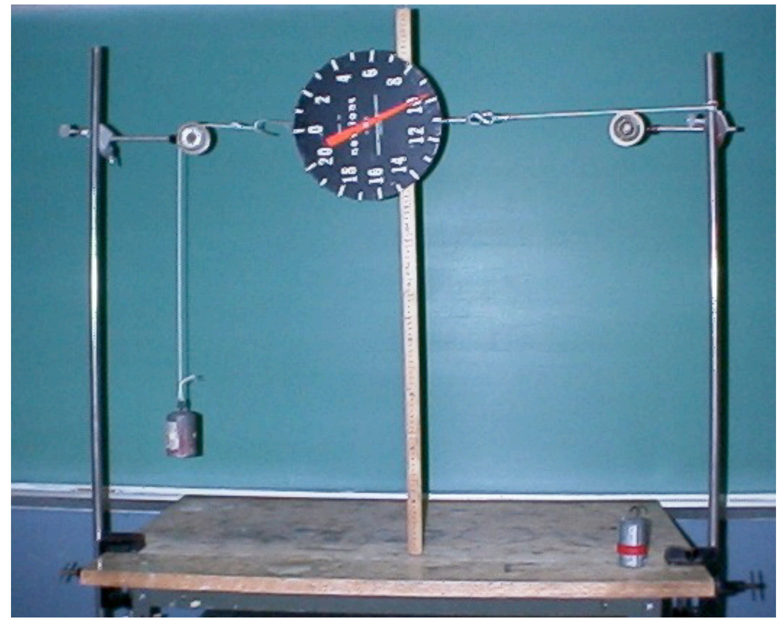
$$\vec{F}_{AB} = -\vec{F}_{BA}$$



Tension

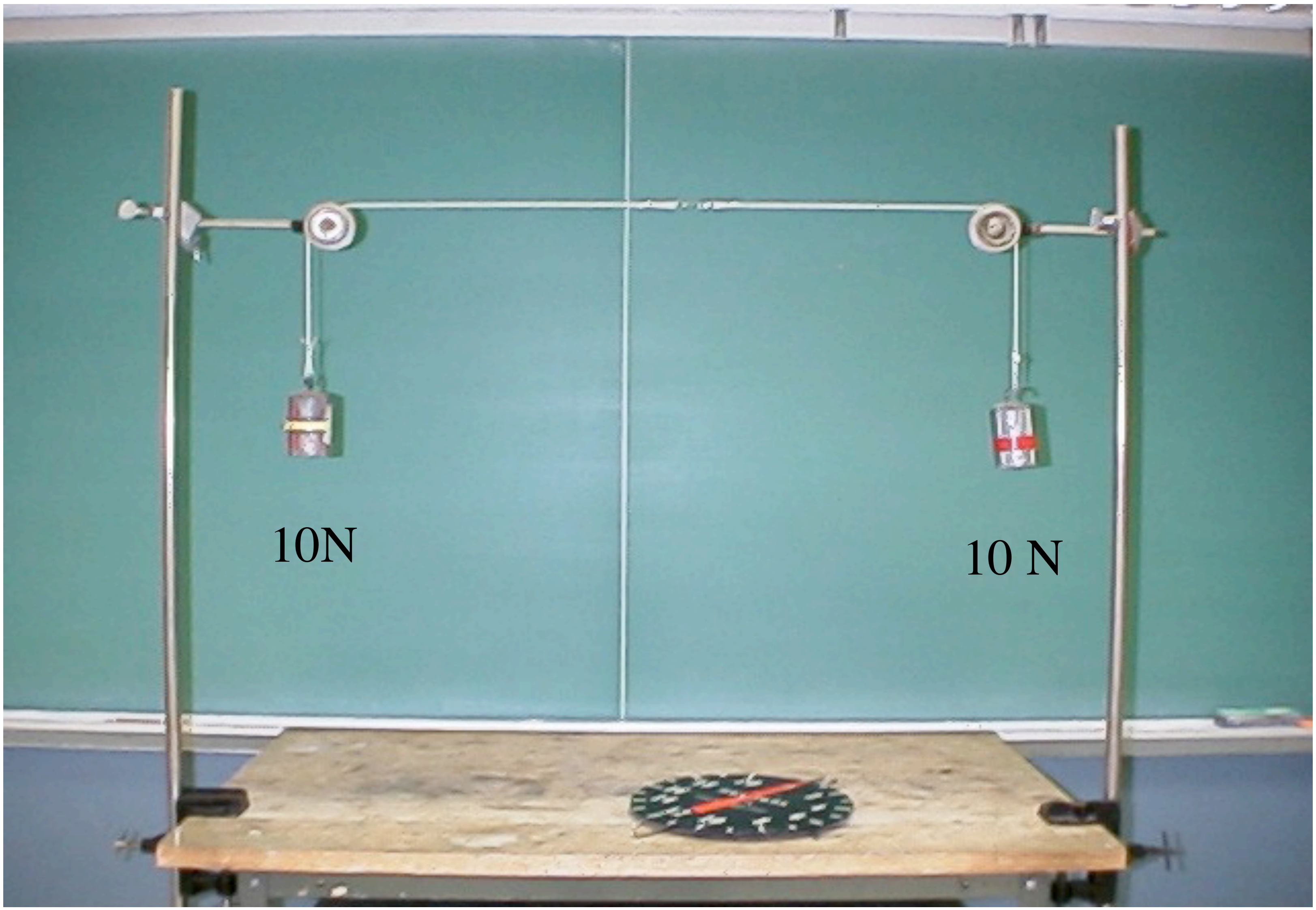
Now lets look at tension in a string



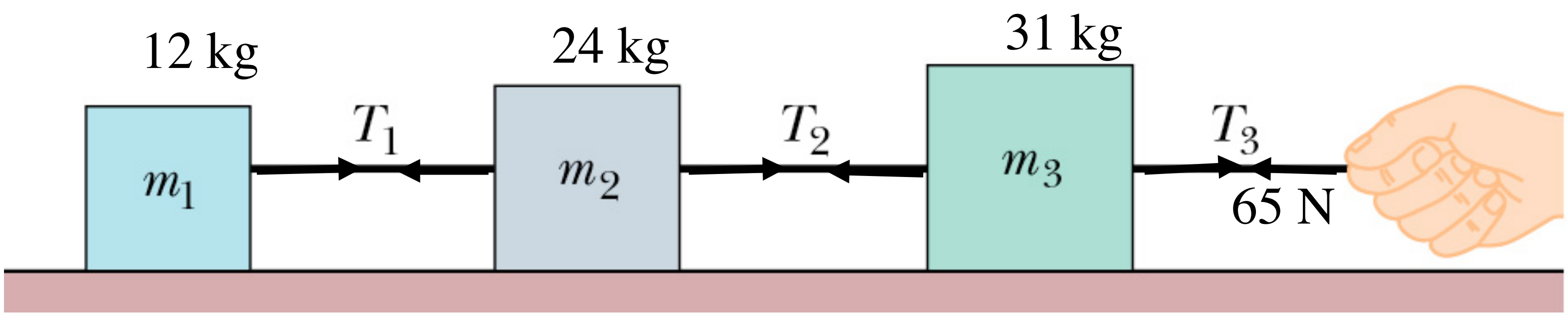


Tension in the string is equal to the weight = 10 N
The scale reads the tension in the string

Is the tension in the string any different when I have weights pulling it down on both sides?



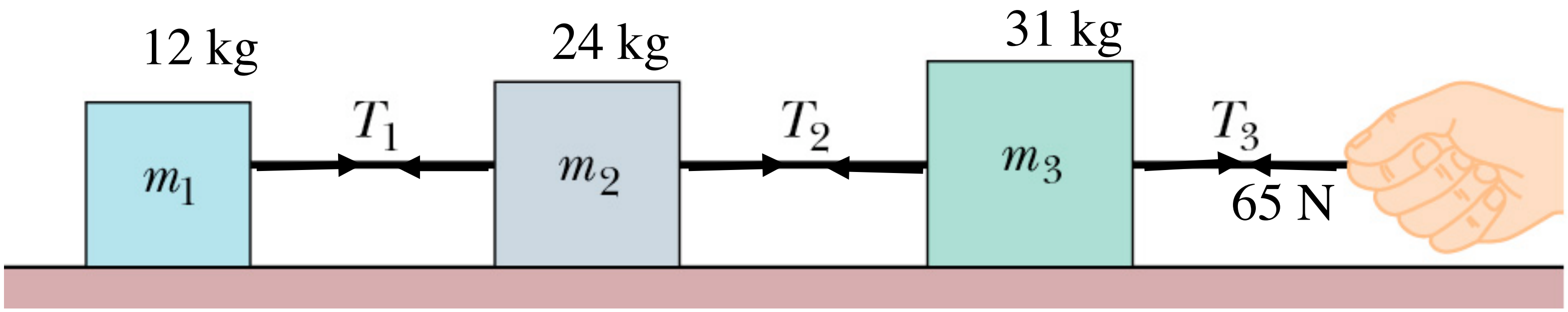
Problem



a) What is the acceleration of the system?
b) Find T_1
c) Find T_2

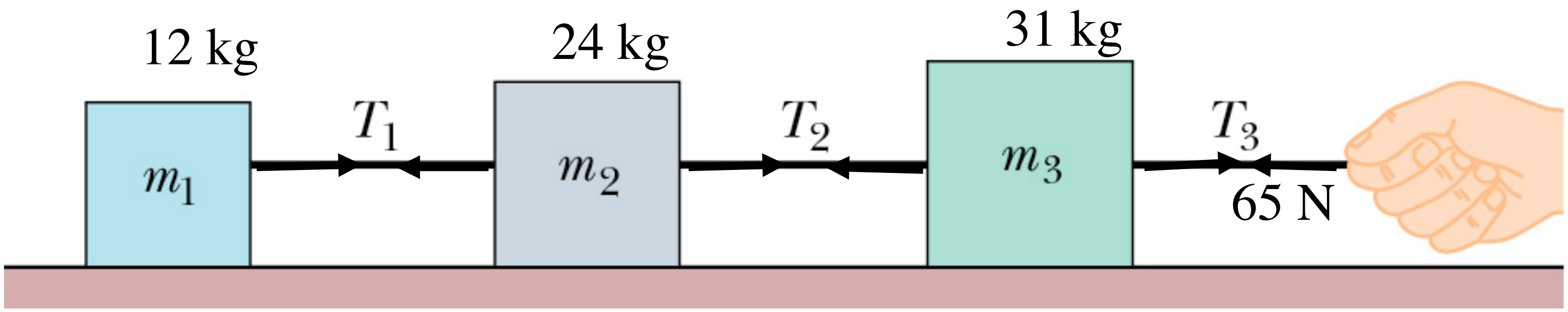
a) $T_3 = m_{sys} a$

Problem



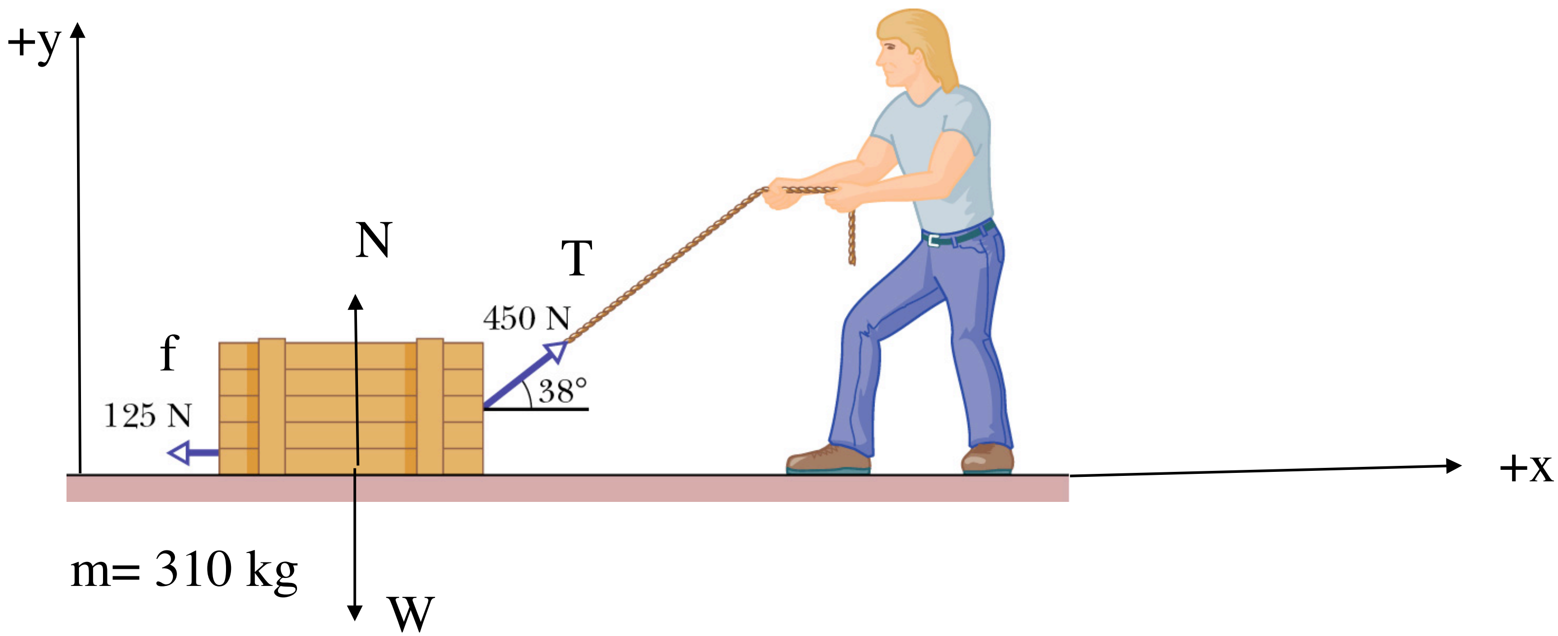
Find T_1

Problem



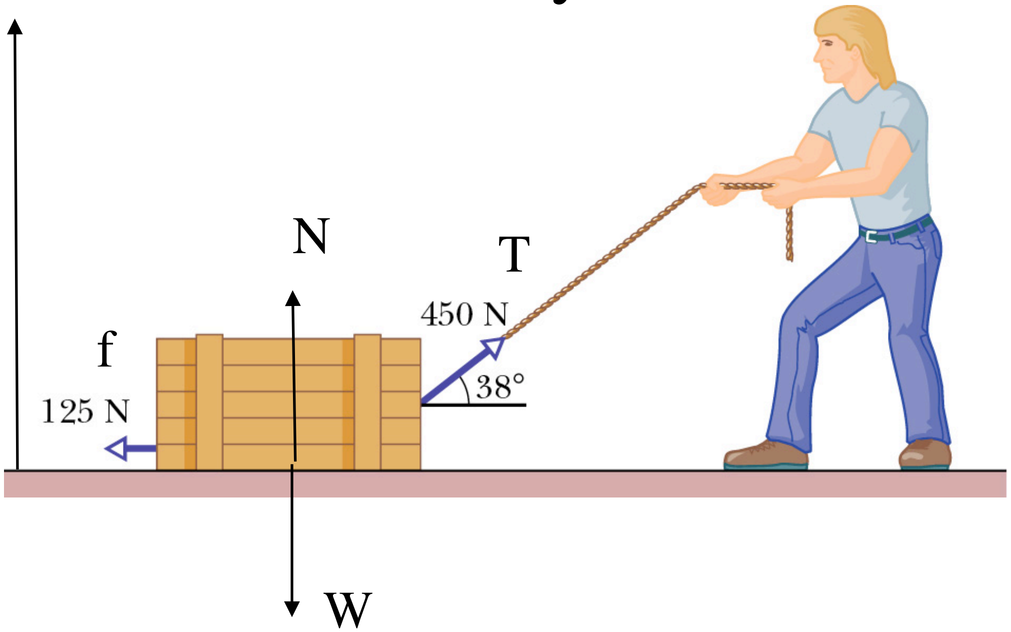
Find T_2

A crate is being pulled by a man as shown in the figure. What is the acceleration of the crate along the x direction? Man does not move.



x component of forces in free body diagram

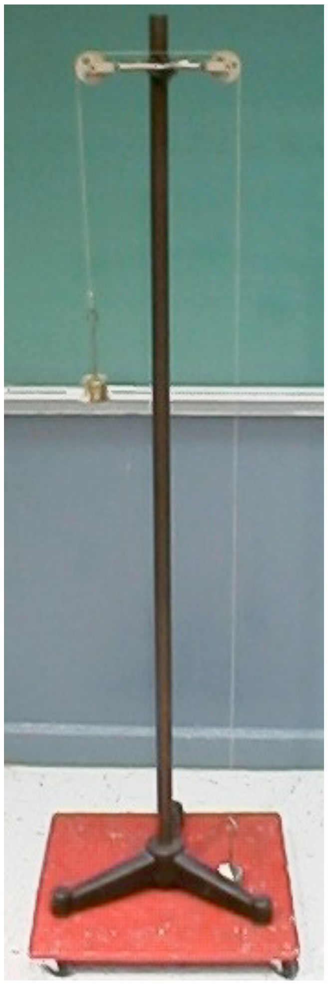
What is the normal force assuming there is no acceleration in the y direction?

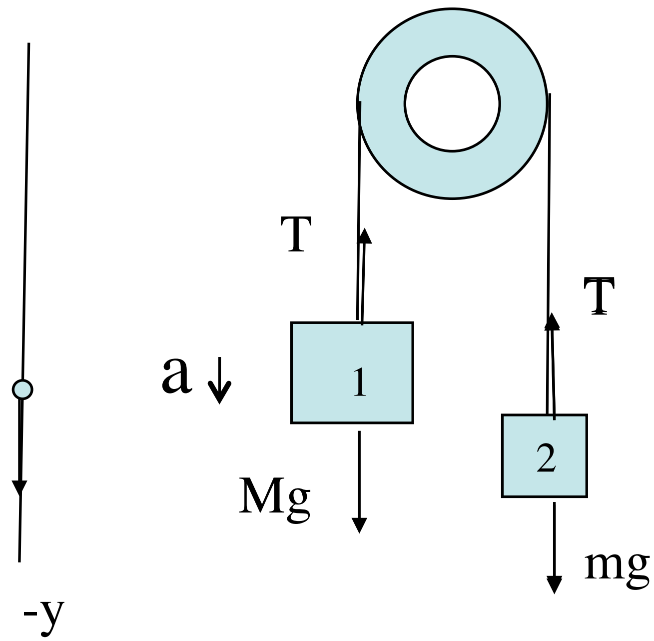


y component of forces in free body diagram

Rev George Atwood's machine 1746 -1807
Find the T and the acceleration a in terms of m, M and g.

Assume left side is moving down in the negative y direction

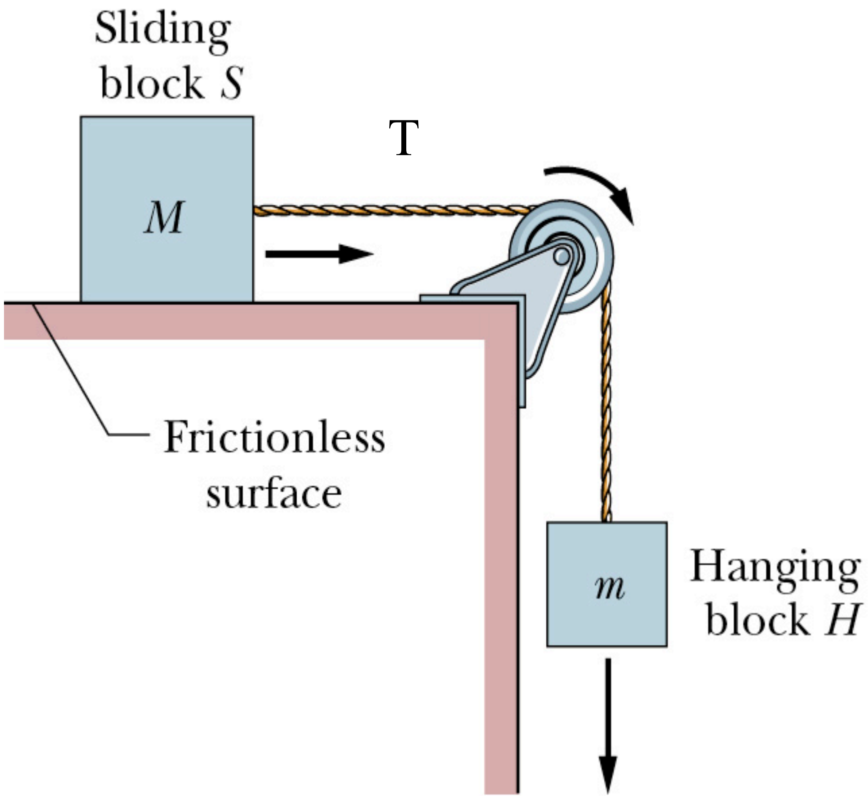


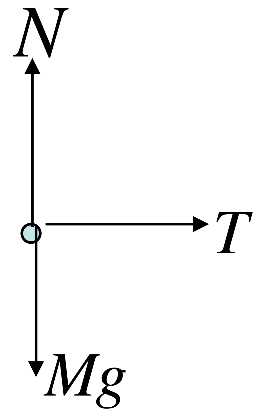


Free body diagram for each mass

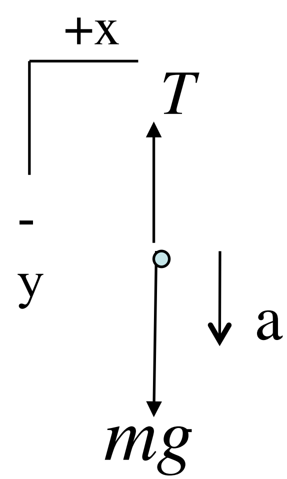
Up is + and down is -

Another example. Find T and acceleration a. Draw free body diagrams of each mass.





$T = Ma$



$$\sum F_y = T - mg = -ma$$

Friction

You are standing still, then begin to walk. What was the external forced that caused you to accelerate?

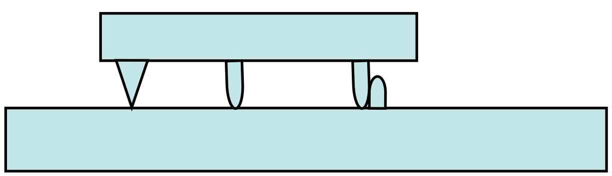
Hint: It is very hard to start walking if you are standing on ice.

What force causes a car to accelerate when a traffic light turns green?

Frictional Forces

Friction is an attractive force between two surfaces that is a result of the vector sum of many electrical forces between the surface atoms of the two different bodies.
Only about 10^{-4} of the surface atoms actually contribute.

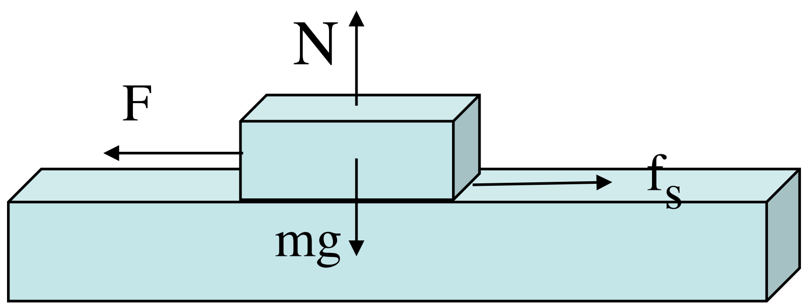
Model of dry friction 3 or 4 asperites support top block. Temporarily weld together



The Friction and Lubrication of Solids
F. P. Bowden and D. Tabor, Oxford University Press
1964

Models of friction
See Chabay and Sherwood
Matter and Ineractions
Volume 1
ISBN 0-471-35491-0

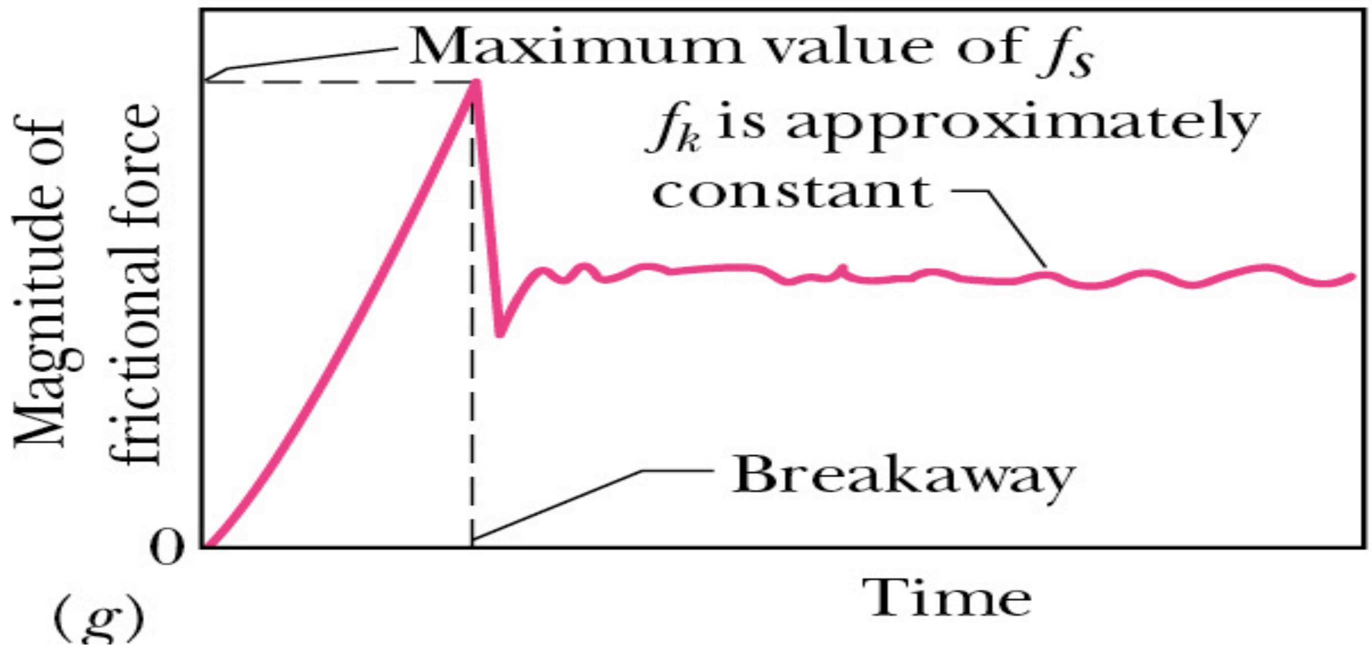
Friction



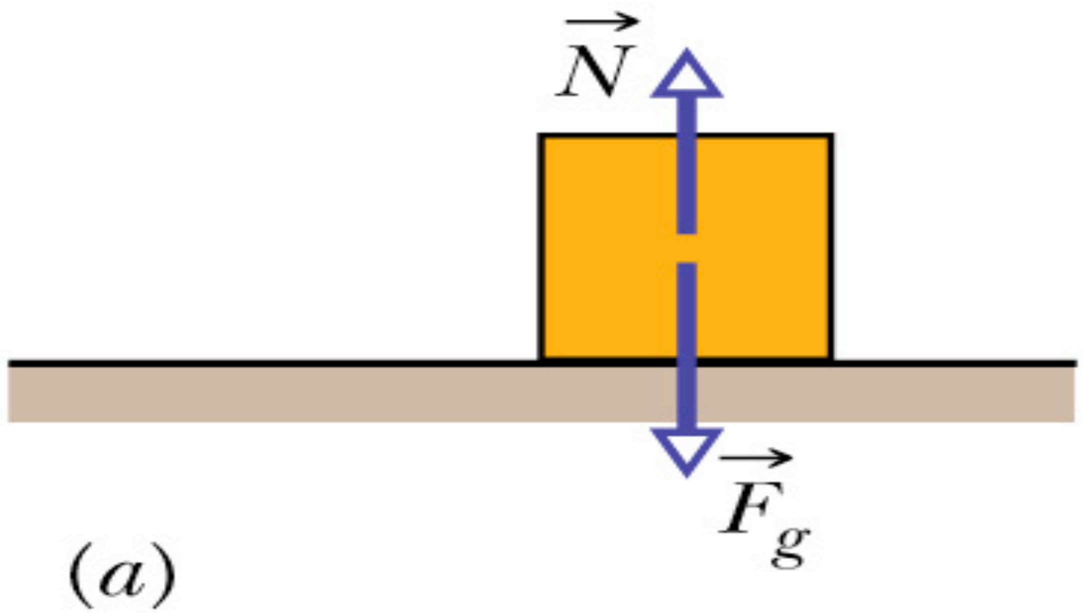
The maximum value equals $f_s = \mu_s N$, where N is the normal force. $f_s = \mu_s$ m g. The coefficient of static friction μ_s ranges from 0 to 1.2

Kinetic Friction: If we increase F until the block starts to move, the friction force decreases to $f_k = \mu_k N$ and remains constant throughout the motion.

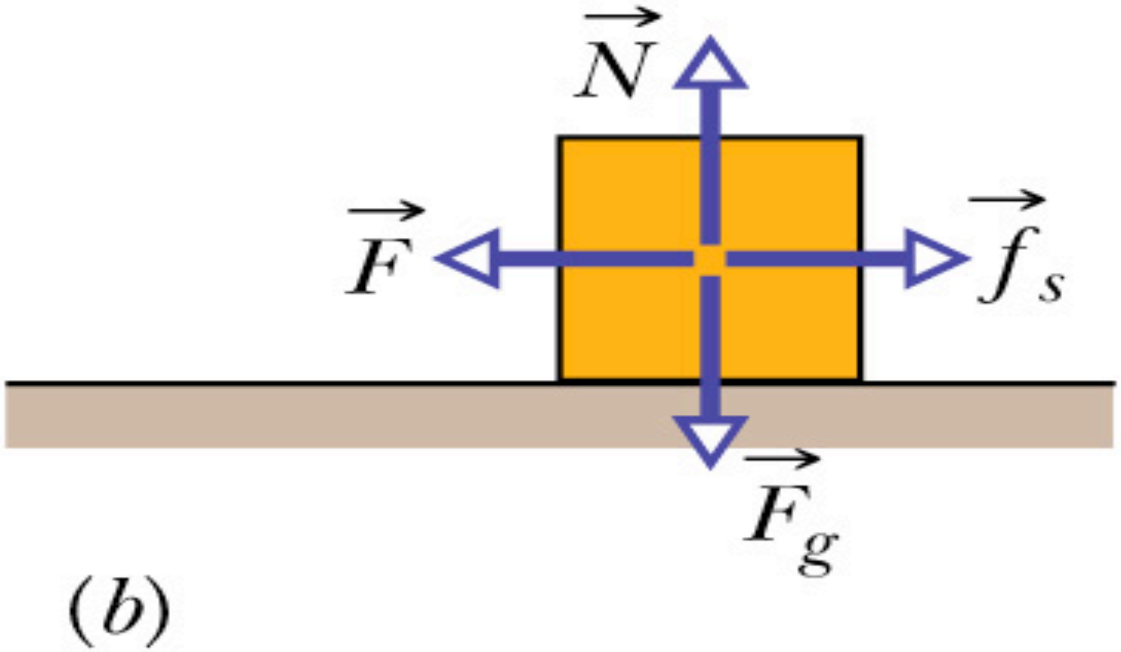
Friction as a function of time



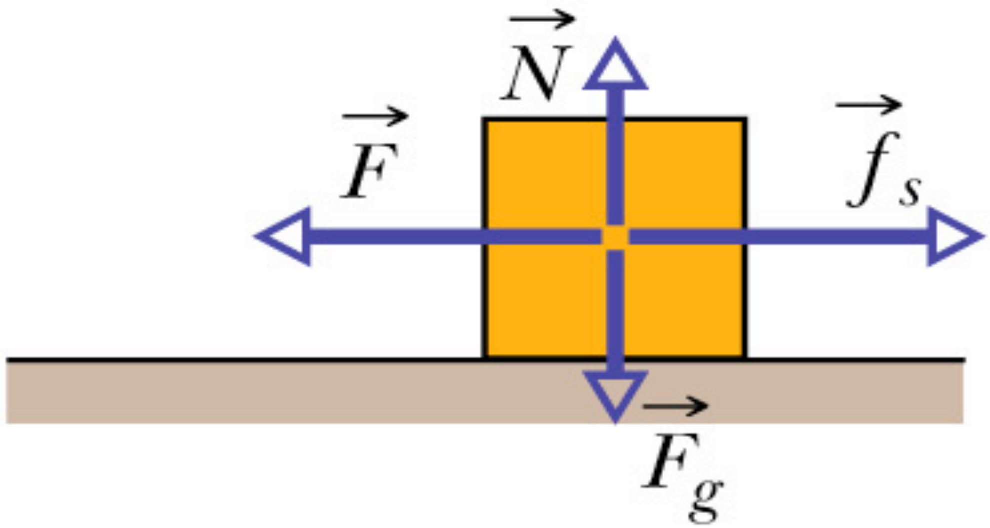
No motion and no horizontal forces



No Motion

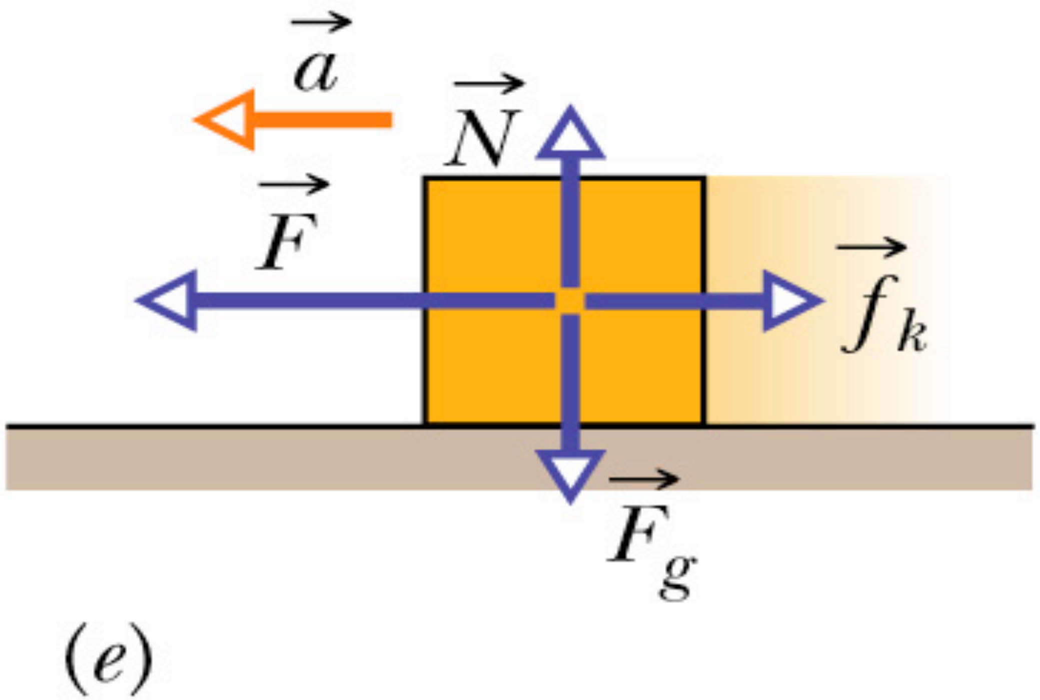


No Motion

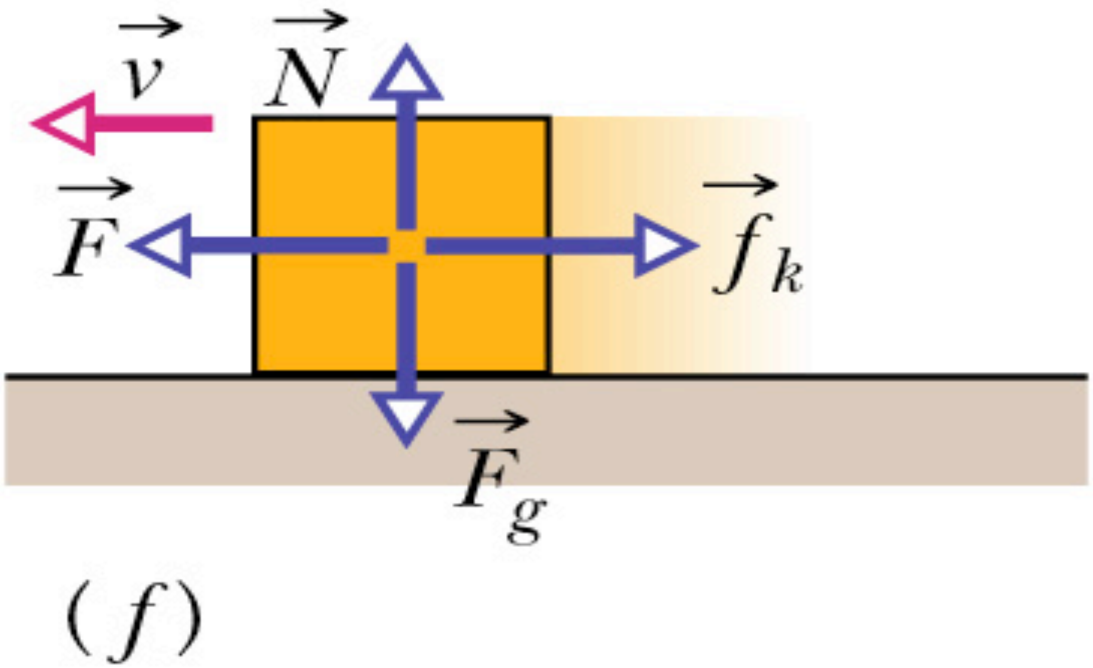


(d)

Acceleration



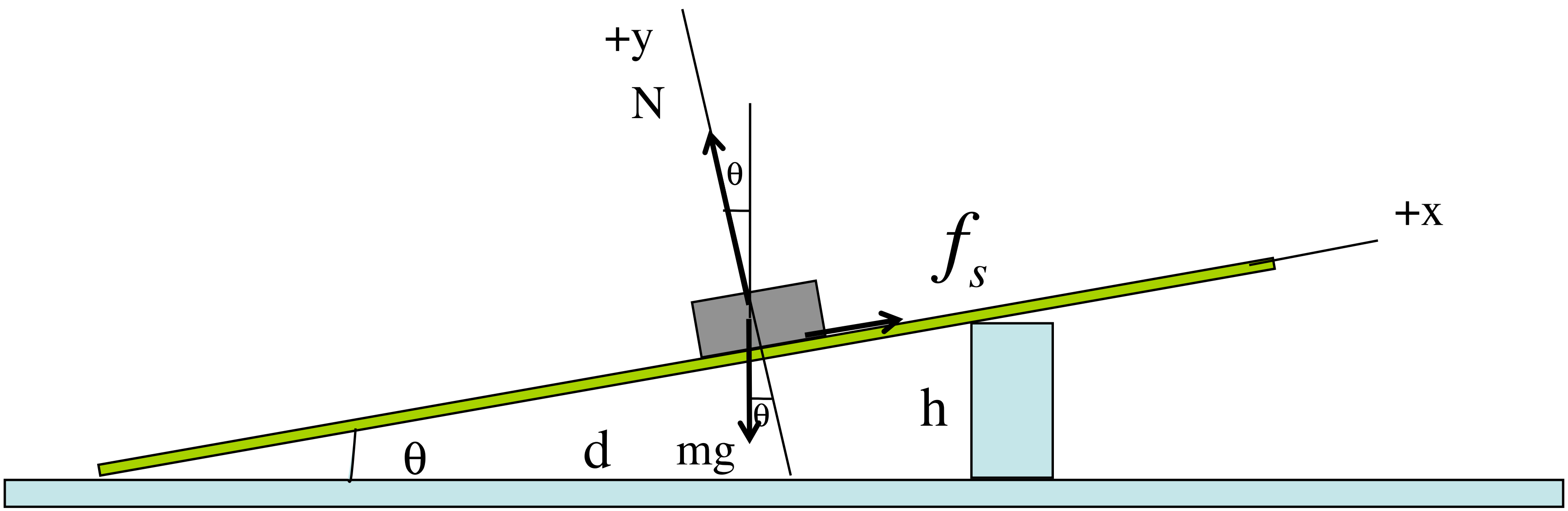
Constant velocity



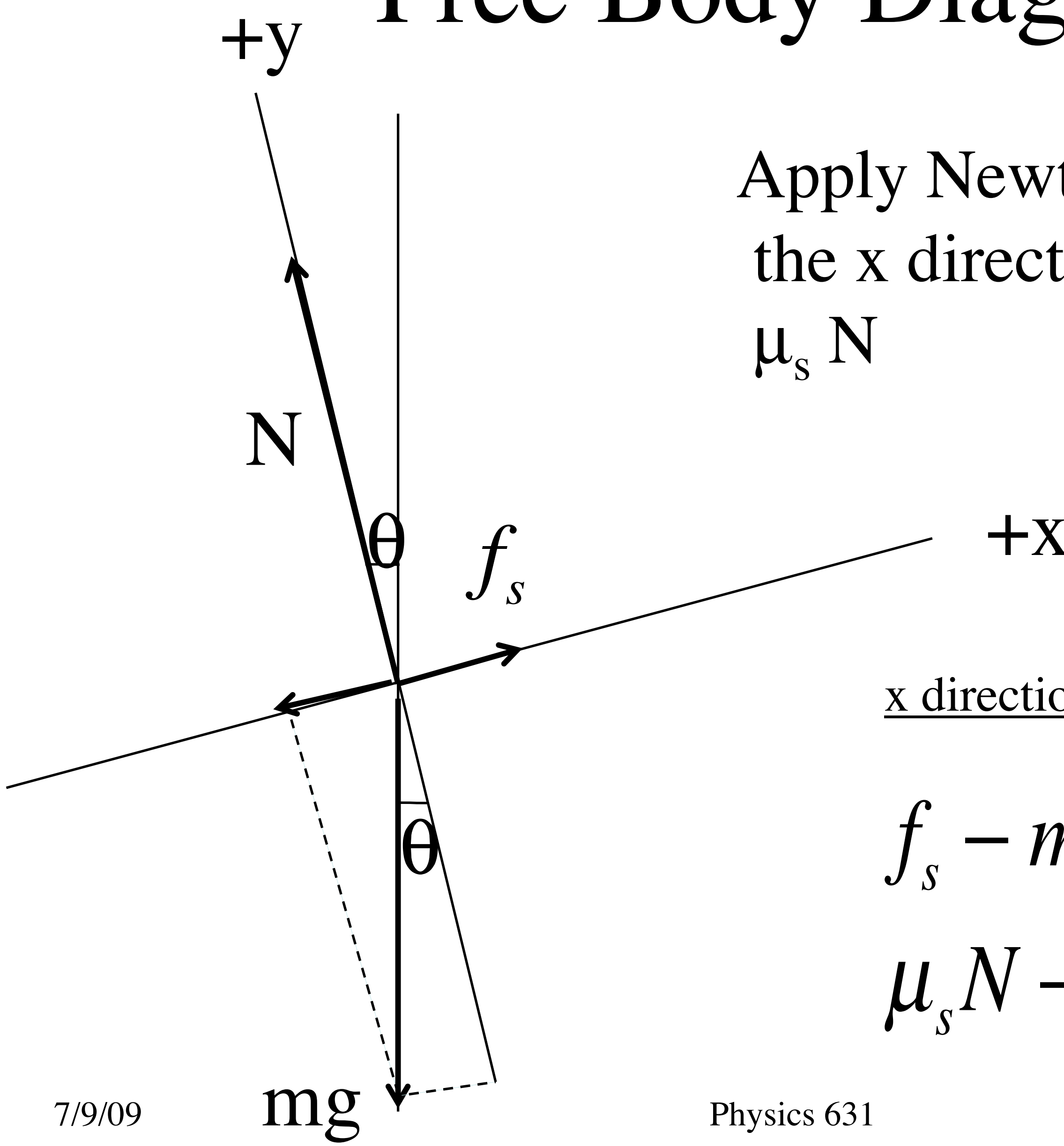
Problem Solving with Newton's 2nd Law involving friction

- Vector sum of external forces in x direction $= ma_x$
- Vector sum of external forces in y direction $= ma_y$
- If no acceleration, then set sum equal to 0

- Mass on incline plane at rest with impending motion. Find an expression for the coefficient of static friction



Free Body Diagram



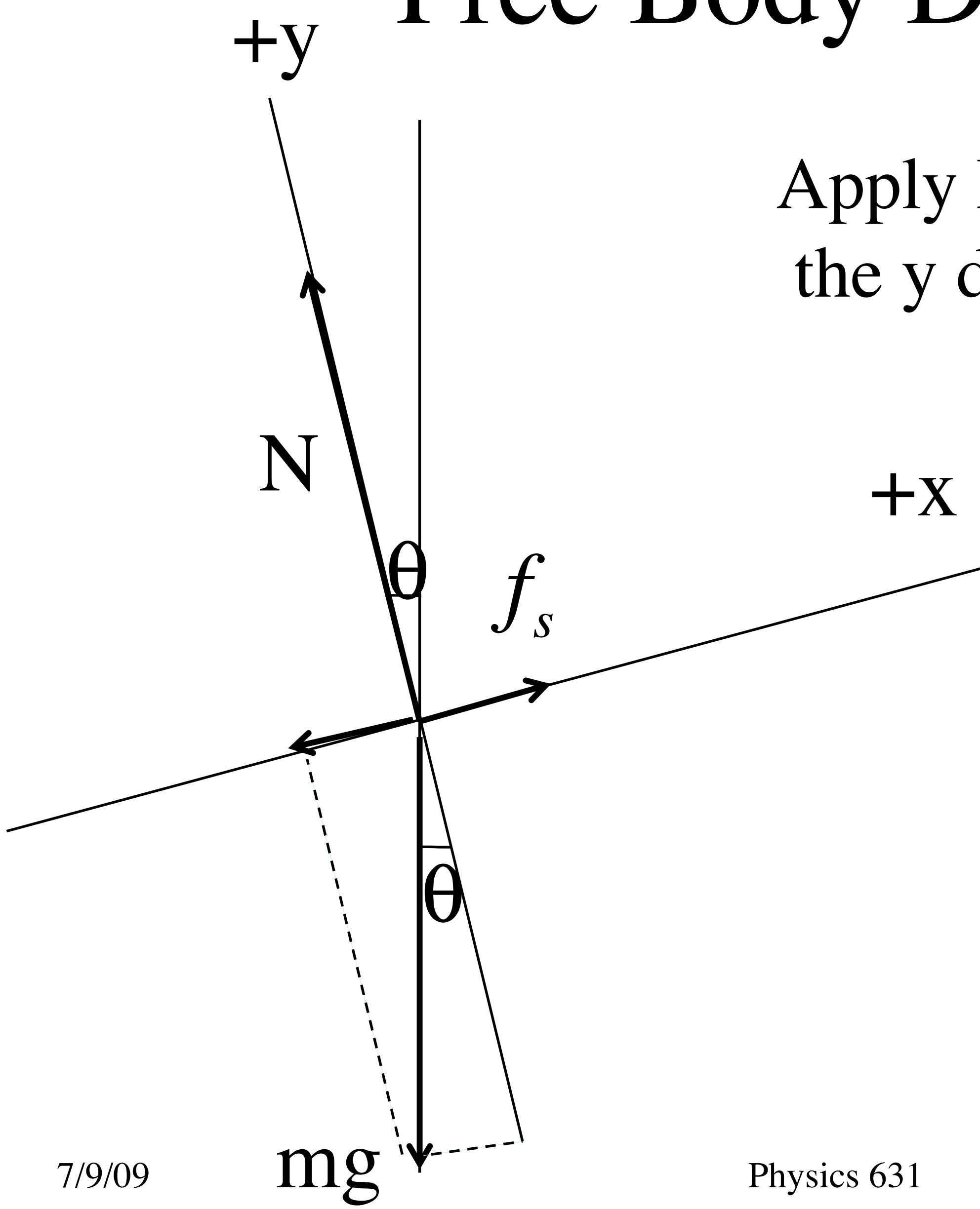
Apply Newton's 2nd law in the x directions and using $f_s = \mu_s N$

x direction

$$f_s - mg\sin\theta = 0$$

$$\mu_s N - mg\sin\theta = 0$$

Free Body Diagram



Apply Newton's 2nd law in the y direction

$$N - mg\cos\theta = 0$$

$$N = mg\cos\theta$$

$$\mu_s = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta$$

$$\tan\theta = \frac{h}{d}$$

Suppose you increase the angle of the inclined plane to some new angle so that the block accelerates down the inclined plate. Find the acceleration

x direction

$$f_s - mg\sin\theta = 0$$

Equilibrium condition

$$\mu_s N - mg\sin\theta = 0$$

$$f_k - mg\sin\theta = ma$$

$$\mu_k N - mg\sin\theta = ma$$

Acceleration condition

$$a = \frac{1}{m}(\mu_k N - mg\sin\theta)$$

·

How big does the mass M have to be to just start the block accelerating up the plane?

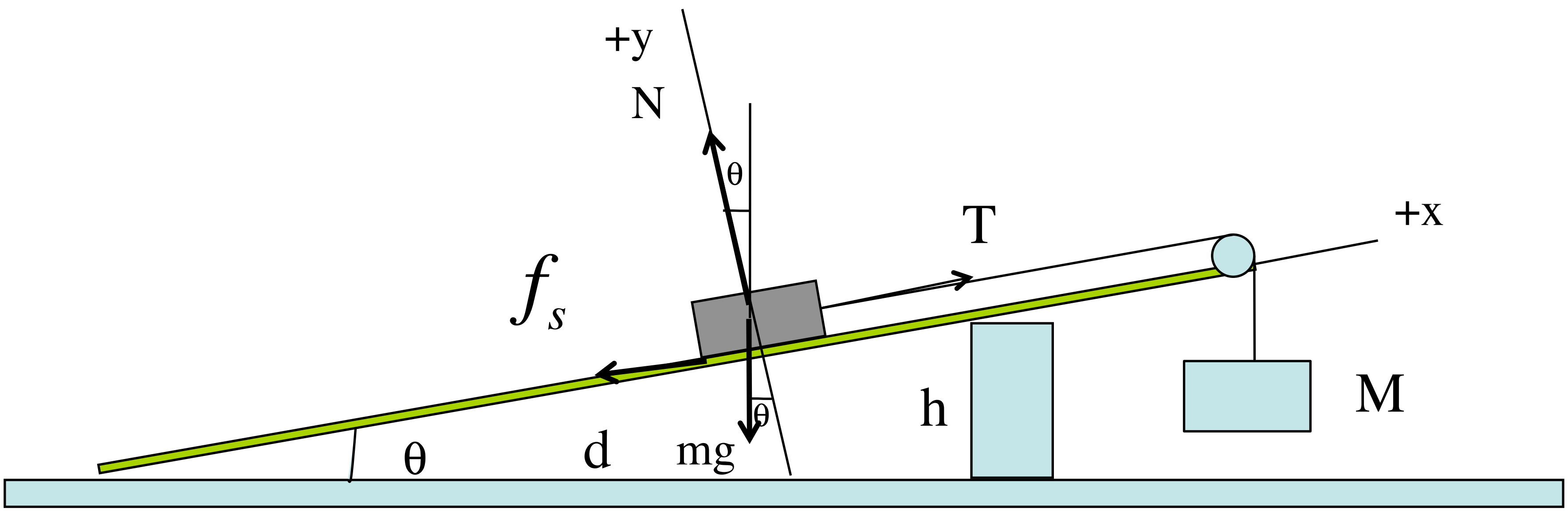
x direction

$$T - f_s - mg\sin\theta = ma$$

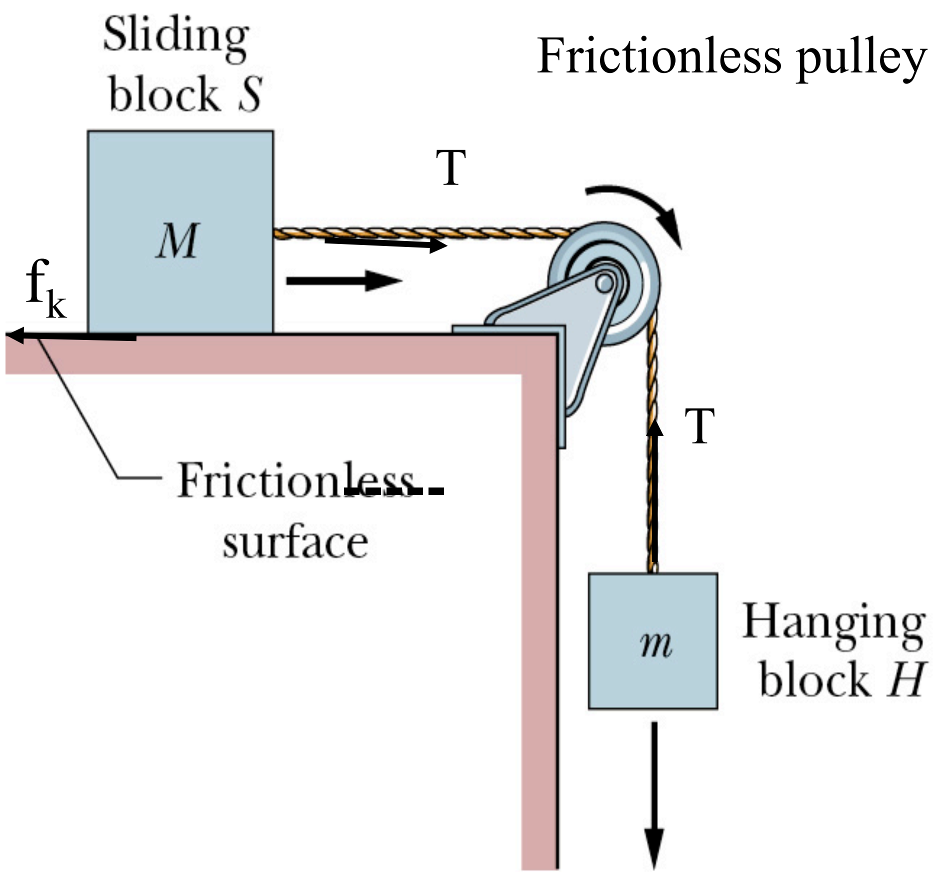
$$T - \mu_s N - mg\sin\theta = ma$$

$$T - Mg = -Ma$$

$$T = M(g - a)$$

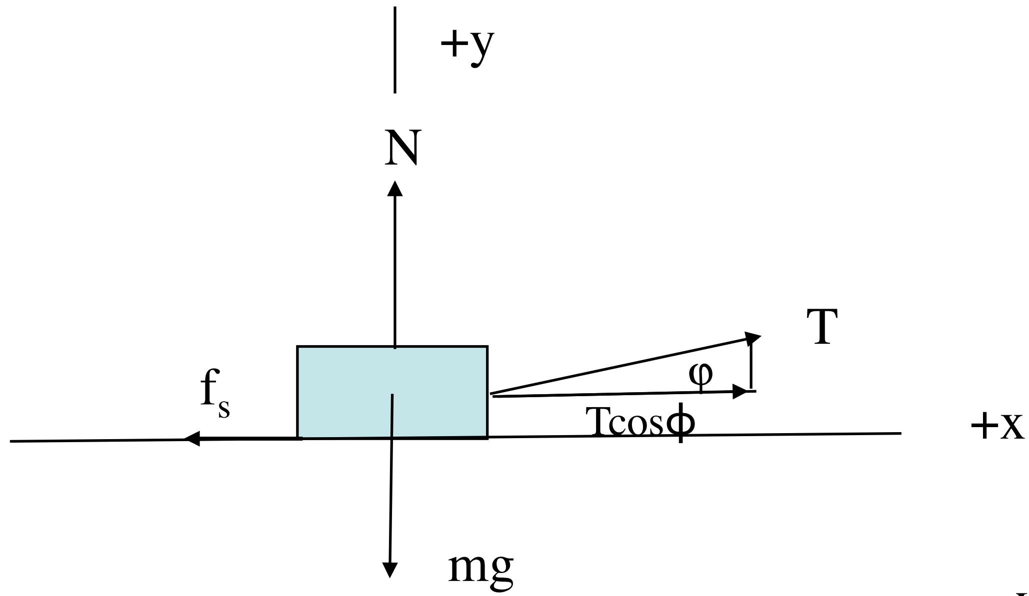


Free Body Diagram of an accelerating system: Atwood's machine with friction. Find a and T.



Problem 13 Chapter 6 edition 6 and 7

Question: What is the minimum magnitude force required to start the crate moving?



$m = 68 kg$

$\phi = 15 \deg rees$

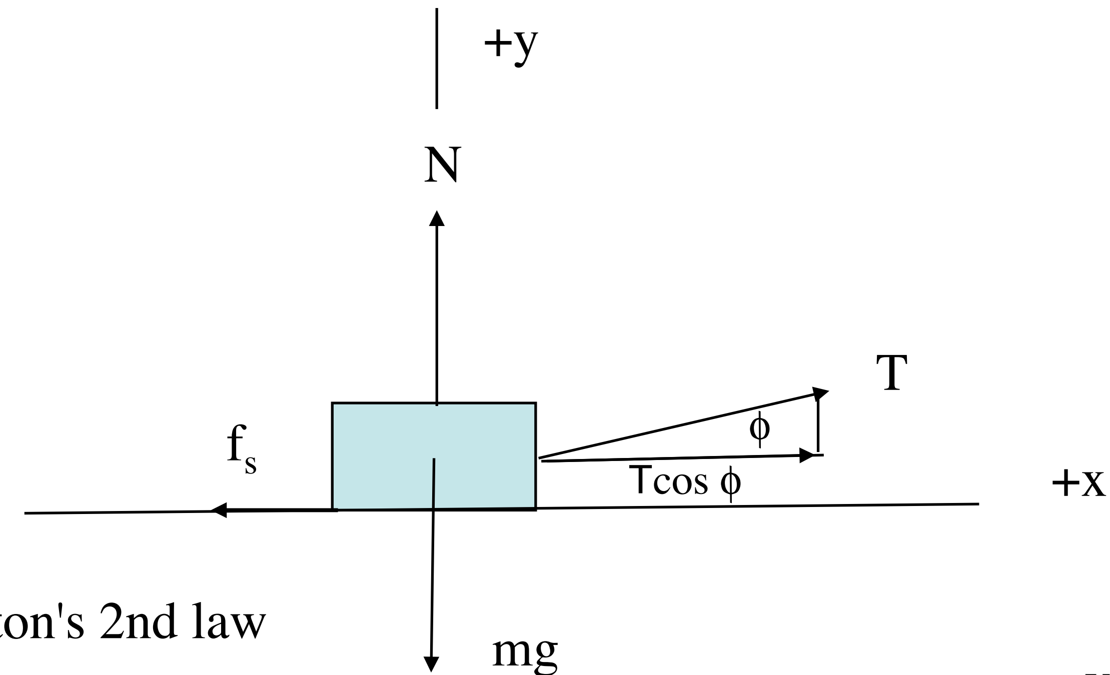
$\mu_s = 0.5$

x components

y components

Problem 13 continued

Question: What is the initial acceleration if μ_k=0.35?



$m = 68kg$

$\phi = 15\text{degrees}$

$\mu_s = 0.35$

Newton's 2nd law

<u>x components</u>

<u>y components</u>

Inertial Drag Force and Terminal Velocity

Drag force: Whenever you have a body like a ball moving through a medium that behaves like a fluid, there will be a drag force opposing the motion.

$$D = \frac{1}{2} C \rho A v^2$$

Imagine a falling ball slowed down due to elastic collisions with air molecules. Simply pushing the air out of the way.

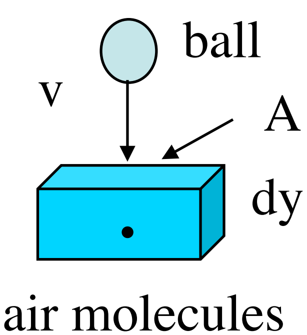
Hand waving argument

$$F = \frac{dp}{dt} = \frac{d(mv)}{dt} = m\frac{dv}{dt} + v\frac{dm}{dt}$$

$$dm = \rho A dy$$

$$\frac{dm}{dt} = \rho A \frac{dy}{dt} = \rho A v$$

$$F = v\rho A v = \rho A v^2$$



Inertial drag

Terminal speeds in air

Using Newton's 2nd law,

$$F = ma = mg - \frac{1}{2} C \rho A v^2$$

where m is the mass of the falling ball

$$mg - \frac{1}{2} C \rho A {v_0}^2 = 0$$

Solve for v_0

$$v_0 = \sqrt{\frac{2mg}{C\rho A}}$$

Stokes-Napier Law

TERMINAL SPEEDS IN AIR

Object `	Speed (m/s)	Speed (mph)
Feather	0.4	0.9
Snowflake	1	2.2
BB	9	20
Mouse	13	29
Tennis ball	31	66
Baseball	42	86
Sky diver	60 -120	134 -268
Cannonball	250	560

Show demo of falling feather in vacuum

How to solve this equation? Two ways

$$F = Mg - \frac{1}{2} C \rho A v^2$$

$$F = Mg - bv^2$$

$$\text{where b} = \frac{1}{2} C \rho A$$

Use Newtons 2nd Law

Initial component of momentum: $p_1 = Mv_1$

Initial force on ball: $F = Mg - bv_1^{\,2}$

Using 2nd Law

$$F = \frac{dp}{dt} = \frac{\Delta p}{\Delta t} = \frac{p_2 - p_1}{\Delta t}$$

Find new p

$$p_2 - p_1 = F\Delta t$$

$$p_2 = p_1 + F\Delta t$$

Substitute in for p_1 *and* F

Use Newtons 2nd Law

Initial velocity v_1

Initial force on ball: $F = mg - bv_1^2$

Using 2nd Law

$$F = m\frac{dv}{dt} = m\frac{\Delta v}{\Delta t} = \frac{m(v_2 - v_1)}{\Delta t}$$

$$m(v_2 - v_1) = F\Delta t$$

Find new v_2

$$v_2 = v_1 + F\Delta t / m$$

substitute in for F

Newtons 2nd Law

$$v_2 = v_1 + [g - (\frac{b}{m})v_1^2]\Delta t$$

$$x_2 = x_1 + v_2\Delta t = x_1 + (\frac{v_1 + v_2}{2})\Delta t$$

Go to Excel Spread Sheet

delta_t=	0.05	
g=	10	gravity
m_1=	0.75	mass
b_1=	0.25	drag coefficient
v_init=	0	initial downward velocity

time	velocity	distance
0	0	0
0.05	0.5	0.0125
0.1	0.995833	0.049896
0.15	1.479305	0.111774
0.2	1.942833	0.197328
0.25	2.379923	0.305397
0.3	2.785522	0.434533
0.35	3.156203	0.583076
0.4	3.490176	0.749235
0.45	3.787154	0.931169
0.5	4.048112	1.12705

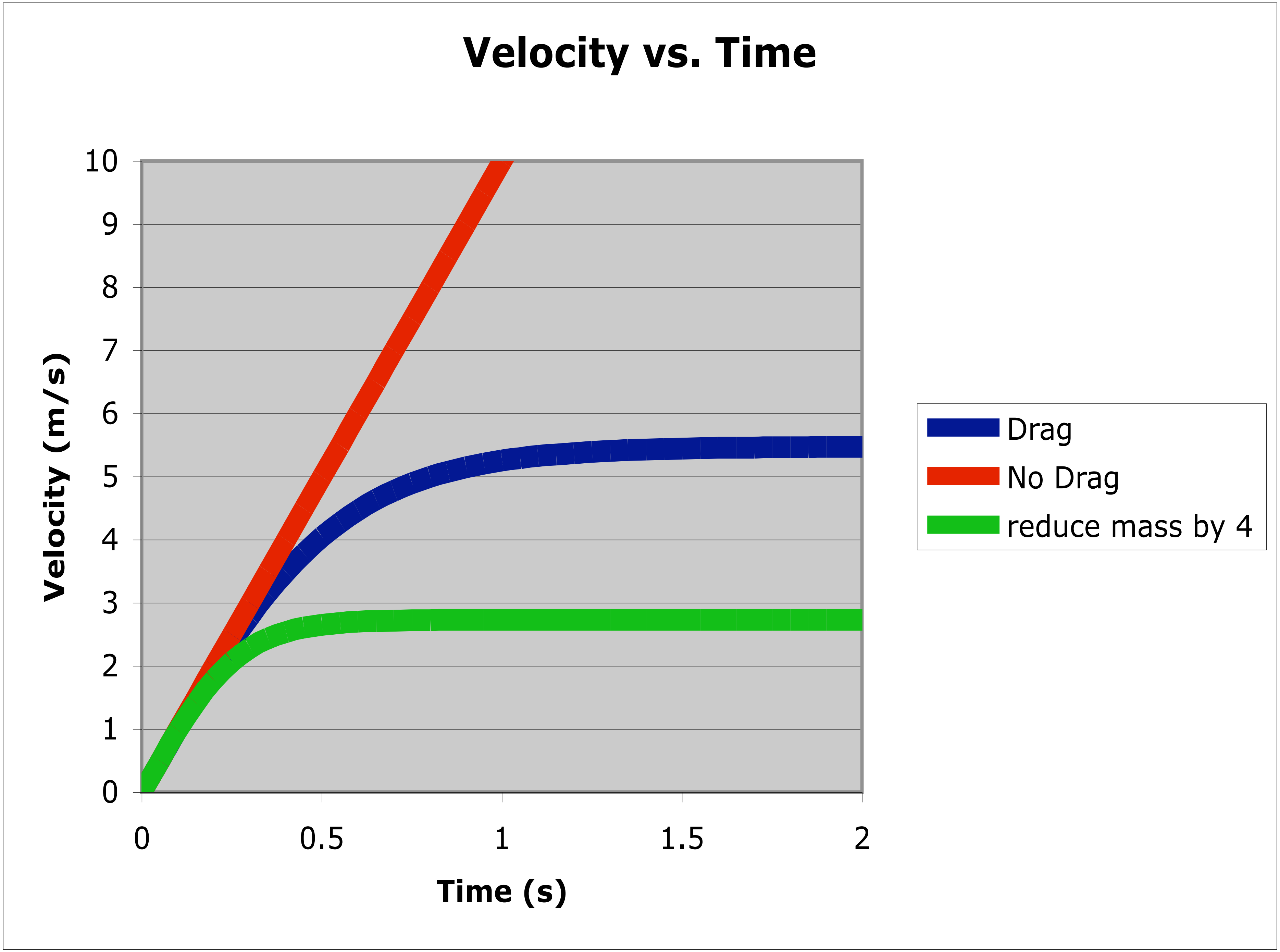
$$v_i = v_{i-1} + \left[g - \left(\frac{b\_1}{m\_1}\right) {v_{i-1}}^2 \right] \Delta t$$

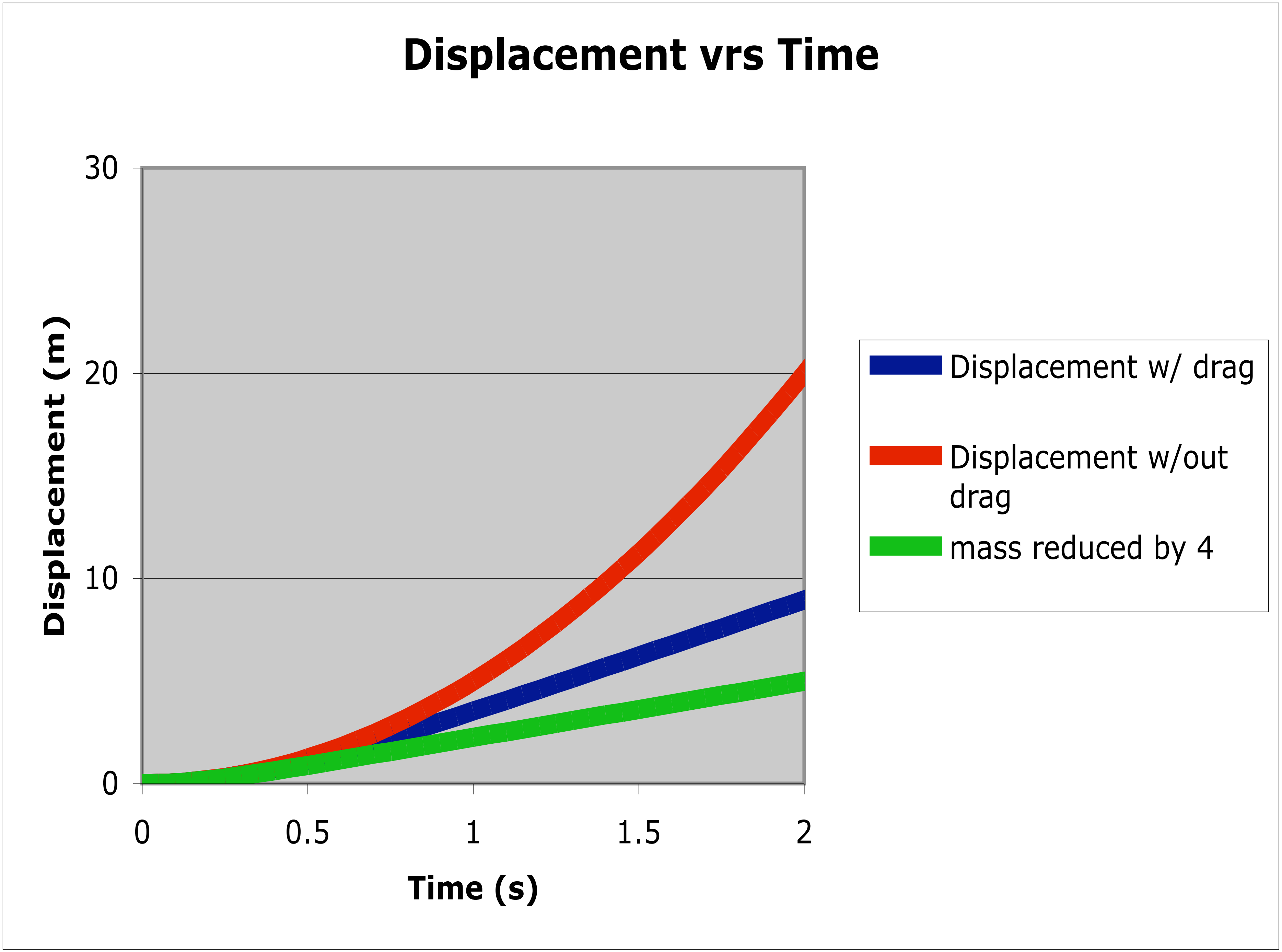
$$x_i = x_{i-1} + \left(\frac{v_{i-1} + v_i}{2}\right) \Delta t$$

See Spread sheet

New velocity = C17=C16+(g-(b_1/m_1)*C16*C16)*delta_t

New position =D17=D16+1/2*(C16+C17)*delta_t


Velocity vs. Time
Velocity (m/s)
10
9
8
7
6
5
4
3
2
1
0
0
0.5
1
1.5
2
Time (s)
Drag
No Drag
reduce mass by 4


Displacement vrs Time
Displacement (m)
30
20
10
0
0
0.5
1
1.5
2
Time (s)
Displacement w/ drag
Displacement w/out drag
mass reduced by 4

We can also solve the equation to get the velocity as a function of time before it reaches terminal velocity. Let b = 1/2CρA

$$F = ma = m\frac{dv}{dt} = mg - bv^2$$

$$m\frac{dv}{dt} = mg - bv^2$$

$$\frac{m}{b}\frac{dv}{dt} = \frac{mg}{b} - v^2$$

$$let\ v_0 = \sqrt{\frac{mg}{b}}$$

$$\frac{m}{b}\frac{dv}{dt} = v_0^2 - v^2$$

$$\frac{1}{v_0^2 - v^2}dv = \frac{b}{m}dt$$

Solving equation continued

$$\frac{1}{v_0^2 - v^2} dv = \frac{b}{m} dt$$

$$\frac{1}{v_0^2 - v^2} = \left(\frac{1}{v^0 - v} + \frac{1}{v^0 + v}\right)\frac{1}{2v_0}$$

$$\frac{dv}{v_0 - v} + \frac{dv}{v_0 + v} = \frac{2v_0 b}{m} dt$$

Integrate both sides

$$\int_0^v \frac{dv}{v_0 - v} + \int_0^v \frac{dv}{v_0 + v} = \frac{2v_0 b}{m} \int_0^t dt$$

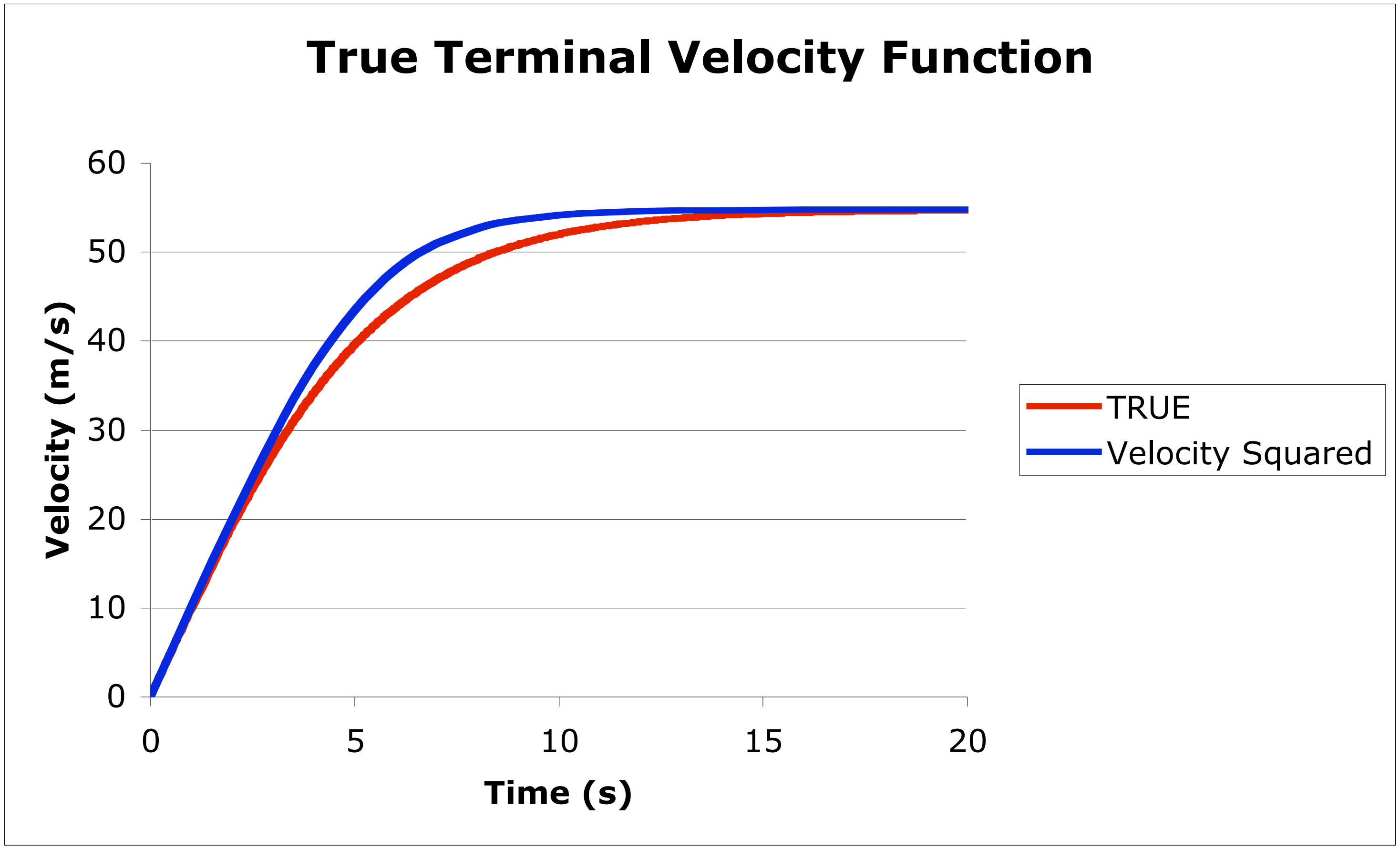
$$\int_0^v \frac{dv}{v_0 - v} + \int_0^v \frac{dv}{v_0 + v} = \frac{2v_0 b}{m} \int_0^t dt$$

This can be integrated, and fixing the constant of integration by the requirement that the velocity be zero at $t = 0$, which is the case for free fall we find:

$$v = v_0 \left(\frac{1 - e^{-\frac{2v_0 b}{m} t}}{1 + e^{-\frac{2v_0 b}{m} t}} \right)$$

Now show comparison of this solution with numerical integration with Excel.

Comparison



The curve modeled by velocity squared for terminal velocity Differs from the true equation due to a large delta t. When delta t becomes small enough the two curves are Indistinguishable.

Water Resistance and Drag Forces

Whenever you have a body moving through a liquid there will be a drag force opposing the motion. Here the drag force is proportional to - kv. Viscous drag.

Stokes Law: terminal velocity is proportional to mass

A 1000 km boat in the water shuts off its engine at 90 km/hr. Find the time required to slow down to 45 km/hr due to a water drag force equal to -70v, where v is the speed of the boat. Let k = 70.

$$ma = -kv$$

$v/v_0 = 45/90 = 1/2$

t = m/k ln 2=1000/70 ln 2 = 9.9 s

$$m\frac{dv}{dt} = -kv$$

$$mdv = -kvdt$$

$$\frac{dv}{v} = -\frac{k}{m}dt$$

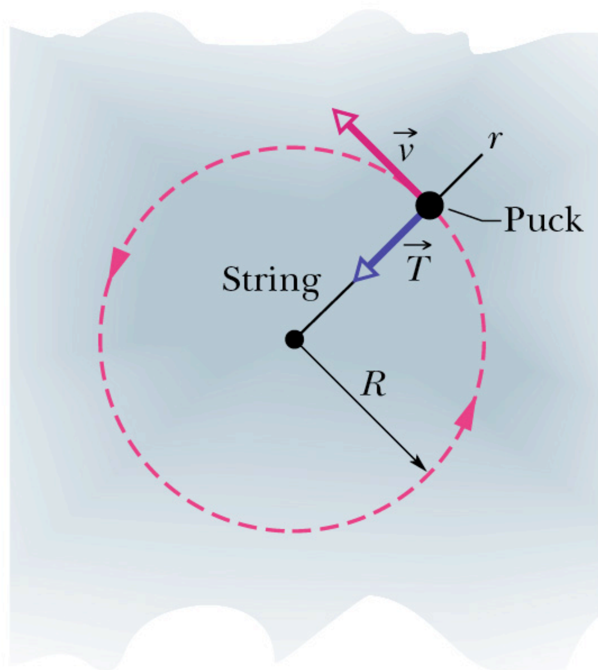
$$\int_{v_0}^{v} \frac{dv}{v} = -\frac{k}{m}\int_0^t dt$$

$$\ln v\Big|_{v_0}^{v} = \ln v - \ln v_0$$

$$\ln\frac{v}{v_0} = -\frac{k}{m}t$$

$$\frac{v}{v_0} = e^{-\frac{k}{m}t}$$

Uniform circular motion



$$T = Ma_c$$

$$a_c = \frac{v^2}{R}$$

Centripetal force is really not a new force like gravity, tension, friction.

Motion of earth around sun – centripetal force is a result of gravity
Rock whirled around on a string – centripetal force is a result of tension
Sometimes it is a result of friction or the normal force

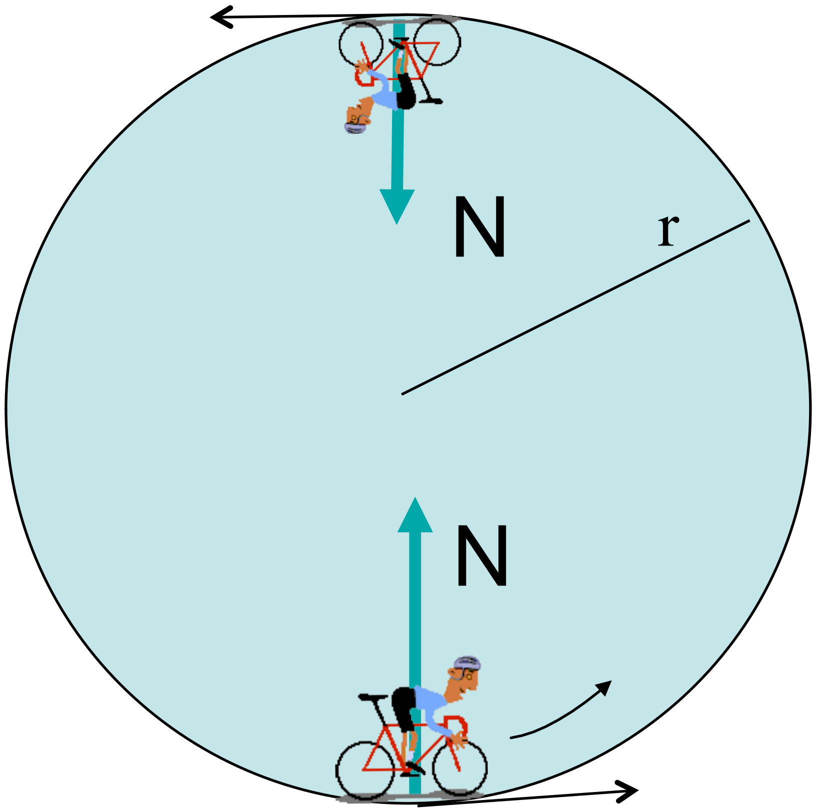
See notes on vertical circular motion

Leads to apparent weight and fictitious forces

VERTICAL CIRCULAR MOTION

Down is negative, Up is positive

At the top: Apparent weight $= N = \dfrac{mv^2}{r} - mg$



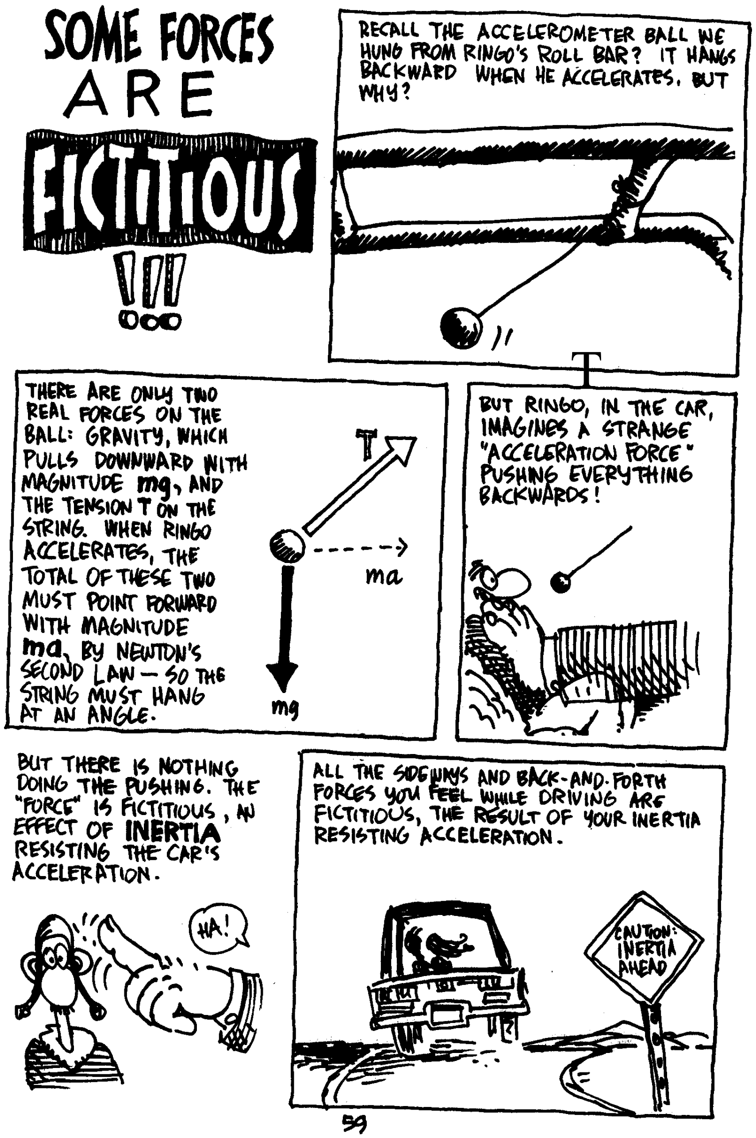
Minimum v for N = 0:
(apparent weightlessness)

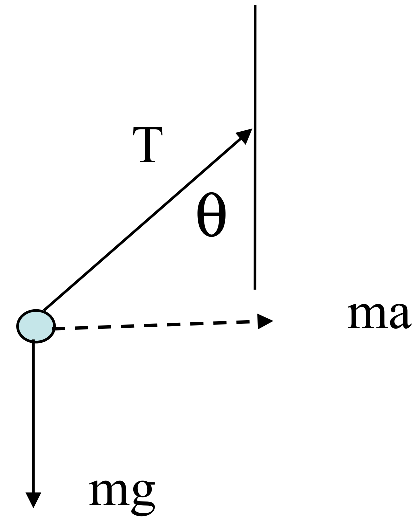
$$\frac{mv^2}{r} = mg \qquad v = \sqrt{rg}$$

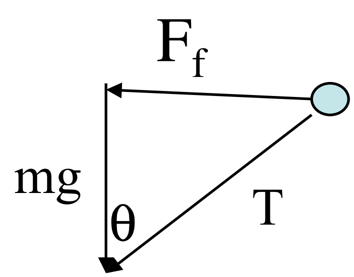
At the bottom:
Apparent weight = N = mv^2/r **+** mg
Weigh more

What do we mean by Fictitious Forces

$F_f = -ma$ (the fictitious force always acts in the opposite direction of acceleration)





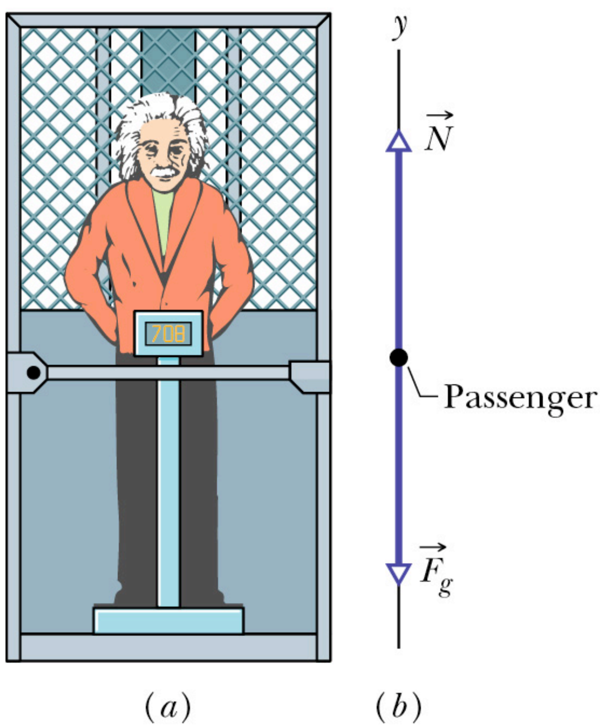


$$\tan\theta = \frac{ma}{mg}$$

$$a = g\tan\theta$$

Example of fictitious force ($F_f = -ma$)

In a vertically accelerated reference frame, eg. an elevator, what is your apparent weight? Apparent weight = N



Upwards is positive
Downwards is negative

Upward acceleration

$$N - mg = ma$$

$$N = mg + ma$$

Downward acceleration

$$N - mg = -ma$$

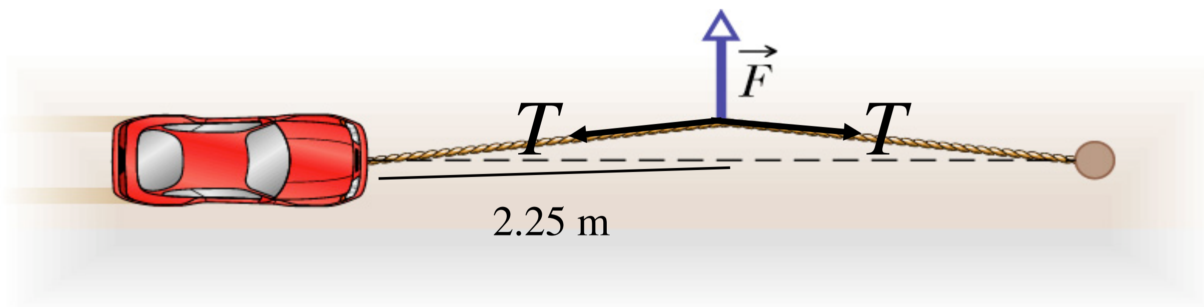
$$N = mg - ma$$

Free fall a = g
N = 0 Weightless condition

Tug-of-war demo illustrates how a small sideways force can produce a large horizontal force

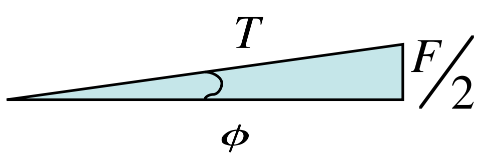
Problem 13-10

Suppose two guys in the tug of war are at 4.5 meters apart and I pull the rope out 0.15 meters. Then $\phi = 4$ degrees.



$$\tan\phi = \frac{0.15}{2.25}$$

$$\phi = 4^o$$



$$\sin\phi = \frac{F}{2T}$$

$$T = \frac{F}{2\sin\phi}$$

Therefore,

$$T = \frac{F}{2\sin(4^o)} = \frac{F}{(2)(0.07)} = 7F$$

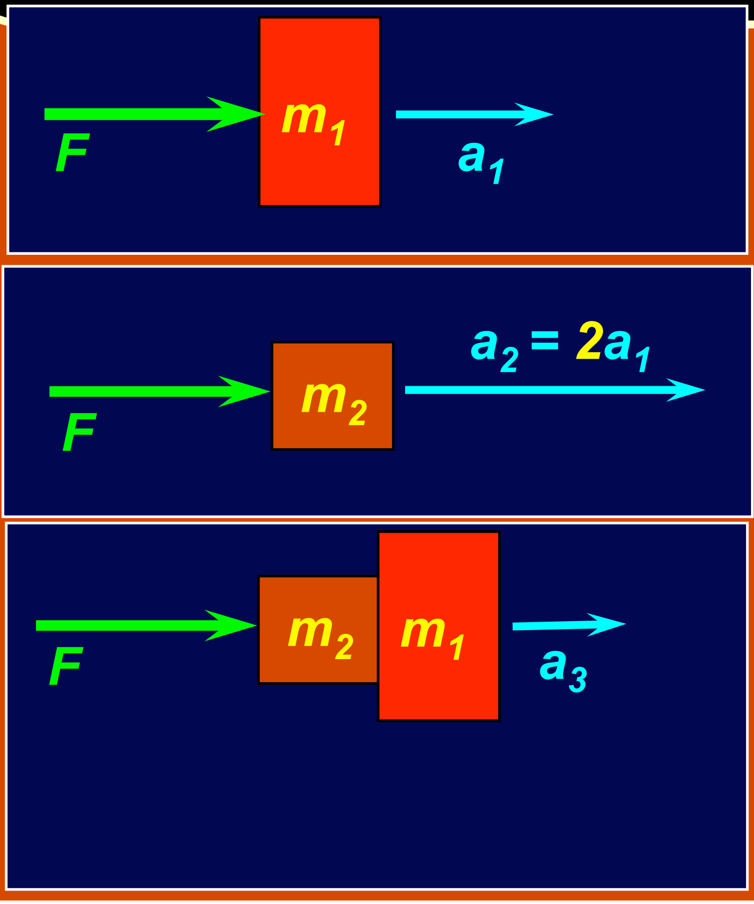
If $F = 100 lbs$ then $T = 700 lbs$

The smaller the angle the larger the magnification

ConcepTest 4.6 Force and Two Masses

A force F acts on mass m_1 giving acceleration a_1. The same force acts on a different mass m_2 giving acceleration $a_2 = 2a_1$. If m_1 and m_2 are glued together and the same force F acts on this combination, what is the resulting acceleration?

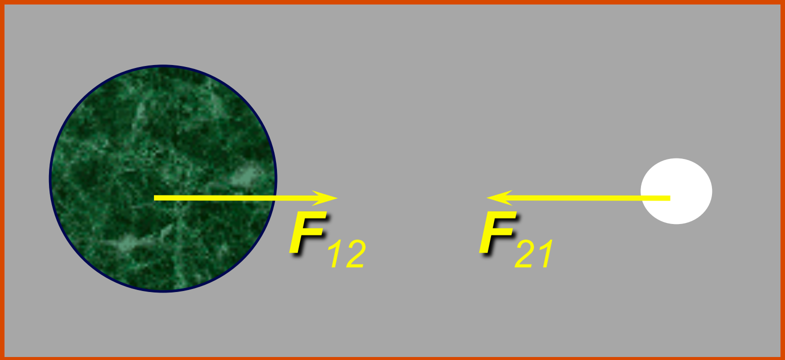
1) $3/4\ a_1$
2) $3/2\ a_1$
3) $1/2\ a_1$
4) $4/3\ a_1$
5) $2/3\ a_1$



ConcepTest 4.8a Bowling vs. Ping-Pong I

In outer space, a **bowling ball** and a **ping-pong ball** attract each other due to gravitational forces. How do the magnitudes of these attractive forces compare?

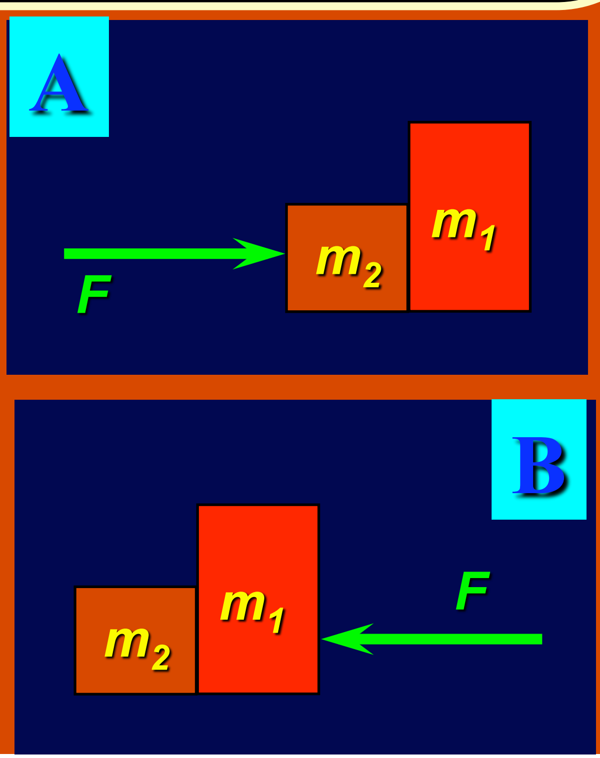
1) The bowling ball exerts a greater force on the ping-pong ball
2) The ping-pong ball exerts a greater force on the bowling ball
3) The forces are equal
4) The forces are zero because they cancel out
5) There are actually no forces at all



ConcepTest 4.10a Contact Force I

If you push with force F on either the heavy box (m_1) or the light box (m_2), in which of the two cases below is the contact force between the two boxes larger?

1) case A
2) case B
3) same in both cases



ConcepTest 5.3b Tension II

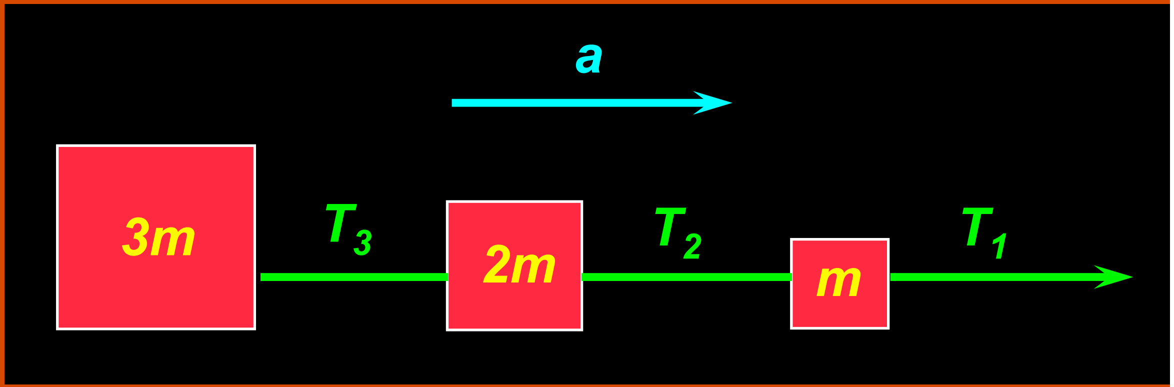
Two tug-of-war opponents each pull with a force of *100 N* on opposite ends of a rope. What is the tension in the rope?

1) *0 N*
2) *50 N*
3) *100 N*
4) *150 N*
5) *200 N*

ConcepTest 5.4 Three Blocks

Three blocks of mass *3m*, *2m*, and *m* are connected by strings and pulled with constant acceleration *a*. What is the relationship between the tension in each of the strings?

1) $T_1 > T_2 > T_3$
2) $T_1 < T_2 < T_3$
3) $T_1 = T_2 = T_3$
4) *all tensions are zero*
5) *tensions are random*



ConcepTest 5.5 Over the Edge

In which case does block *m* experience a larger acceleration? In (1) there is a *10 kg* mass hanging from a rope and falling. In (2) a hand is providing a constant downward force of *98 N*. Assume massless ropes.

1) *case 1*
2) *acceleration is zero*
3) *both cases are the same*
4) *depends on value of m*
5) *case 2*