Find dy/dx

$$y = x^5$$

$$y = x^n$$

$$y = a$$

$$y = 8x^2 + 12x + 3$$

$$y = \sin x$$

$$y = \ln x$$

Integrate (Anti-derivative)

$$\int x^n \, dx$$

$$\int dx$$

$$\int a \, dx$$

$$\int (v_0 + at) \, dt$$

$$\int (ay^3 \pm by^2) dy$$

$$\int x \, dx$$

$$\int \cos\theta \, d\theta$$

Three Important Rules of Differentiation

Power Rule

$$y = cx^n$$

$$dy / dx = ncx^{n-1}$$

$$y = 30x^5$$

$$\frac{dy}{dx} = 5(30)x^4 = 150x^4$$

Three Important Rules of Differentiation

Product Rule

$$y(x) = f(x)g(x)$$

$$\frac{dy}{dx} = \frac{df}{dx}g(x) + f(x)\frac{dg}{dx}$$

$$y = 3x^2(\ln x)$$

$$f(x) = 3x^2$$
$$g(x) = \ln x$$

$$\frac{dy}{dx} = 2(3)x(\ln x) + 3x^2(\frac{1}{x}) = 6x\ln x + 3x$$
$$\frac{dy}{dx} = 3x(2\ln x + 1)$$

Three Important Rules of Differentiation

Chain Rule

$$y(x) = y(g(x))$$

$$\frac{dy}{dx} = \frac{dy}{dg}\frac{dg}{dx}$$

$y = (5x^2 - 1)^3 = g^3$ where $g = 5x^2 - 1$

$$\frac{dy}{dg} = 3g^2$$

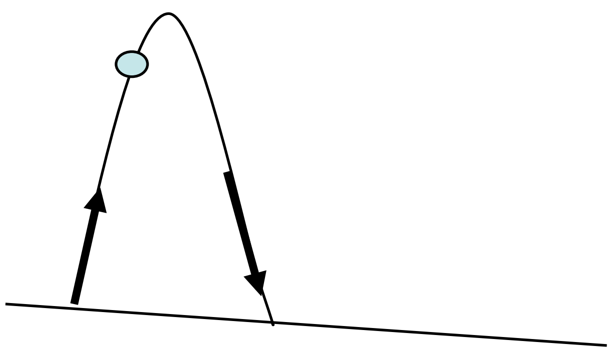
$$\frac{dg}{dx} = 10x$$

$$\frac{dy}{dx} = 3g^2(10x)$$

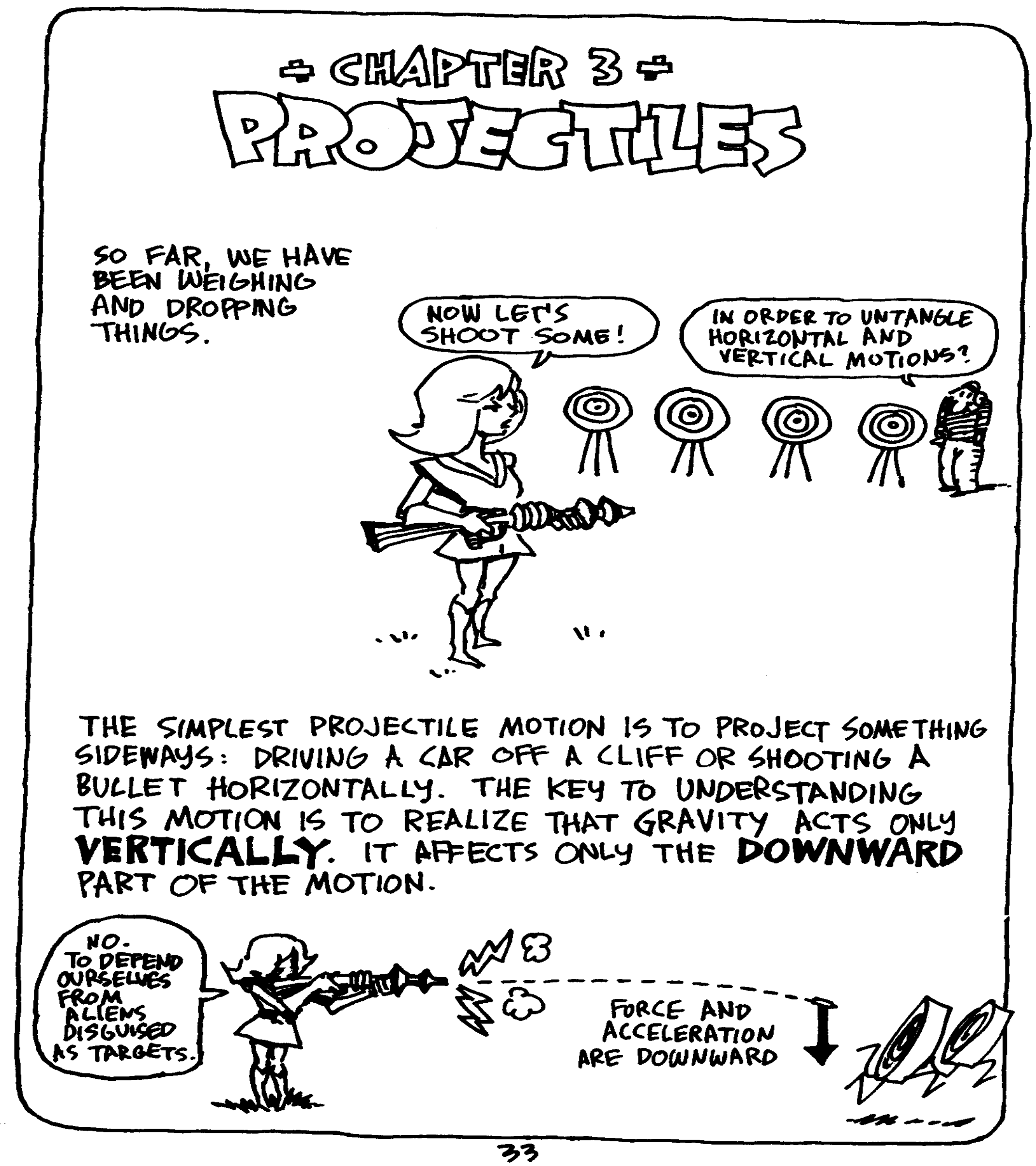
$$\frac{dy}{dx} = 30x(5x^2 - 1)^2$$

Lecture 2: Chapter 4

Motion in two and three dimensions



x and y components



1D Motion

3 independent equations

$$v = v_0 + at$$

$$v_{avg} = \frac{1}{2}(v_0 + v)$$

$$x = x_0 + v_{av}t$$

Derive these 2 from the other 3

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

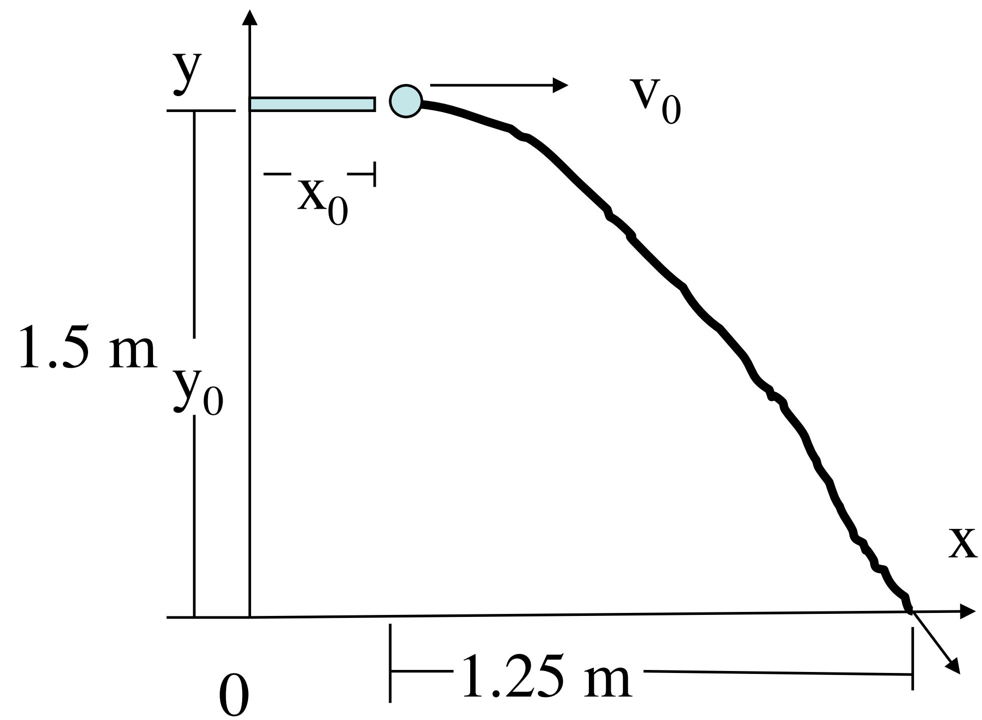
Demo Problem

A projectile at 1.5 m high is shot horizontally with speed v_0 that causes it to land 1.25 m away.

What is the time it is in the air

What is v_0?

What is v_x and v_y and the speed v when it lands?



x direction

3 independent equations

$$v_x = v_{0x} + a_x t$$

$$v_{avgx} = \frac{1}{2}(v_{0x} + v_x)$$

$$x = x_0 + v_{avgx} t$$

Derive these 2 from the other 3

$$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$$

y direction

3 independent equations

$$v_y = v_{0y} + a_y t$$

$$v_{avgy} = \frac{1}{2}(v_{0y} + v_y)$$

$$y = y_0 + v_{avgy} t$$

Derive these 2 from the other 3

$$y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$$

$$v_y^2 = v_{0y}^2 + 2a_y (y - y_0)$$

Now we want to use what we know about x and y components of a vector to help understand motion in the xy plane: We call this Projectile Motion

- Galileo separated motion into Horizontal and Vertical
- Horizontal: v is constant provided a or F = 0
- Vertical: a is constant provided F = constant

(air resistance or friction is 0)

Demo of projectile balls

Use SUPERPOSITION to combine x and y motion to find resultant. Consider a ball projected horizontally with initial speed v_0

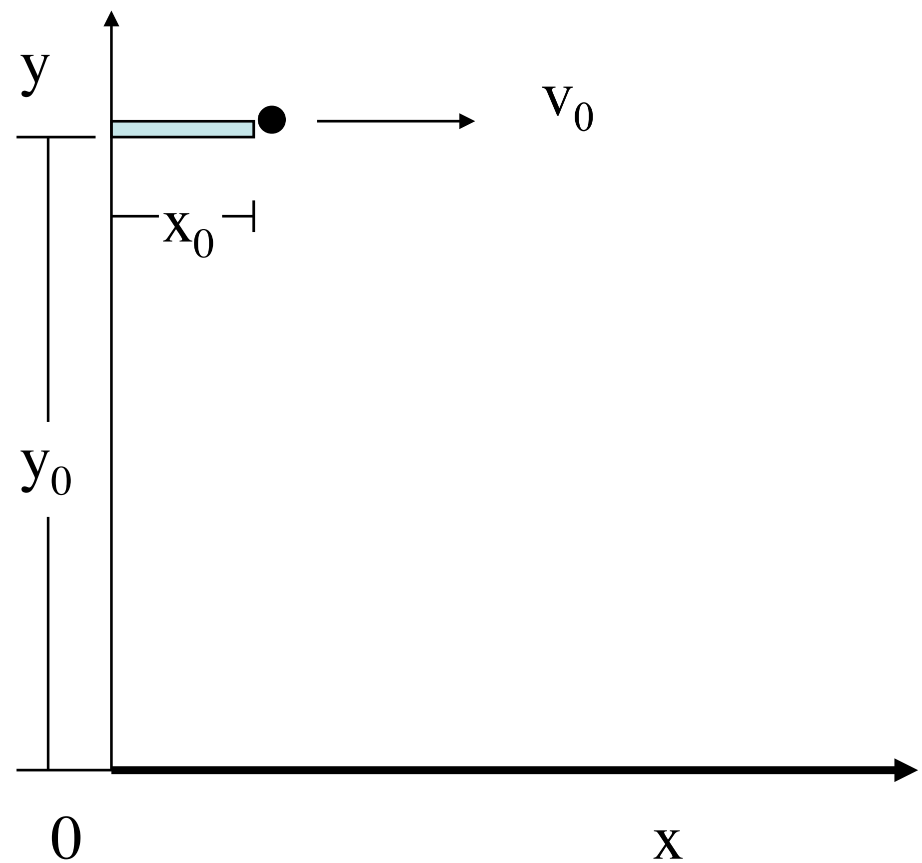
$$x = x_0 + v_{0x} t$$

$$y = y_0 - \frac{1}{2} g t^2$$

$$v_y = -gt$$

$$v_x = v_{0x}$$

$$v_{0y} = 0$$



This is called projectile motion. You can eliminate t to find trajectory in terms of x and y. Get a parabola.

Demo Problem

A projectile at 1.5 m high is shot horizontally with speed v_0 that causes it to land 1.25 m away.

What is time in the air?

t = time in the air:

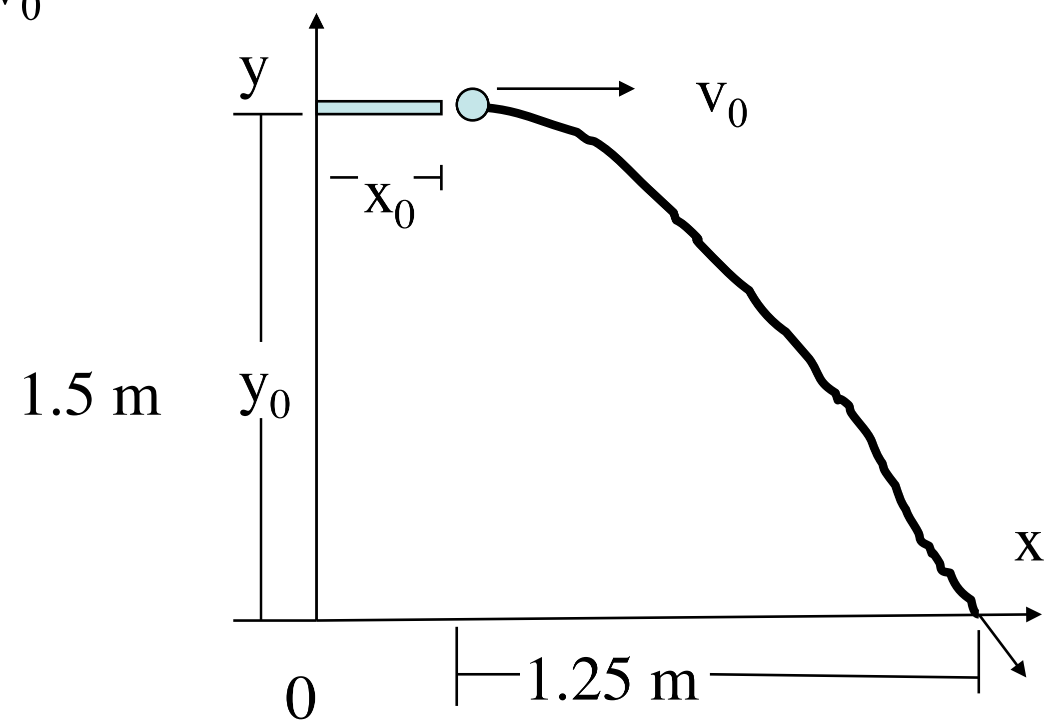
$$y = y_o - \frac{1}{2} g t^2$$

$$0 = 1.50 - 5t^2$$

$$t^2 = 0.30$$

$$t = 0.55$$

What is v_0?



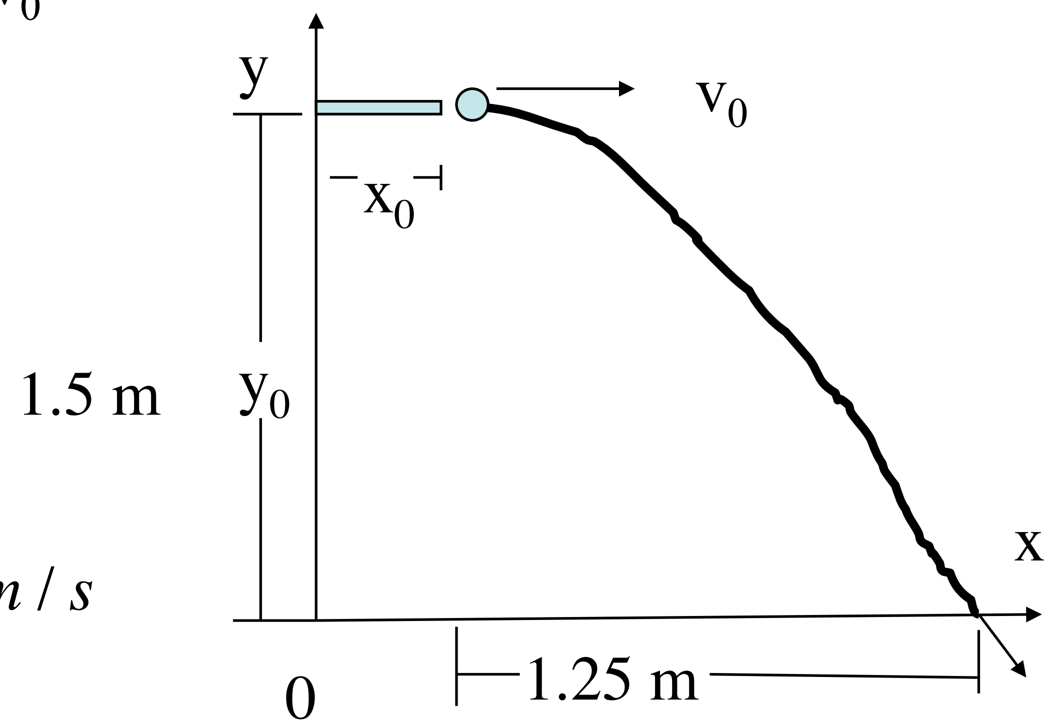
Demo Problem

A projectile at 1.5 m high is shot horizontally with speed v_0 that causes it to land 1.25 m away.

What is v_0?

$$v_x = v_0$$

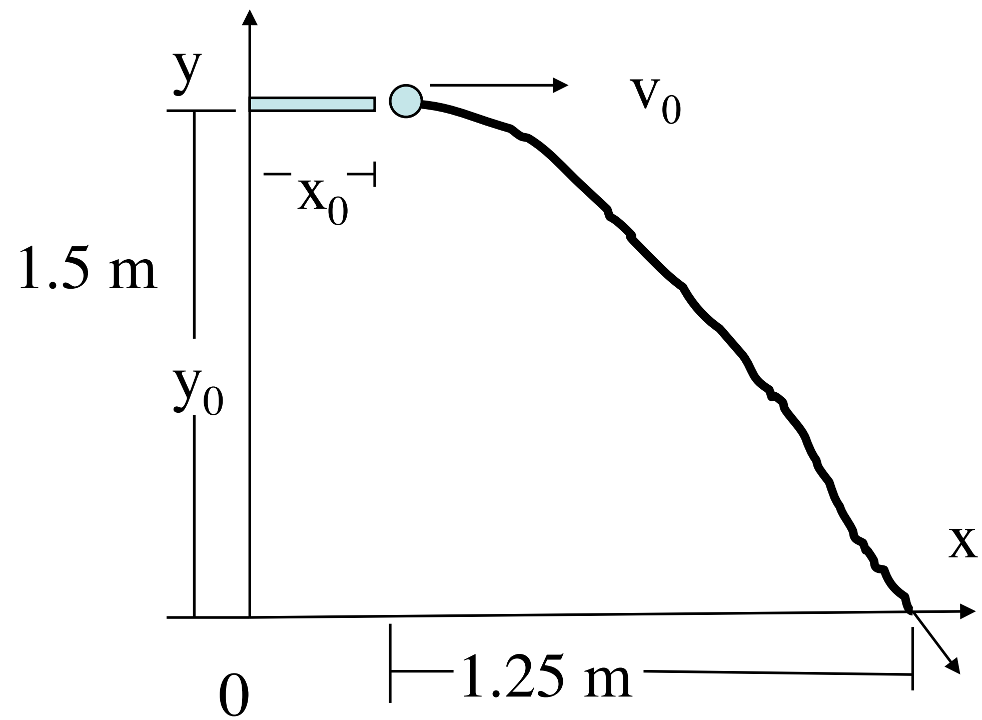
$$v_o = x / t = 1.25 / 0.55 = 2.27 m / s$$



Demo

A projectile at 1.5 m high is shot horizontally with speed v_0 that causes it to land 1.25 m away?

Find v_y and the speed v when it lands?



$$v_y = -gt = -10(0.55) = -5.5 m/s$$

$$v = \sqrt{v_x^2 + v_y^2}$$

$$v = \sqrt{(2.27)^2 + (-5.50)^2}$$

$$v = 5.05 \ m/s$$

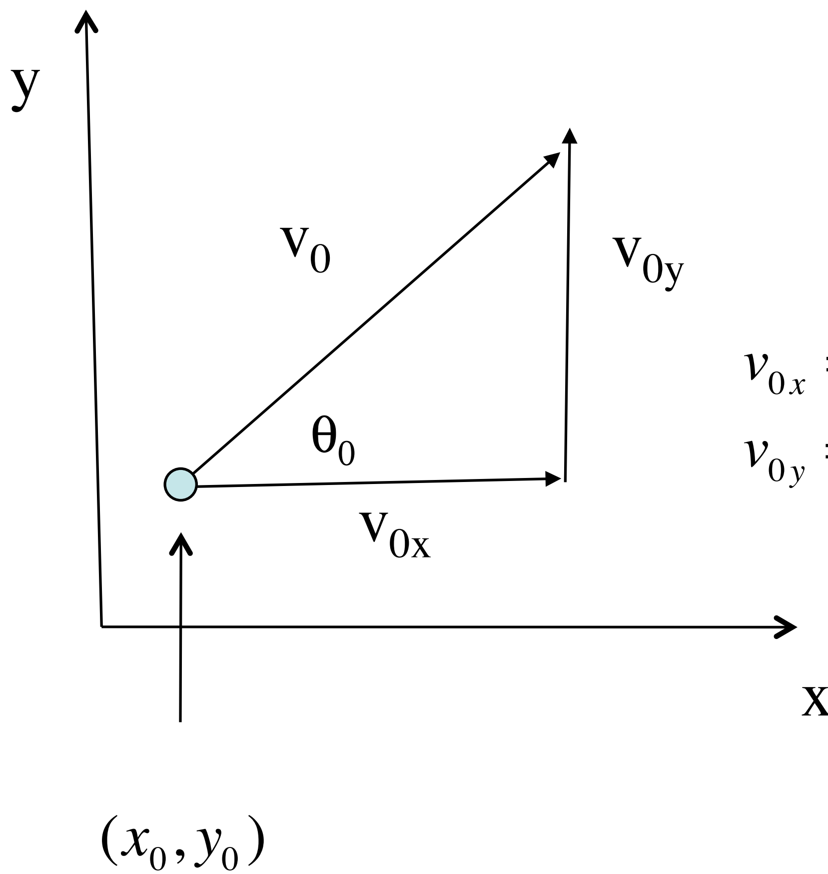
Projectile Motion

$$x = x_0 + v_{0x}t$$

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

$$v_y = v_{0y} - gt$$

$$v_x = v_{0x}$$



$$v_{0x} = v_0 \cos\theta_0$$

$$v_{0y} = v_0 \sin\theta_0$$

$$x - x_0 = (v_0 \cos\theta_0)t$$

$$y - y_0 = (v_0 \sin\theta_0)t - \tfrac{1}{2}gt^2$$

$$v_y = v_0 \sin\theta_0 - gt$$

$$v_y^2 = (v_0 \sin\theta_0)^2 - 2g(y - y_0)$$

$$v_x = v_0 \cos\theta_0$$

Projectile Motion

$$x - x_0 = (v_0 \cos\theta_0)t$$

$$y - y_0 = (v_0 \sin\theta_0)t - \tfrac{1}{2}gt^2$$

$$v_y = v_0 \sin\theta_0 - gt$$

$$v_y^2 = (v_0 \sin\theta_0)^2 - 2g(y - y_0)$$

$$v_x = v_0 \cos\theta_0$$

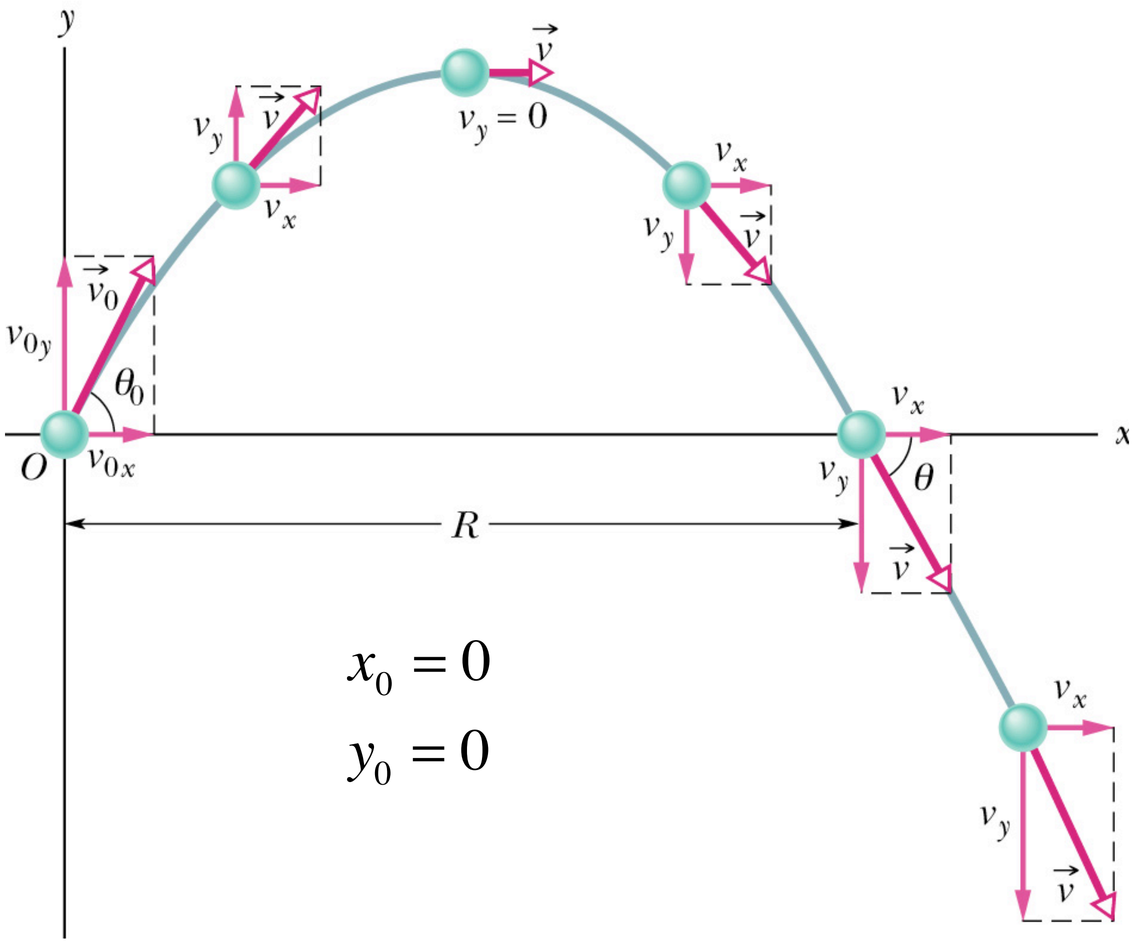
$$x = (v_0 \cos\theta_0)t$$

$$y = (v_0 \sin\theta_0)t - \tfrac{1}{2}gt^2$$

$$v_y = v_0 \sin\theta_0 - gt$$

$$v_y^2 = (v_0 \sin\theta_0)^2 - 2g(y - y_0)$$

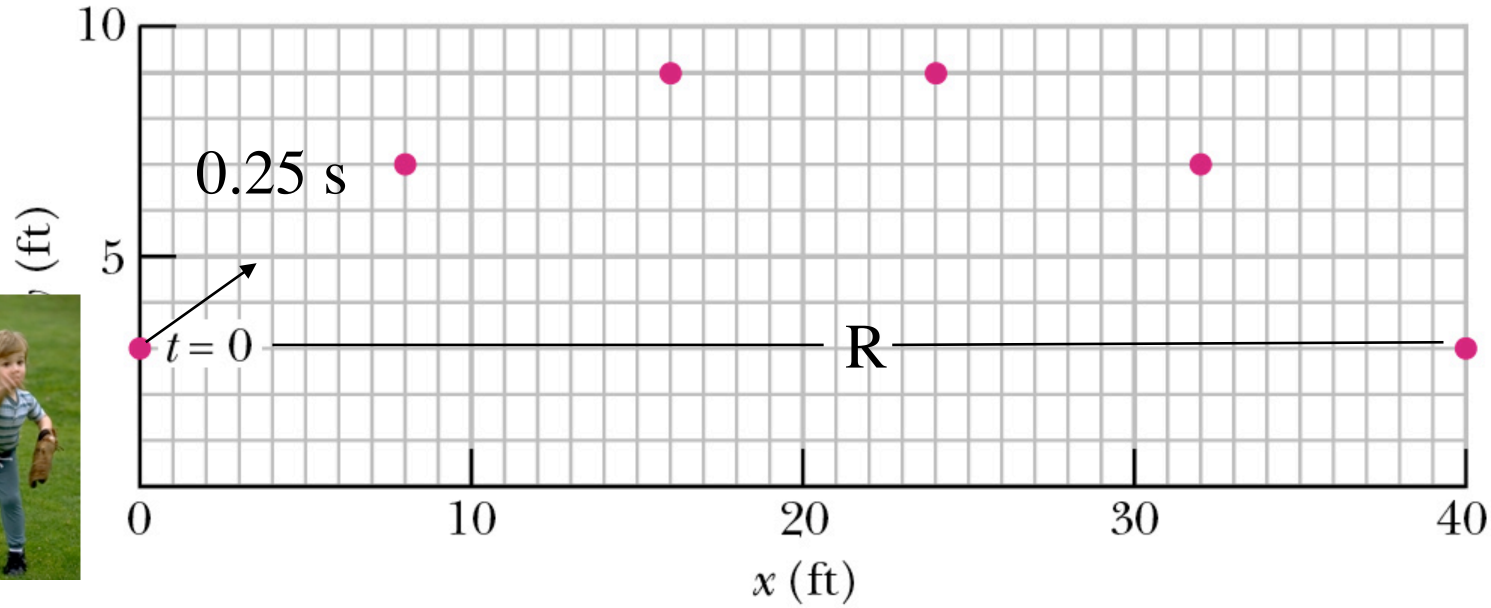
$$v_x = v_0 \cos\theta_0$$



$$x_0 = 0$$

$$y_0 = 0$$

Slow Pitch Problem #106 ed. 7



A baseball is released at 3 ft above ground level. A stroboscopic plot shows the position of the ball every 0.25 sec. Answer the following questions.

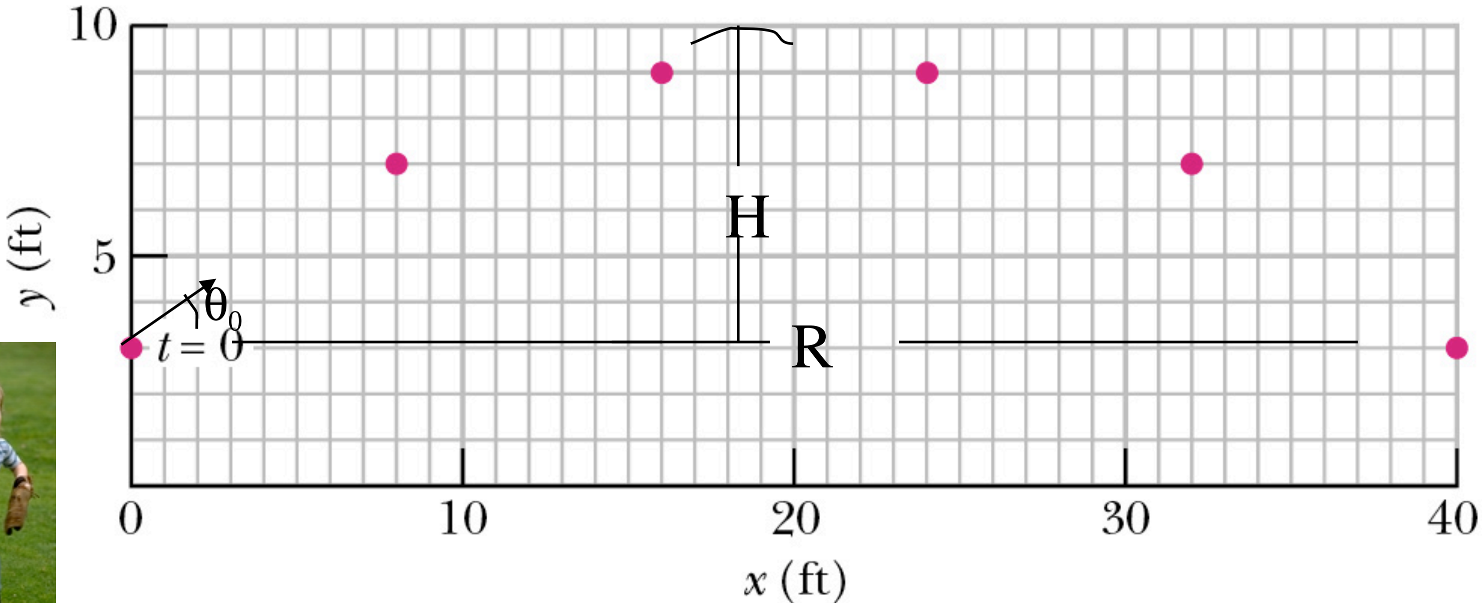
a) How far does it go when it reaches the batter?

R=40 ft

b) How long does it take the ball to reach the bat?

t = 1.25 s

c) What is the initial speed of the ball?

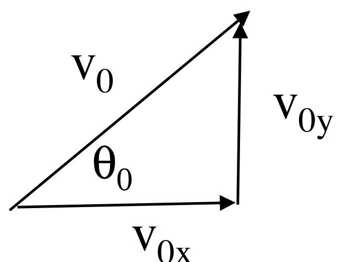


c) What is the initial speed of the ball?

$v_0 = (v^2_{0x} + v^2_{0y})^{1/2}$

Find v_{0x}

$$v_{0x} = R / t$$



$$v_{0x} = 40\,ft / 1.25\,s = 32\,ft / s$$

To find v_{0y}. Consider what happens when the ball reaches the batter.

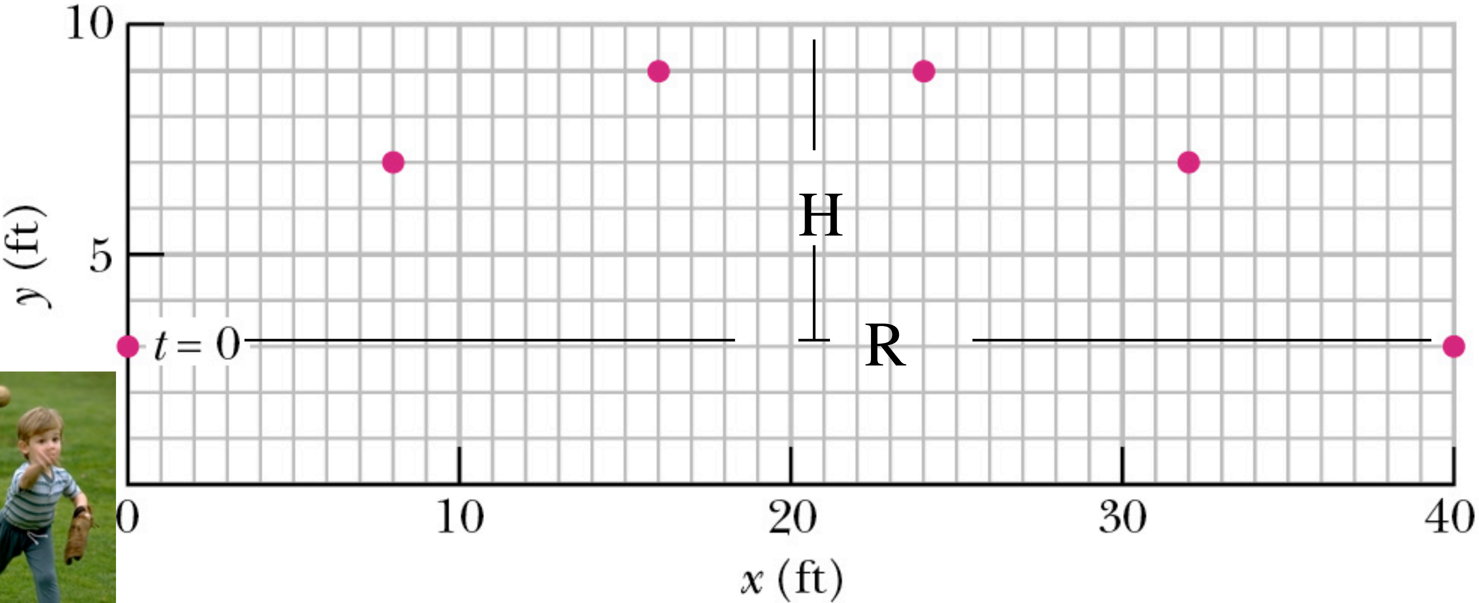
$$y - y_0 = (v_{0y})t - \tfrac{1}{2}gt^2$$

$$0 = v_{0y}t - \tfrac{1}{2}gt^2$$

$$v_{0y} = \tfrac{1}{2}gt$$

$$v_{0y} = \frac{1}{2}(32)(1.25) = 20\,ft / s$$

FINALLY $\quad v_0 = \sqrt{20^2 + 32^2} = 38\,ft / s$



d) How high does the ball go above the ground? It reaches the maximum when $v_y = 0$.

$$v_y = v_{0y} - gt = 0$$

$$t = v_{0y} / g$$

$$H = v_{0y}t - \tfrac{1}{2}gt^2$$

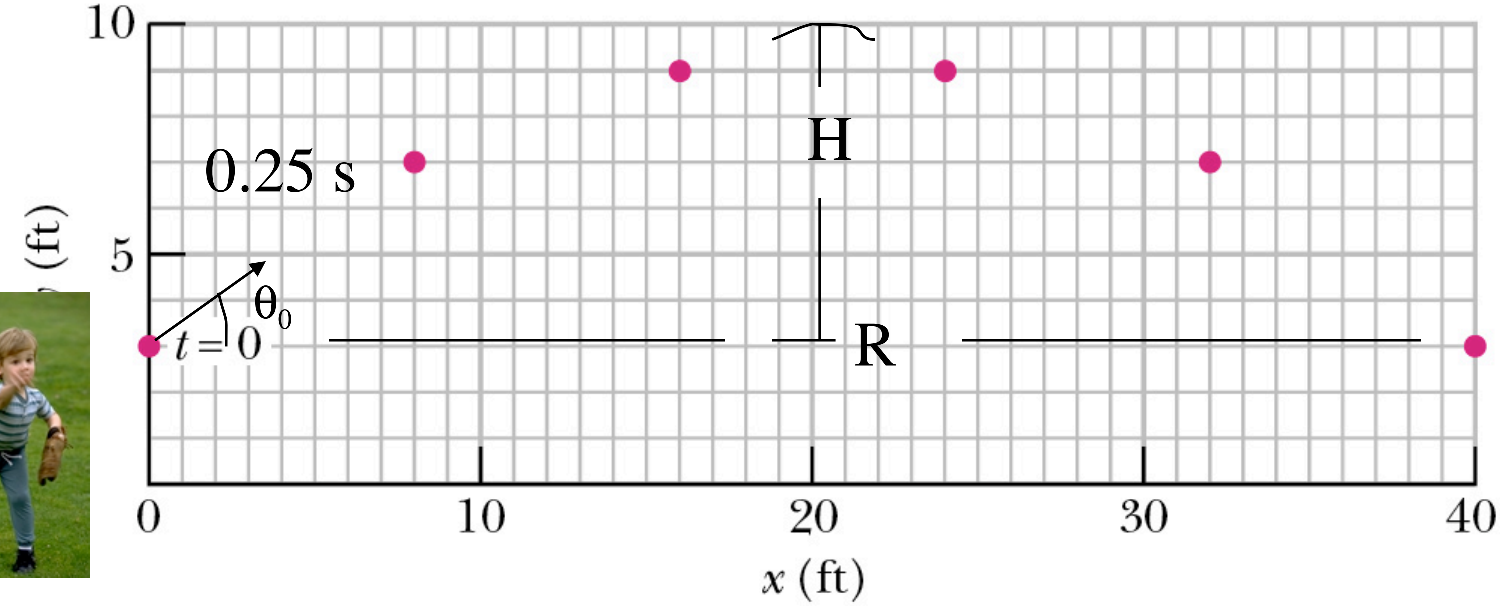
$$H = (v_{0y})^2 / g - \tfrac{1}{2}g(v_{0y} / g)^2$$

$$H = {v_{0y}}^2 / 2g$$

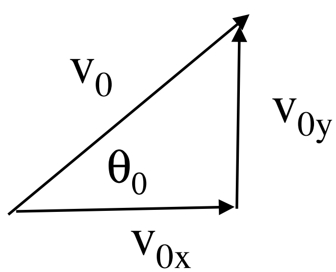
$$H = \frac{20^2}{2 * 32} = 6.25\,ft$$

$y = 6.25 + 3 = 9.25\,ft$ (Above the ground)

Slow Pitch Problem



e) What is the launch angle?

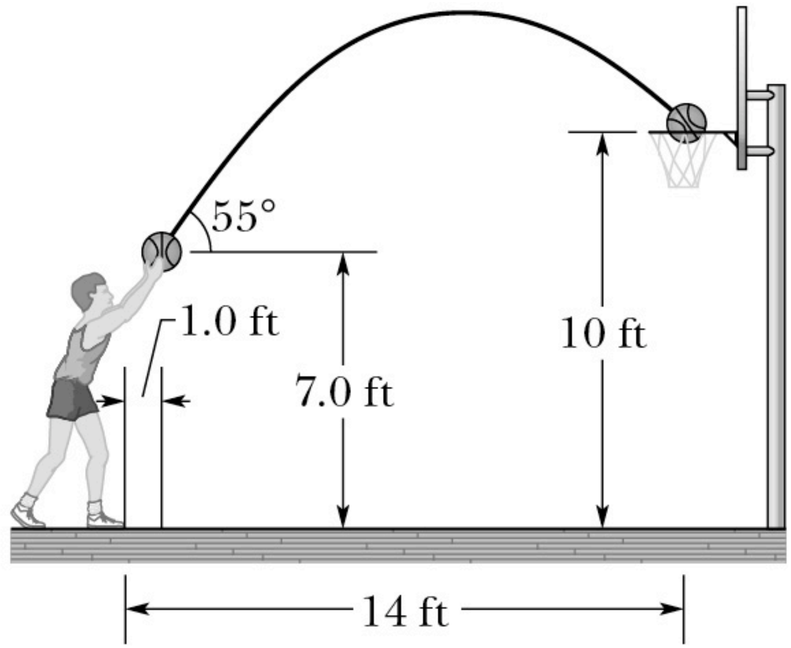


$$\sin\theta_0 = \frac{v_{0y}}{v_0}$$

$$\theta_0 = \sin^{-1}\left(\frac{v_{0y}}{v_0}\right) = \sin^{-1}\left(\frac{20}{38}\right) = 31.8^o$$

Foul Shot problem

Find the initial speed and the time it takes for the ball to go through the basket.



$$x - x_0 = (v_0 \cos\theta_0)t$$

$$y - y_0 = (v_0 \sin\theta_0)t - \tfrac{1}{2}gt^2$$

$$x - x_0 = 14 - 1 = 13\,ft \quad\Longrightarrow\quad 13 = (v_0 \cos 55)t$$

$$y - y_0 = 10 - 7 = 3\,ft \quad\Longrightarrow\quad 3 = (v_0 \sin 55)t - \tfrac{1}{2}gt^2$$

$$\theta_0 = 55^o$$

Foul Shot Continued

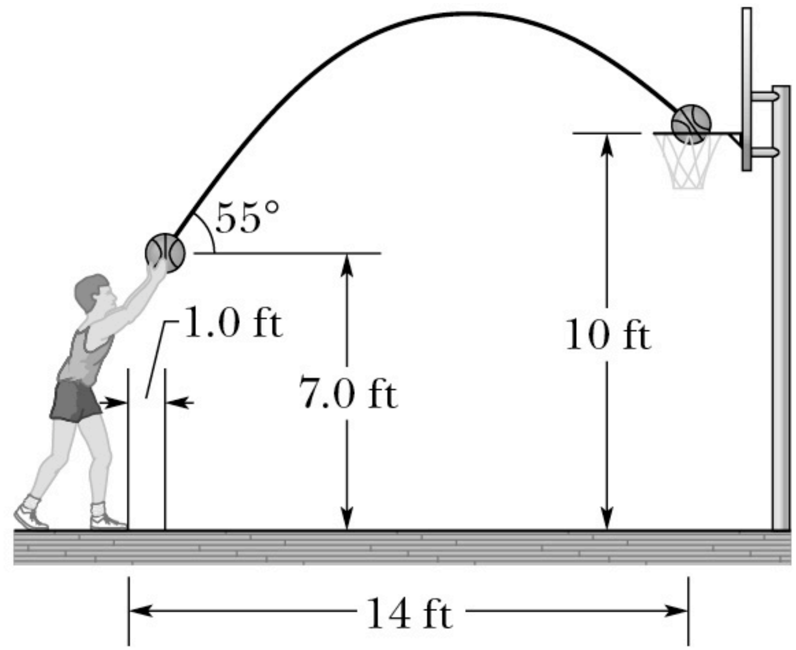
$$13 = (v_0 \cos 55)t$$

$$3 = (v_0 \sin 55)t - \tfrac{1}{2} g t^2$$

$$13 = 0.574 v_0 t, \quad v_0 t = 22.65$$

$$3 = 0.819 v_0 t - \tfrac{1}{2} g t^2$$

$$3 = 0.819(22.65) - \tfrac{1}{2} g t^2$$



$$-15.55 = -16t^2$$

$$t^2 = 0.972$$

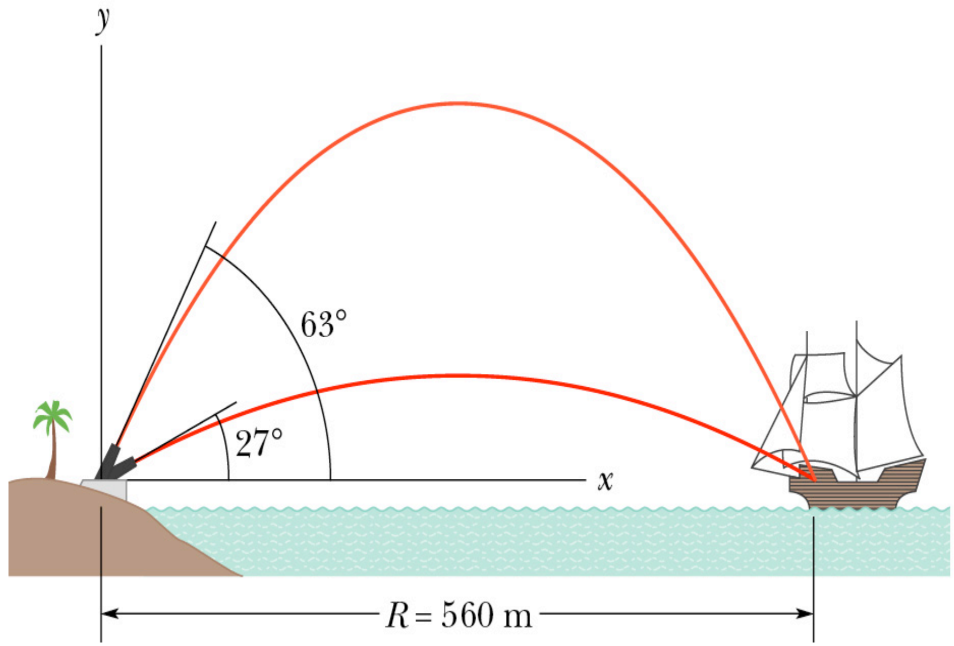
$$t = 0.986$$

$$v_0 = 22.65 / 0.986$$

$$v_0 = 22.97\, ft/s$$

Find the initial speed and find the time it takes for the ball to go through the basket

Horizontal Range formula



A cannon ball is fired with speed v_0 = 82 m/s at a ship 560 m from shore. What are the two launch angles needed to hit the ship?

x component $R = v_0 \cos\theta_0 t$

y component $0 = v_0 \sin\theta_0 t - \frac{1}{2} g t^2 \quad 0 = v_0 \sin\theta_0 - \frac{1}{2} g t$

Solve for t $t = \dfrac{2 v_0 \sin\theta_0}{g}$ $\qquad R = \dfrac{2 v_0^2 \sin\theta_0 \cos\theta_0}{g} = \dfrac{v_0^2 \sin 2\theta_0}{g}$

Horizontal Range continued

$$R = \frac{2v_0^2 \sin\theta_0 \cos\theta_0}{g} = \frac{v_0^2 \sin 2\theta_0}{g}$$

$$2\theta_0 = \sin^{-1}(\frac{Rg}{v_0^2})$$

$$2\theta_0 = \sin^{-1}(\frac{560 * 9.8}{82}) = \sin^{-1}(.816)$$

$$2\theta_0 = 54.7^o$$

$\theta_0 = 27.3$ degrees

second solution $2\theta_0 = 180 - 54.7 = 125.3$ degrees

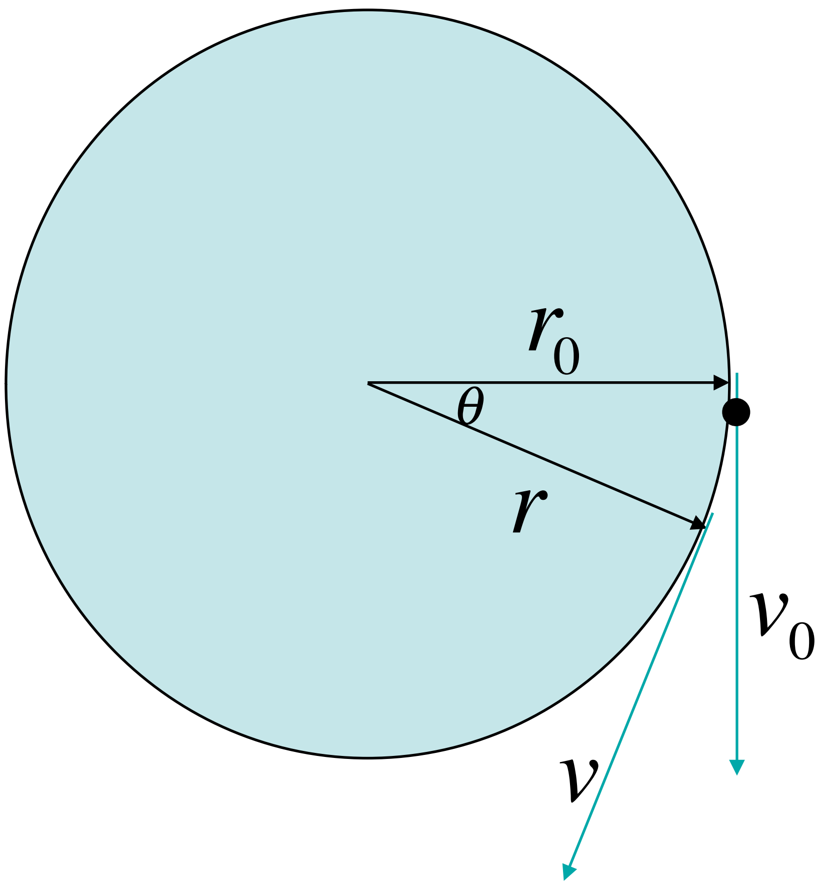
$\theta_0 = 62.6$ degrees

UNIFORM CIRCULAR MOTION

- Centripetal Acceleration: accelerates a body by changing the direction of the body's velocity without changing the speed. There must be a force also pointing radially inward to make this motion.
- Examples:
  - Ball on a string : show demo: Force is produced by the weight of the mass and transmitted by the tension in the string.
  - Moon in orbit around the earth: gravitational force
  - A car making a sharp turn: friction
  - A carousel; friction and contact forces

- Demo: pushing bowling ball with broom in a circle

CENTRIPETAL ACCELERATION:

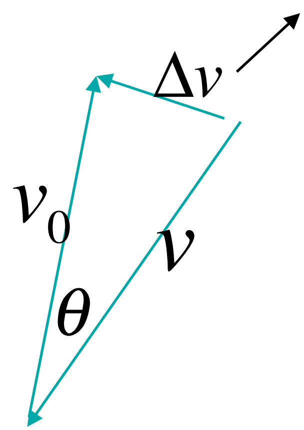


$$a_c = \Delta\vec{v} / \Delta t$$

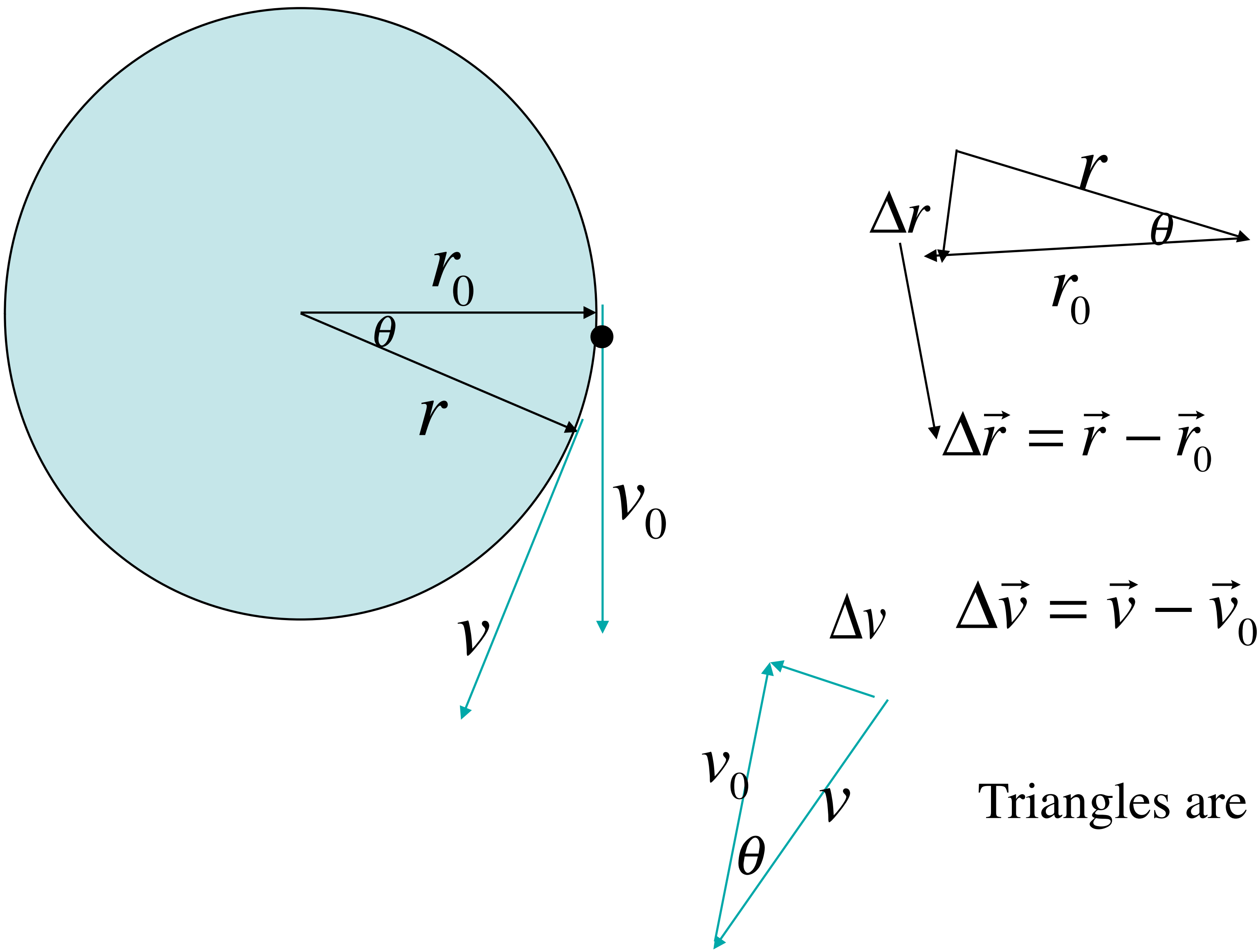
Δv points radially inward

Find $\Delta\vec{v}$

$$\Delta\vec{v} = \vec{v} - \vec{v}_0$$



CENTRIPETAL ACCELERATION



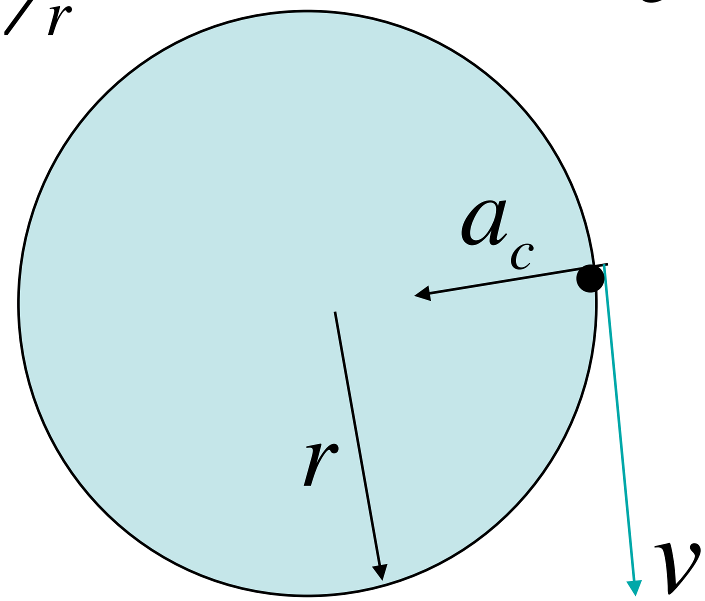
$$\Delta\vec{r} = \vec{r} - \vec{r}_0$$

$$\Delta\vec{v} = \vec{v} - \vec{v}_0$$

Triangles are similar

Centripetal Acceleration

Magnitudes are related by $|\Delta\vec{v}|/v = |\Delta\vec{r}|/r$ due to similar triangles



And, $\Delta r = v\Delta t$ so

$$\frac{\Delta v}{v} = \frac{(v\Delta t)}{r}$$

$$\frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

Magnitude of $a_c = \frac{v^2}{r}$

Period of the motion

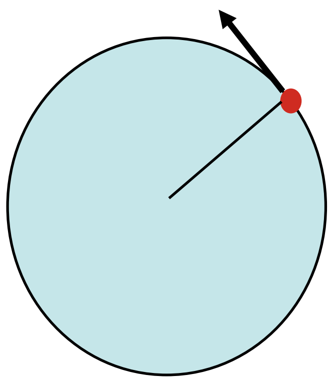
$$T = \frac{2\pi r}{v}$$

What is the magnitude of a_c and its direction for a radius of r = 0.5 m and a period of T= 2 s,

- $a_c = \frac{v^2}{r}$ Need to find v

$$v = \frac{2\pi r}{T} = \frac{2\pi * 0.5}{2} = 1.57 m/s$$

$$a_c = \frac{v^2}{r} = \frac{1.57^2}{0.5} = 4.92 m/s^2$$

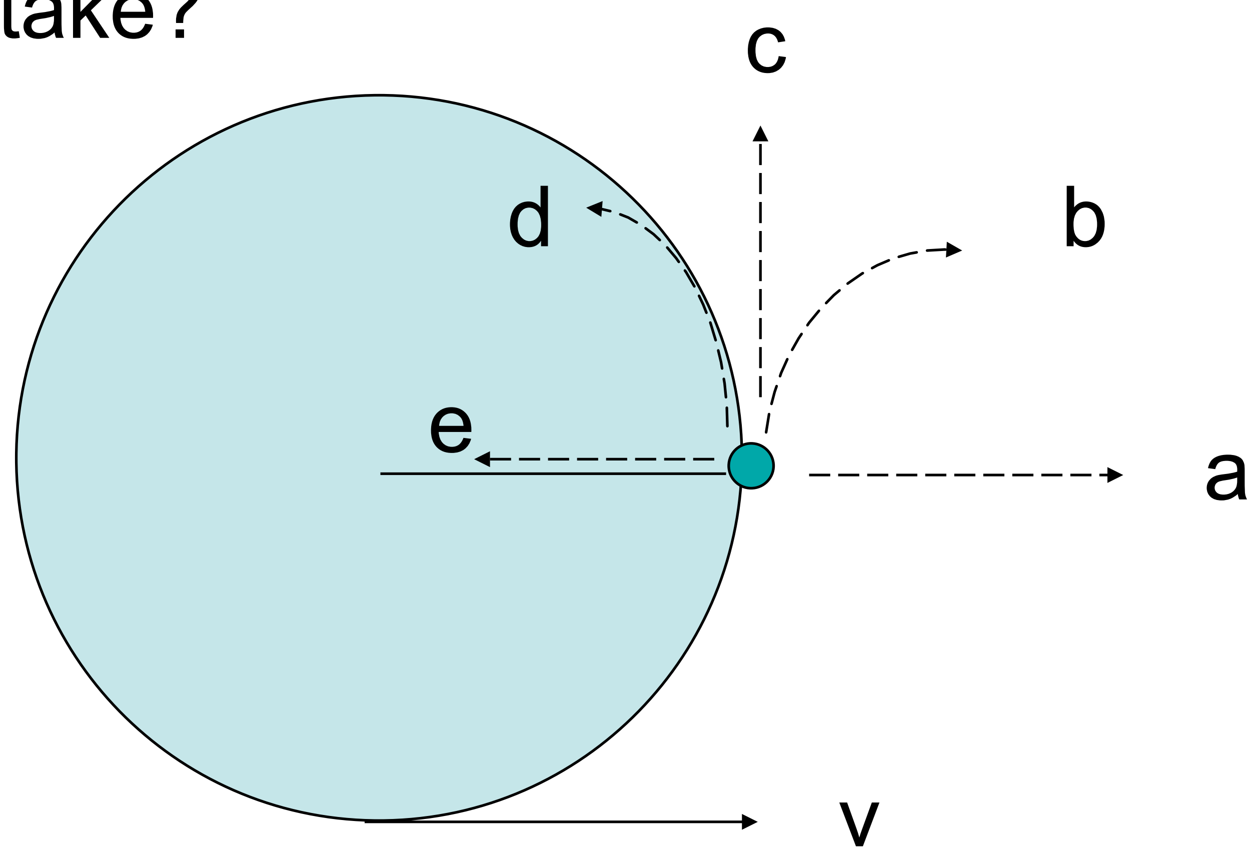


- What is the direction of a_c ?

INWARD

QUALITATIVE QUIZ

A ball is being whirled around on a string. The string breaks. Which path does the ball take?



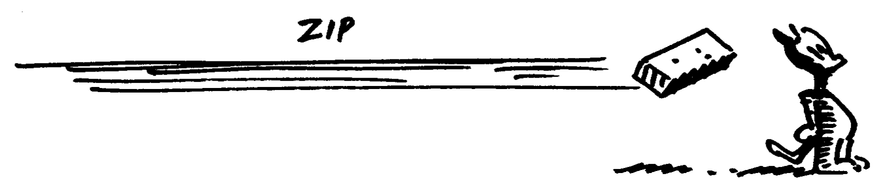
Chapter 5
Force and Motion I

ISAAC **NEWTON** (1642-1727) SUMMARIZED GALILEO'S IDEA AS **NEWTON'S FIRST LAW:**

AN OBJECT AT REST TENDS TO STAY AT REST. AN OBJECT IN MOTION TENDS TO CONTINUE IN MOTION AT CONSTANT SPEED IN A STRAIGHT LINE.

(HE ALSO SAID: "IF I HAVE SEEN FAR, IT IS BECAUSE I HAVE STOOD ON THE SHOULDERS OF GIANTS," MEANING GALILEO OF COURSE...)

IN THE TERMINOLOGY WE DEVELOPED IN CHAPTER ONE, WE WOULD SAY THAT WHEN THERE ARE NO FORCES, OBJECTS MOVE WITH **CONSTANT VELOCITY.**

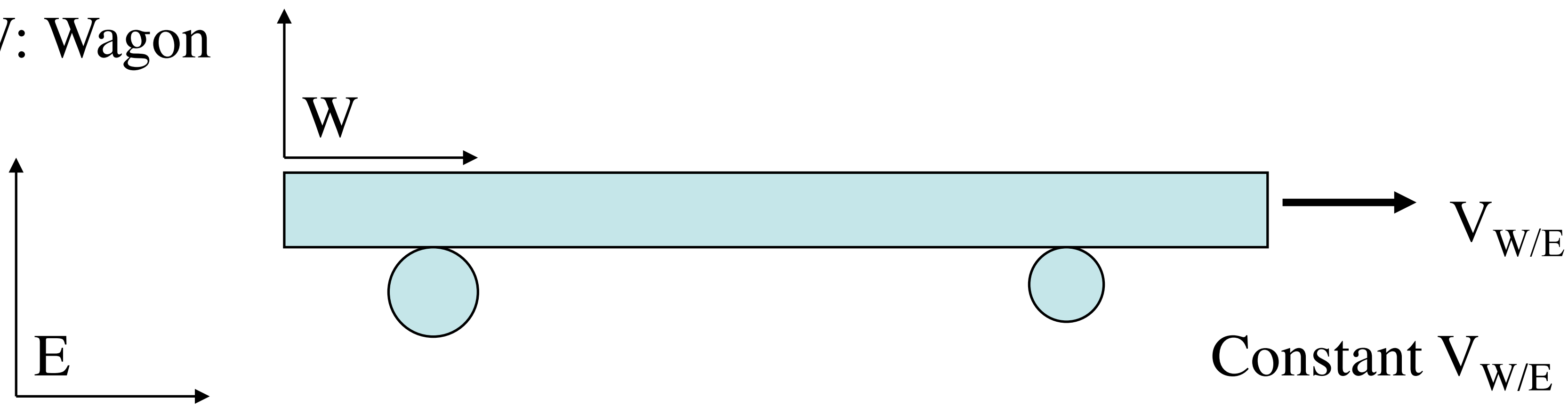


Important Concepts to Know

- **Principle of superposition** - net force = vector sum of all external forces acting on a body.
- **Inertial reference frame** - Where Newton's Laws hold true.
  one that is not accelerating- not the earth
- **Mass** - a characteristic that relates force to acceleration-
  leads to Newton's second Law $m = F/a$
- **Free body diagram** (modeling)
  - important in isolating forces on a body in order to determine its motion

Two Reference frames moving at a RELATIVE SPEED

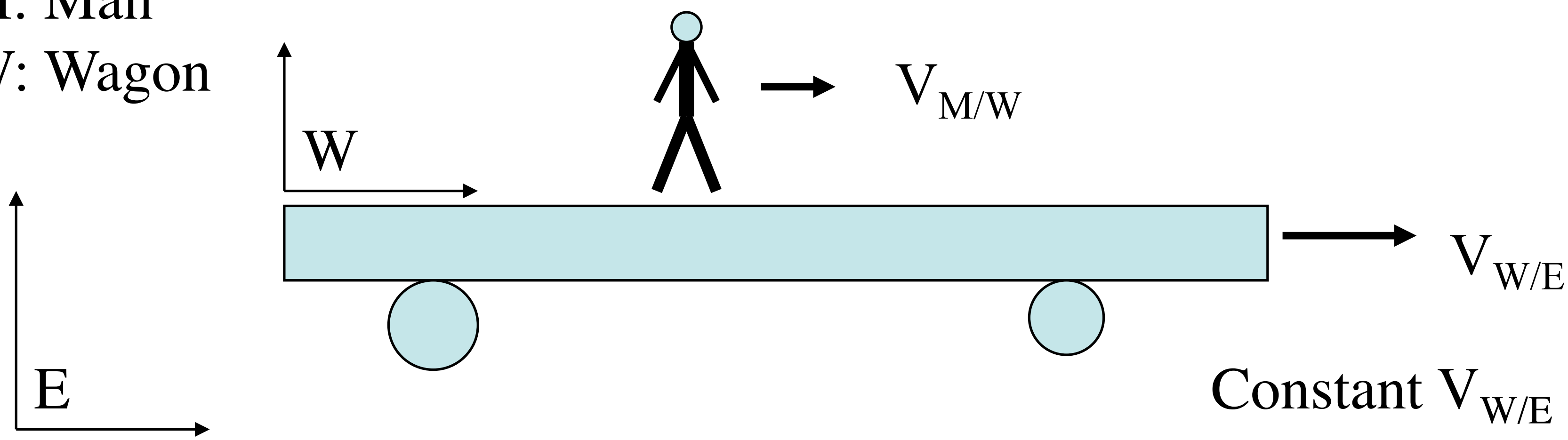
E: Earth
M: Man
W: Wagon



Constant $V_{W/E}$
$a_{W/E} = 0$

Two Reference frames moving at a RELATIVE SPEED

E: Earth
M: Man
W: Wagon

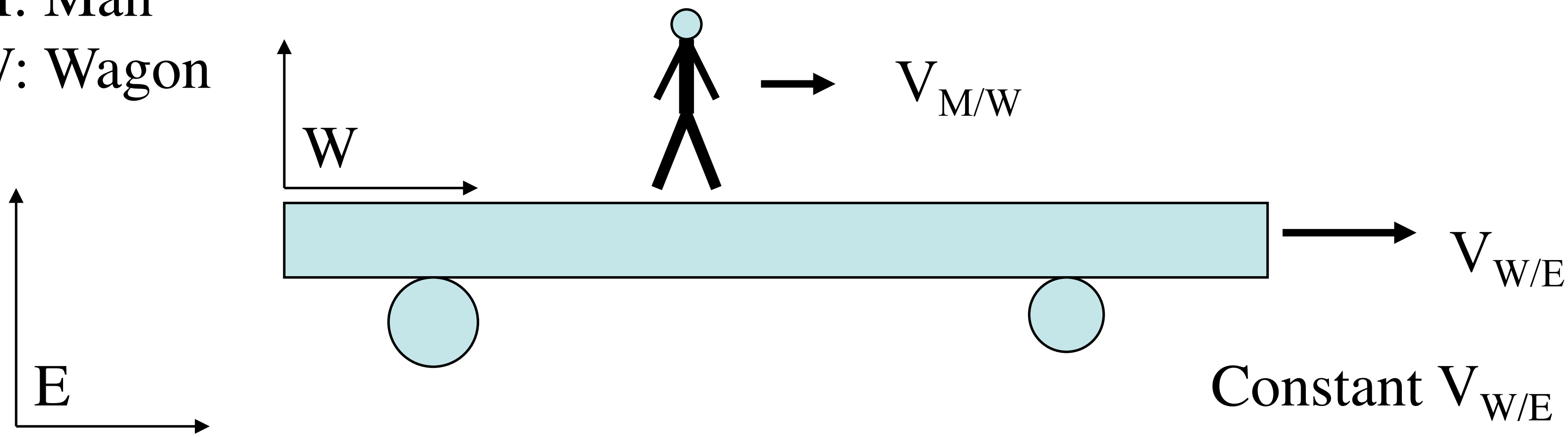


Constant $V_{W/E}$
$a_{W/E} = 0$

$$V_{M/E} = V_{M/W} + V_{W/E}$$

Two Reference frames moving at a RELATIVE SPEED

E: Earth
M: Man
W: Wagon



Constant $V_{W/E}$
$a_{W/E} = 0$

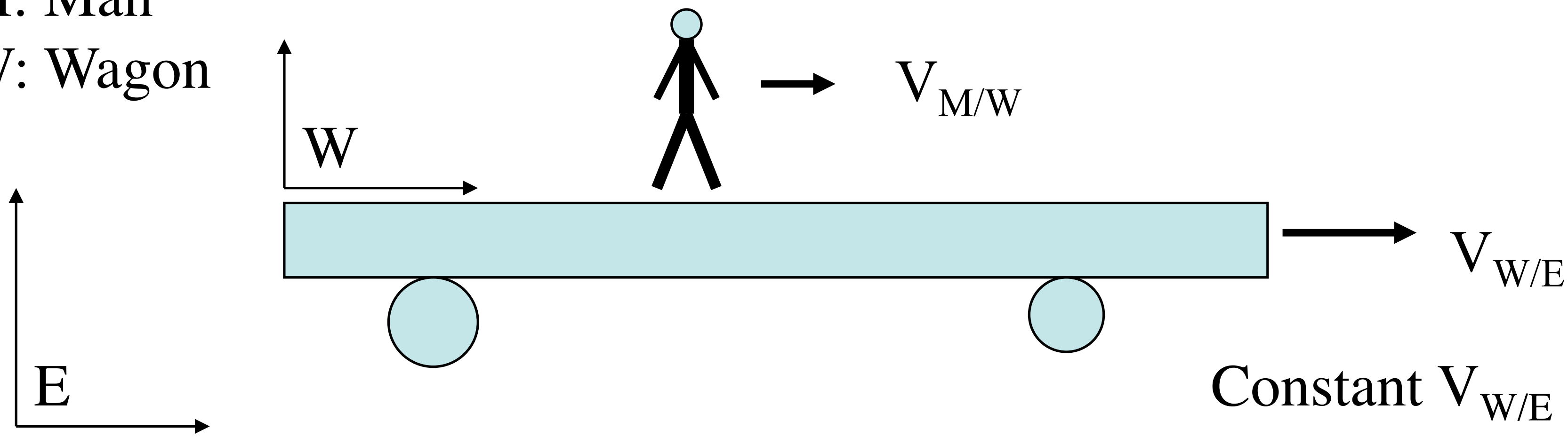
$$V_{M/E} = V_{M/W} + V_{W/E}$$

$$a_{M/E} = \frac{dv_{M/E}}{dt} = \frac{dv_{M/W}}{dt} + \overset{0}{\cancel{\frac{dv_{W/E}}{dt}}}$$

$$a_{M/E} = a_{M/W}$$

Two Reference frames moving at a RELATIVE SPEED

E: Earth
M: Man
W: Wagon



Constant $V_{W/E}$
$a_{W/E} = 0$

$$V_{M/E} = V_{M/W} + V_{W/E}$$

$$a_{/E} = {}_{M}a_{M/W}$$

Acceleration and forces are the same in Earth and Wagon reference frames as long as one frame moves at constant speed relative to the other.

GALILEAN RELATIVITY

Person on a moving ship drops a rock. They see it fall straight down to land at their feet.

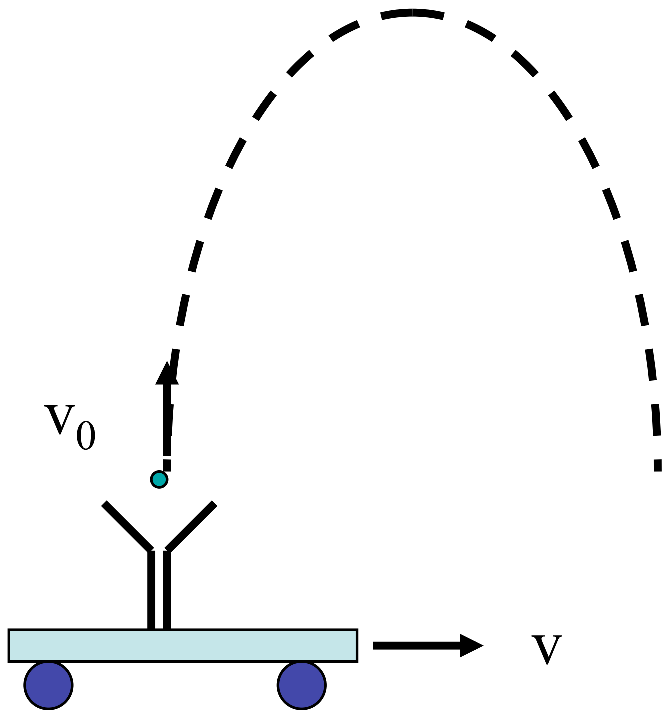
Someone on shore sees the rock continue to move horizontally as it falls, and says the trajectory is parabolic.

Another observer on another ship sees it move horizontally with a different speed and sees a different parabola.

But they all see it land at the same time. Galileo concluded that the horizontal motion **cannot** influence the vertical motion, and that <u>what takes place on the ship is independent of the ship's motion</u>. He argued this applies to all physical and biological processes.

The same principle applies to what takes place on a moving Earth.

RELATIVITY CAR



Force and Motion I

- Objects undergo accelerations. This is caused by an interaction between bodies. Such interactions are called forces. Recall most of our forces are contact forces
- Examples of Contact Forces:
  - A push or pull can be a force
  - Normal force,
  - Tension in a string
  - Friction
  - Two balls colliding.
- Example of Noncontact forces
  - Gravitation
  - Coulombs Law or Electric
- Newton's laws of motion: Published Newton's Principia.

Newton (1642)

In any direction: v = constant when $F = 0$

Forces cause acceleration

Newton's 3 Laws

Newton's Universal Law of Gravitation

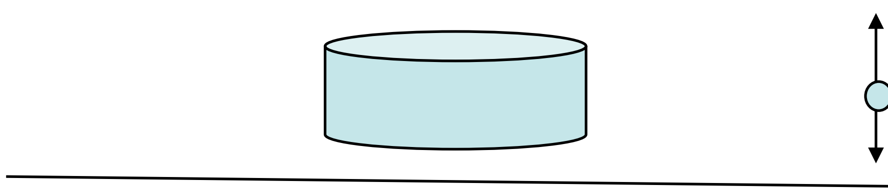
NEWTON'S FIRST LAW

An object continues in a state of rest or of motion at constant speed in a straight line unless acted upon by a net force.

If you don't push it, it won't move. A body has inertia.

Example : Hovering puck of mass m (Demo)

sum of vertical forces are zero



$$F_{air} = F_g$$

Inertia

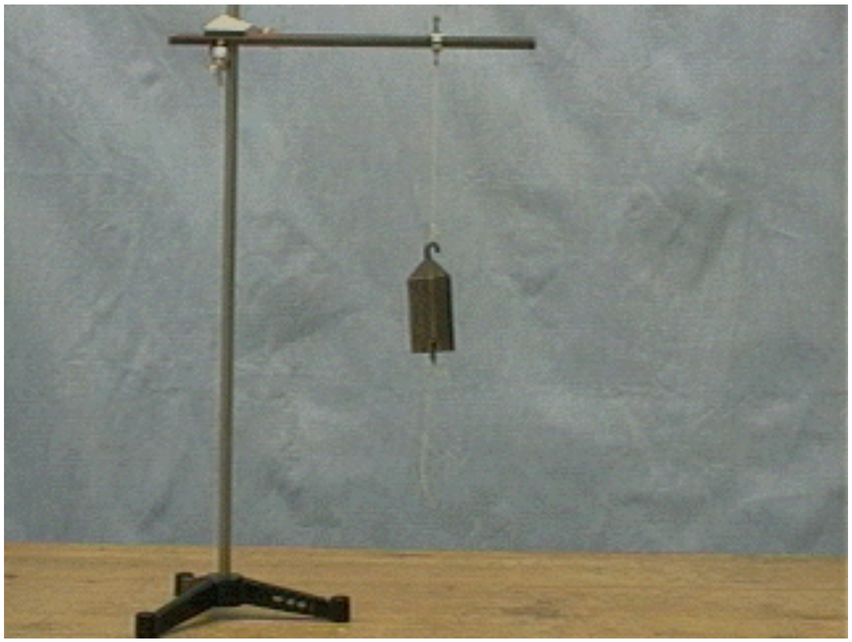
Another way of understanding the First Law

Inertia is a bodies resistance to change due to forces. Inertia is related to mass.

Some examples of inertia

- Hit a nail in a piece of wood on an anvil sitting on your head
- Table cloth jerk(Demo)
- Mass on string (Demo)

Two different ways of breaking the string: Inertia and Tension



Upper string breaks when you pull slowly because tension is greater

Lower string breaks when you pull quickly because of inertia

First pull fast see where it breaks

Then pull slowly see where it breaks

Explain

NEWTON'S SECOND LAW

$$\vec{a} = \frac{\vec{F}}{m} = \frac{1}{m}\sum_i \vec{F}_i$$

$\vec{F}$ = sum of all external forces acting on the body = net force

System	Mass	Acceleration	Force
SI	kg	m/s^2	Newton (N)
CGS	g	cm/s^2	dyne (dyn)
BE	slug (sl)	ft/s^2	pound (lb)

Now let's combine Newton's Second Law F=ma
and his Law of Gravitation to find the acceleration of gravity.

$$F = \frac{GmM}{R^2}$$ Law of Gravitation

$g = 9.8\ m/s^2$ Acceleration of gravity

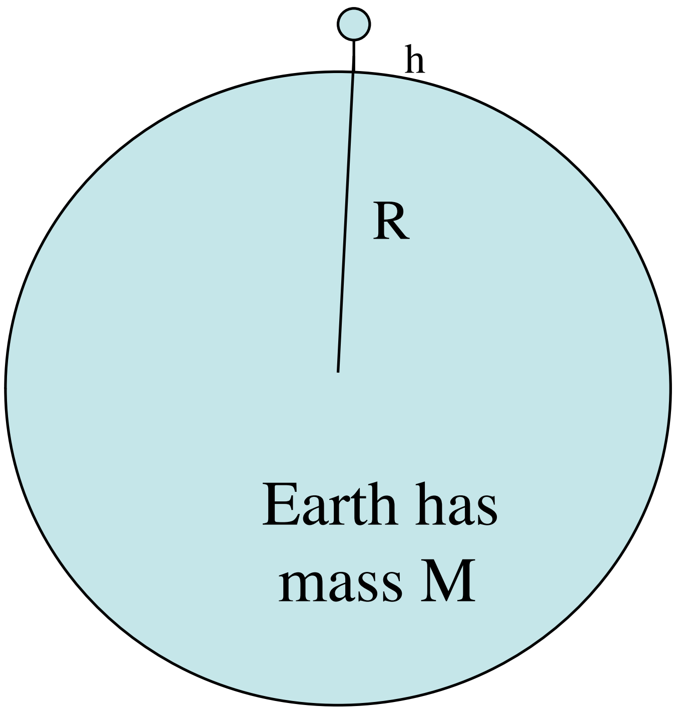
Where does the value of g come from?

Newton's Law of Gravitation and Weight

$$F = \frac{GmM}{r^2}$$

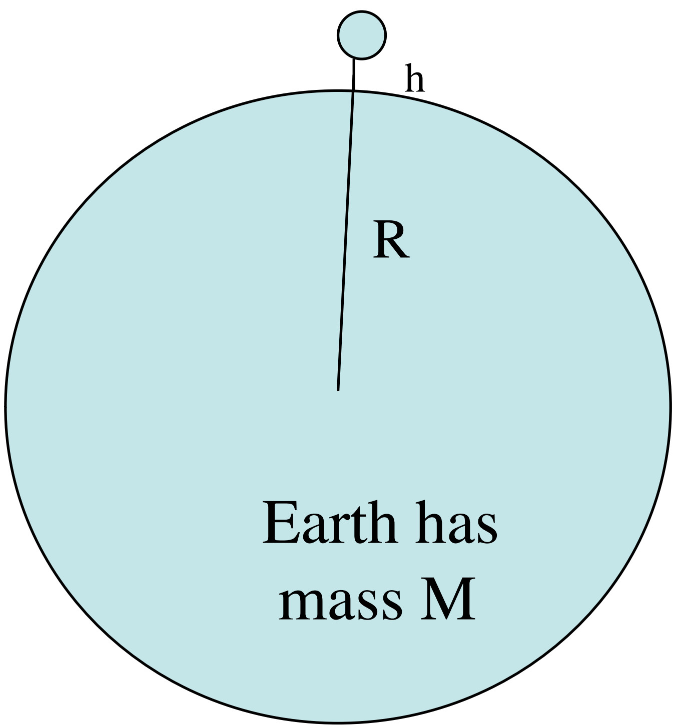
Mass m at height h

$$r = R + h$$



Where does the value of g come from?

Mass m at height h



$$F = \frac{GmM}{(R+h)^2}$$

$$R = 6.37 \times 10^6 m$$

$$M = 5.97 \times 10^{24} kg$$

$$G = 6.67 \times 10^{-11} m^3 / s^2 \cdot kg$$

h is small compared to R

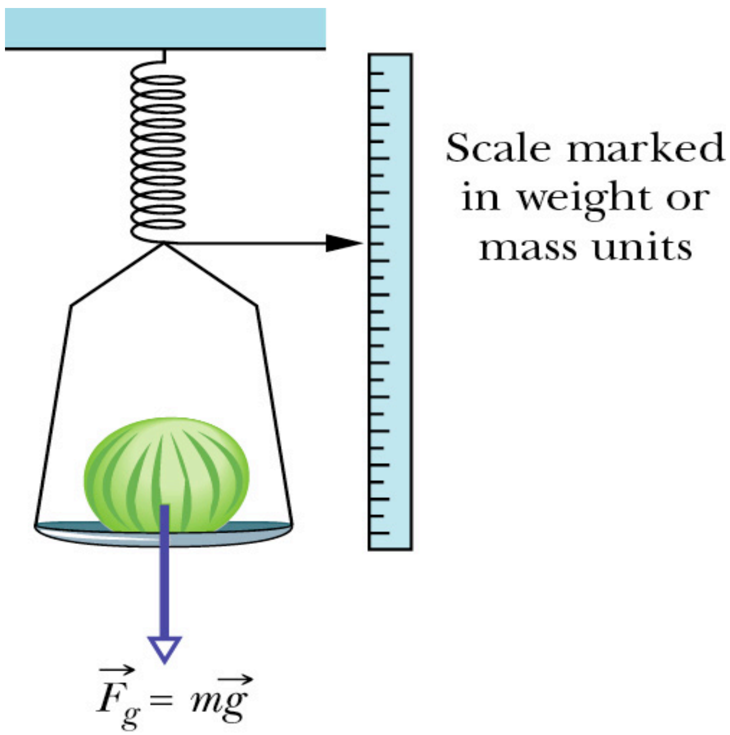
$$F = \frac{GmM}{R^2}$$

$$F = ma = \frac{GmM}{R^2}$$

$$a = \frac{GM}{R^2} = g$$

where g= $\frac{GM}{R^2} = 9.81 m / s^2$

This gives us our weight.
Weight = F=mg



200 lbs

1 lb = 4.45 N

200 lbs = 890 N

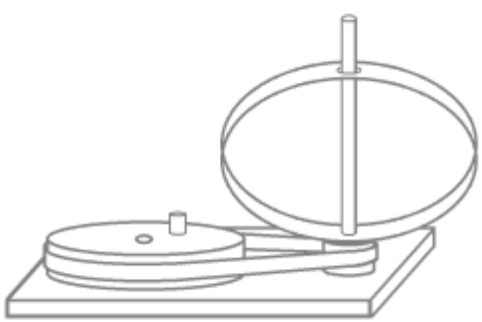
mg= 890 N

m=890/9.81=90.7 kg

At the equator the earths effective g is 9.7805 m/s^2. The acceleration at the equator due to the earths rotation is 0.0339 m/s^2. This means that the true gravitational acceleration is 9.8144=9.7805+0.0339 m/s^2

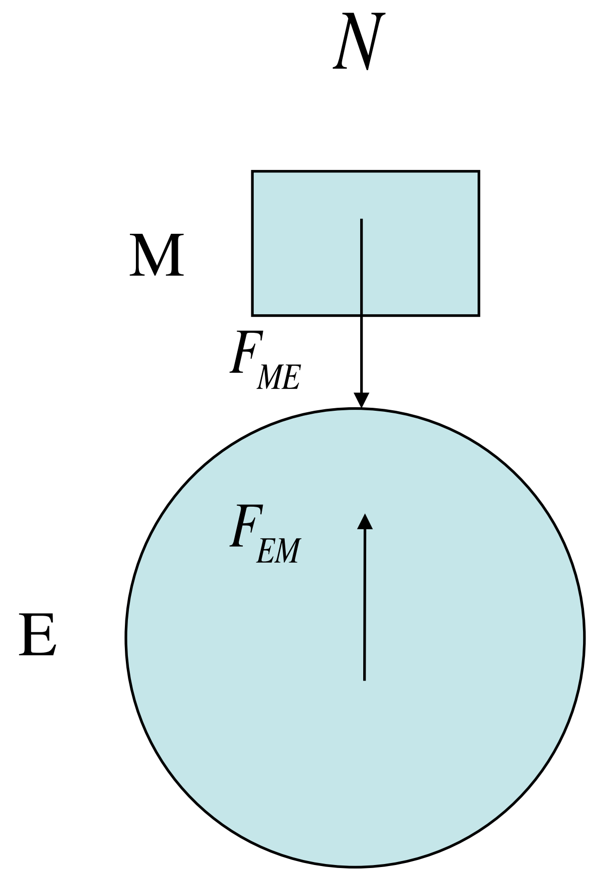
At the poles the Gravitational acceleration is 9.8322 m/s^2 The difference between the poles and the equator is that objects on the Equator are still about 21 km away from the center of the earth.

In summary the difference between the poles and the equator is 70% is due to the acceleration and 30% is due to the oblateness



NEWTON'S THIRD LAW

When two bodies interact, the forces on the bodies due to each other are always equal in magnitude and opposite in direction.



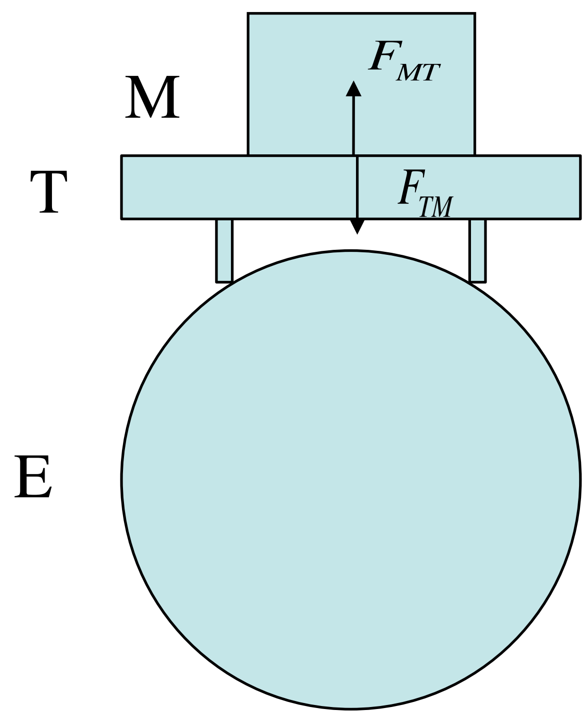
$$\vec{F}_{ME} = -\vec{F}_{EM}$$

Gravitational Interaction

NEWTON’S THIRD LAW

When two bodies interact, the forces on the bodies due to each other are always equal in magnitude and opposite in direction.

N



$$\vec{F}_{MT} = -\vec{F}_{TM}$$

Contact Interaction

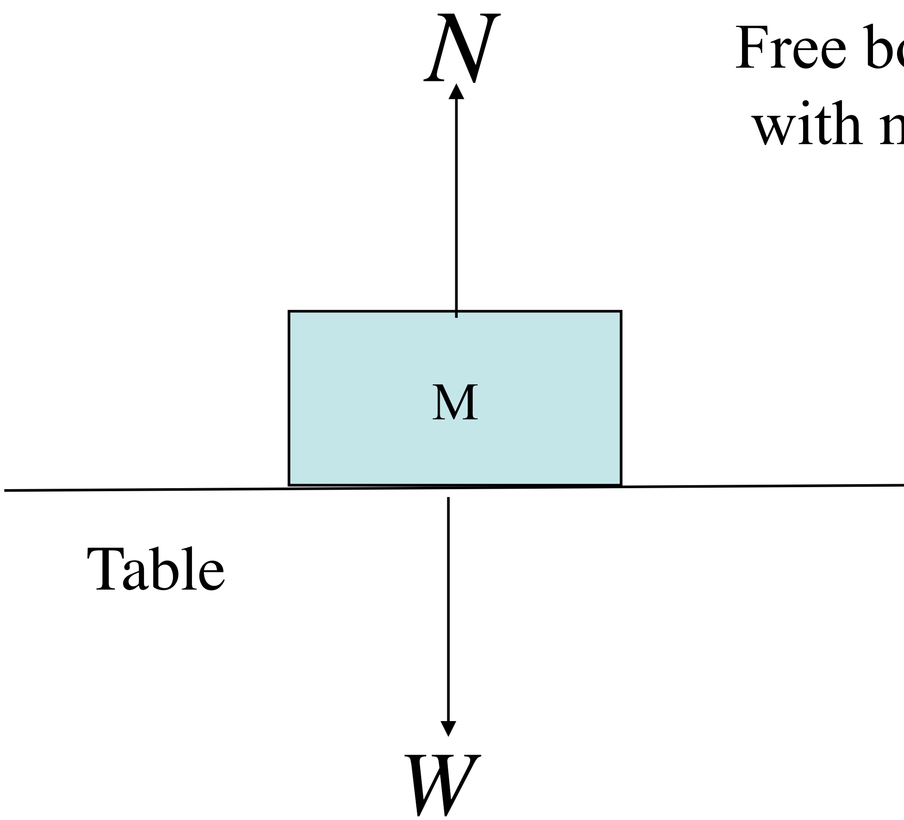
Rules for drawing free body diagrams.
Isolates the forces acting on one body

1) Represent the body by a point.
2) Each force acting on the body is represented by a vector with tail at the point and the length of vector indicating the approximate magnitude of the force.
3) A coordinate system is optional.
4) If the situation consist of several bodies which are rigidly connected, you can still represent all the bodies by a point and use the total mass. Internal forces are not included.

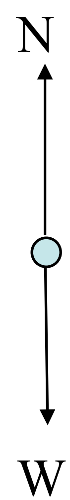
What is the free body diagram of the block at rest on the table?

$$F_{MT} = N$$

$$F_{ME} = W$$



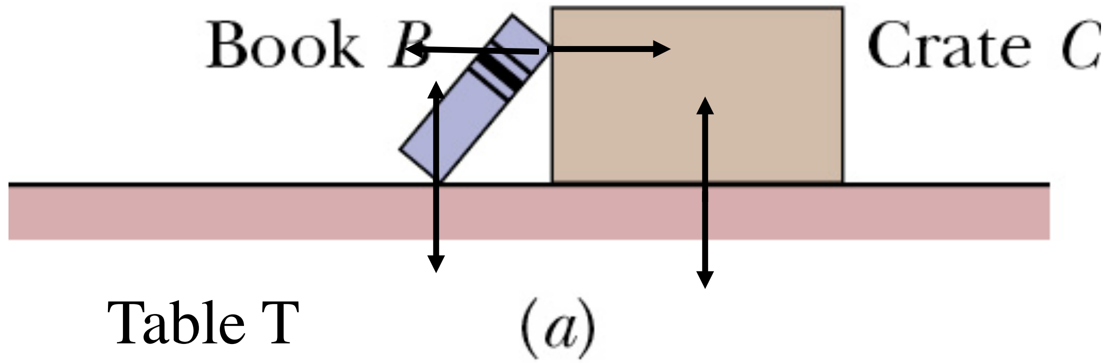
Free body diagram of object with mass M



$$\vec{N} = -\vec{W}$$

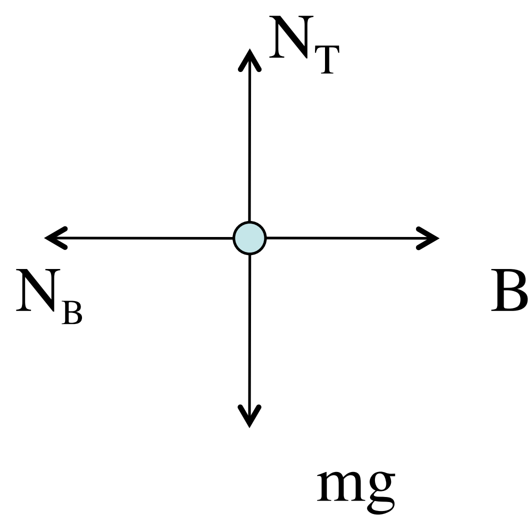
$$W = Mg$$

Book leaning against a crate on a table at rest. What are the action –reaction pairs?

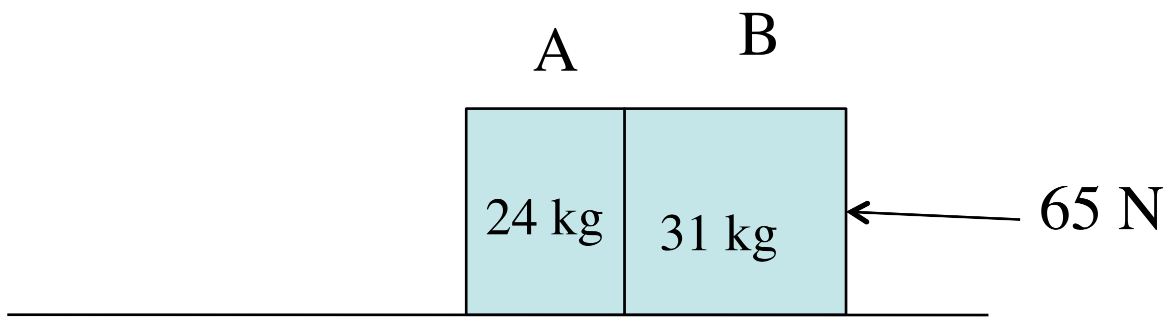


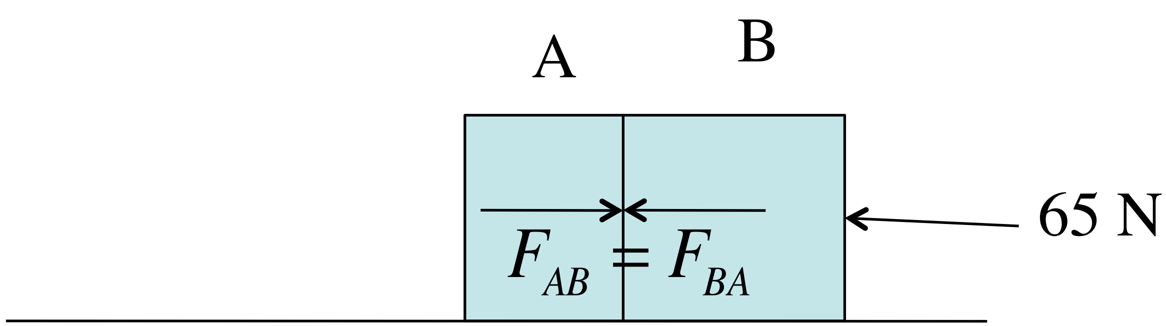
(a)

1) Draw a free body diagram of the forces acting on the crate

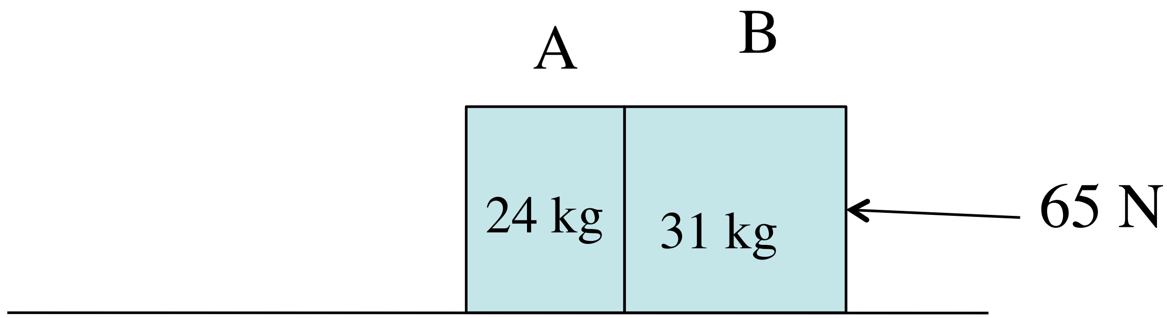


Problem: What is the acceleration of the system of the two blocks and the contact force between the blocks? What is the net force on Block B?





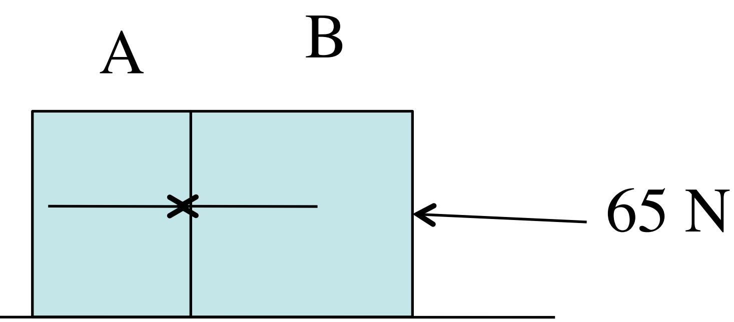
Problem: What is the acceleration of the system of the two blocks and the contact force between the blocks? What is the net force on Block B?



$$a = F / m = 65N / (24kg + 31kg) = 65N / 55kg$$

$$a = 1.18 \text{ m/s}^2$$

$$F_{AB} = F_{BA}$$

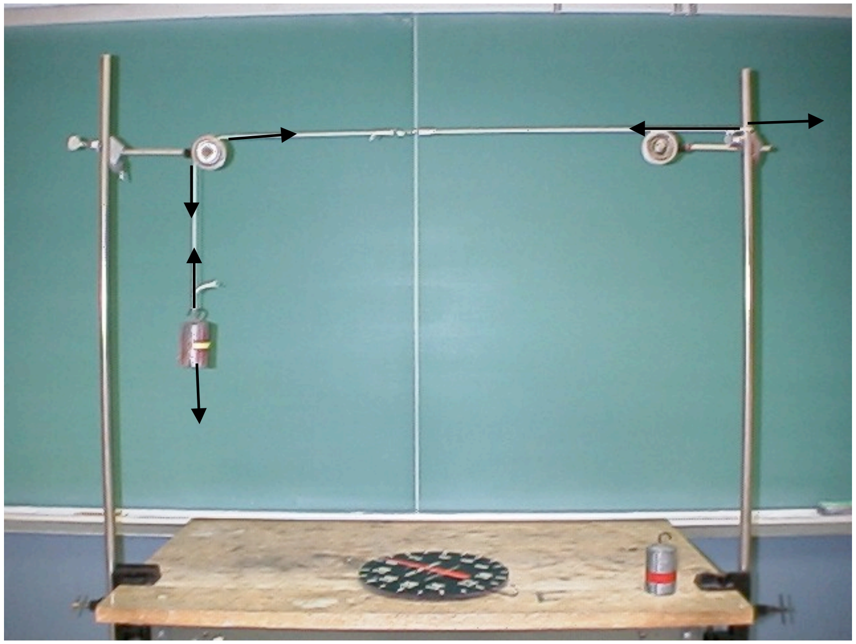


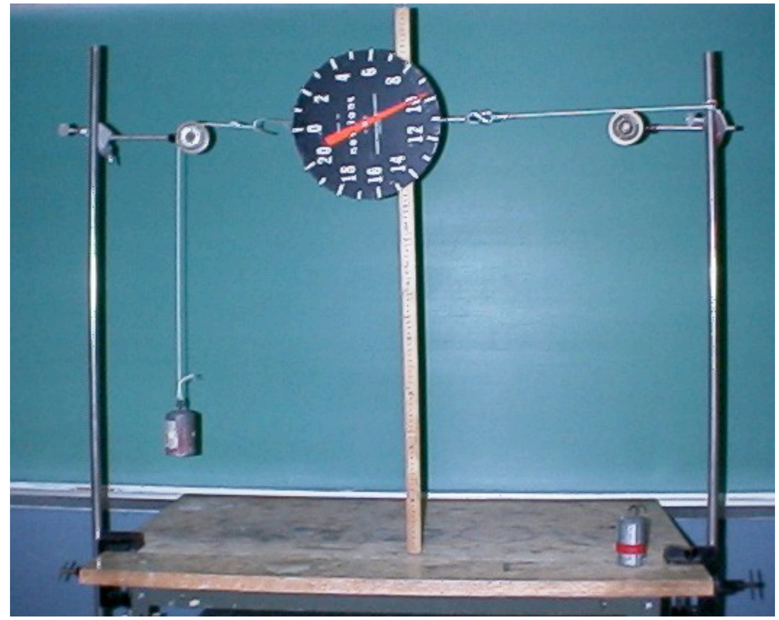
$$F_{BA} = m_A a_A$$

$$F_{BA} = 24 \times 1.18 = 28.3\ N$$

Net Force on Block B=65.0 N - 28.3 N= 36. 6 N

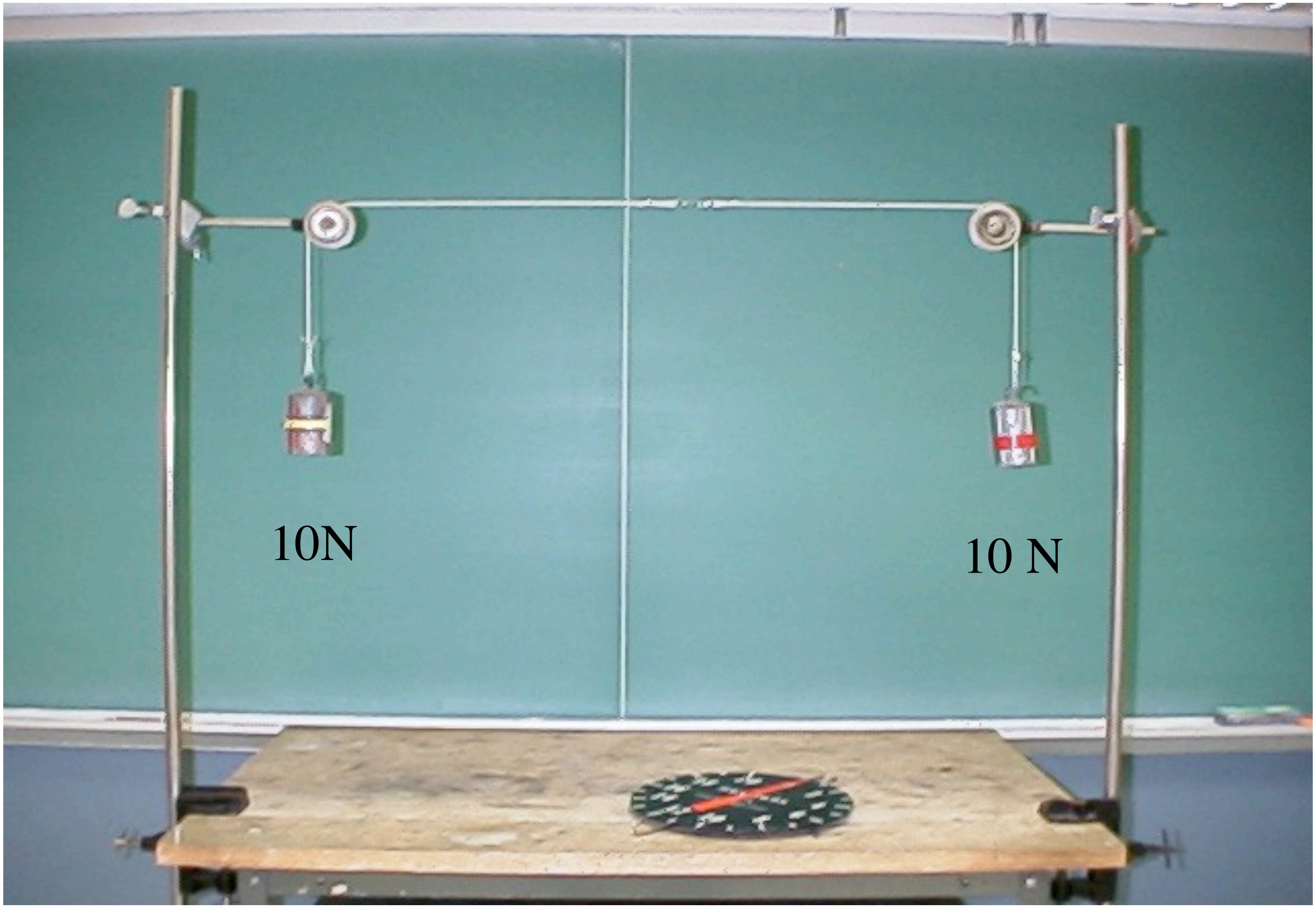
Now lets look at tension in a string



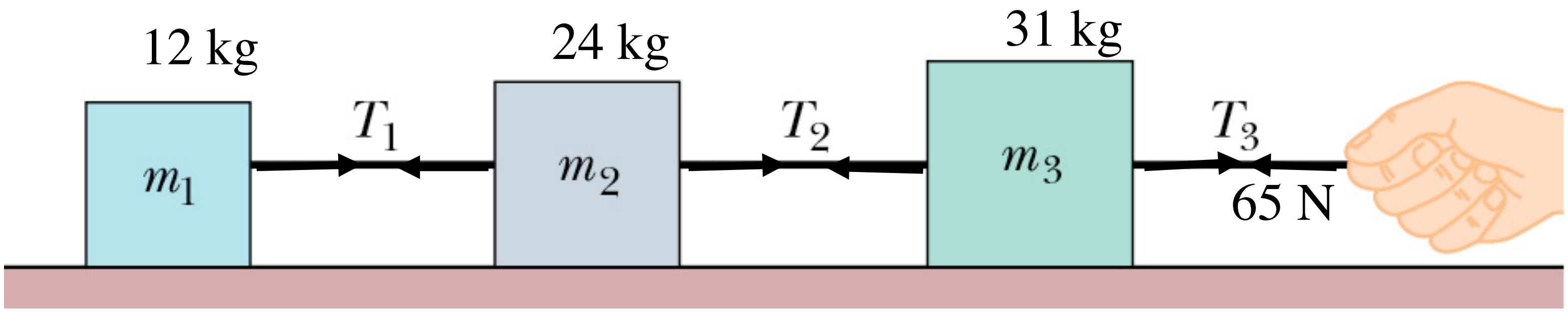


Tension in the string is equal to the weight = 10 N
The scale reads the tension in the string

Is the tension in the string any different when I have weights pulling it down on both sides?



Problem



a) What is the acceleration of the system?
b) Find T_1
c) Find T_2

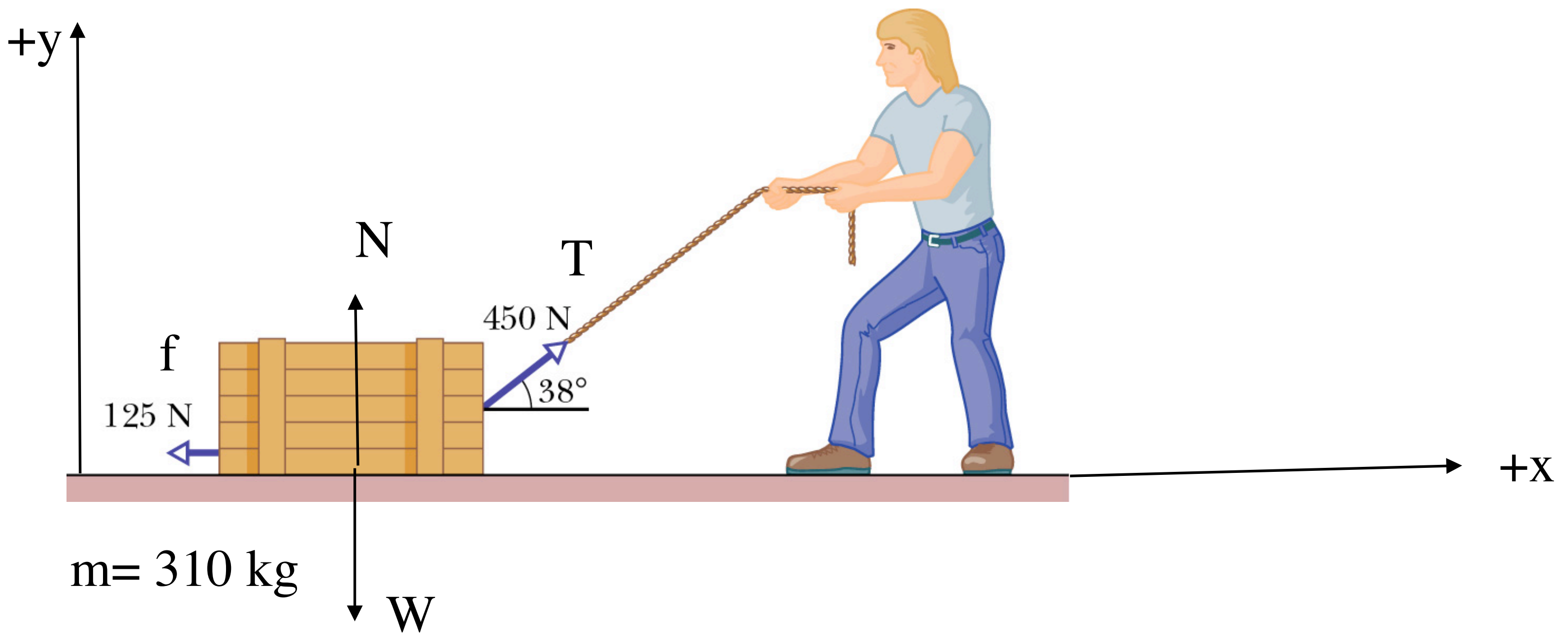
a) $T_3 = m_{sys}a$ $\qquad a = \dfrac{T_3}{m_{sys}} = \dfrac{65N}{(12+24+31)kg} = 0.97 m/s^2$

b) $T_1 = m_1 a$ $\qquad T_1 = (12kg)(0.97 m/s^2) = 11.6N$

c) $T_2 = (m_1 + m_2)a$ $\qquad T_2 = (12+24kg)(0.97 m/s^2) = 34.9N$

$$T_2 = T_3 - m_3 a = 65 - (31)(0.97) = 65 - 30.1 = 34.9N$$

A crate is being pulled by a man as shown in the figure. What is the acceleration of the crate along the x direction? Man does not move.



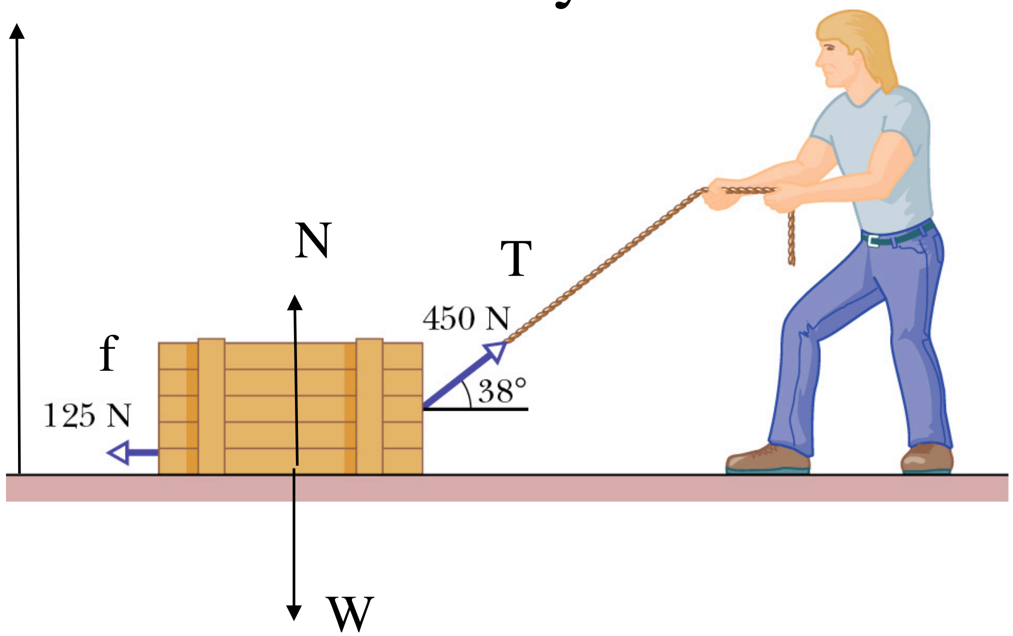
$$\sum F_x = T\cos(38^o) - f = ma$$

x component of forces in free body diagram

$$450\cos(38^o) - 125 = 310a$$

$$a = (450\cos(38^\circ) - 125)/310 = 0.74 m/s^2$$

What is the normal force assuming there is no acceleration in the y direction?



$$\sum F_y = N + T\sin(38^o) - W = 0$$

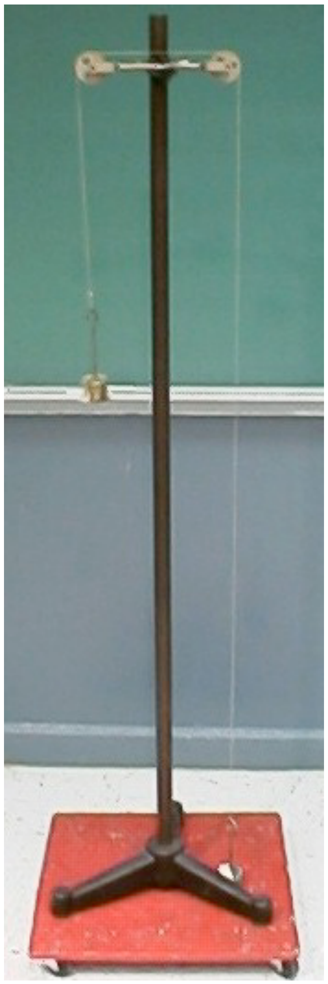
y component of forces in free body diagram

$$N = W - T\sin(38)$$

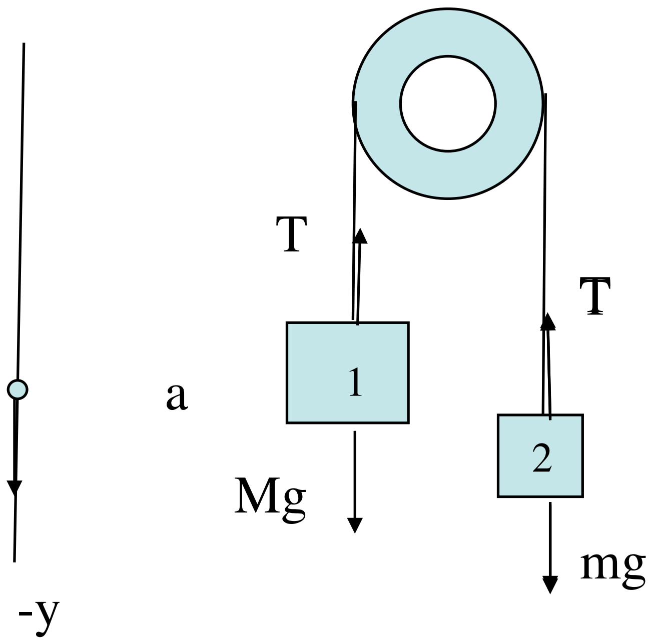
$$N = 310(9.8) - 450(.616) = 3038 - 277.1$$

$$N = 2761.0 \ Newtons$$

Rev George Atwood's machine 1746 -1807
Tutor Trinity College, Cambridge



Assume left side is moving down in the negative y direction



Free body diagram for each body

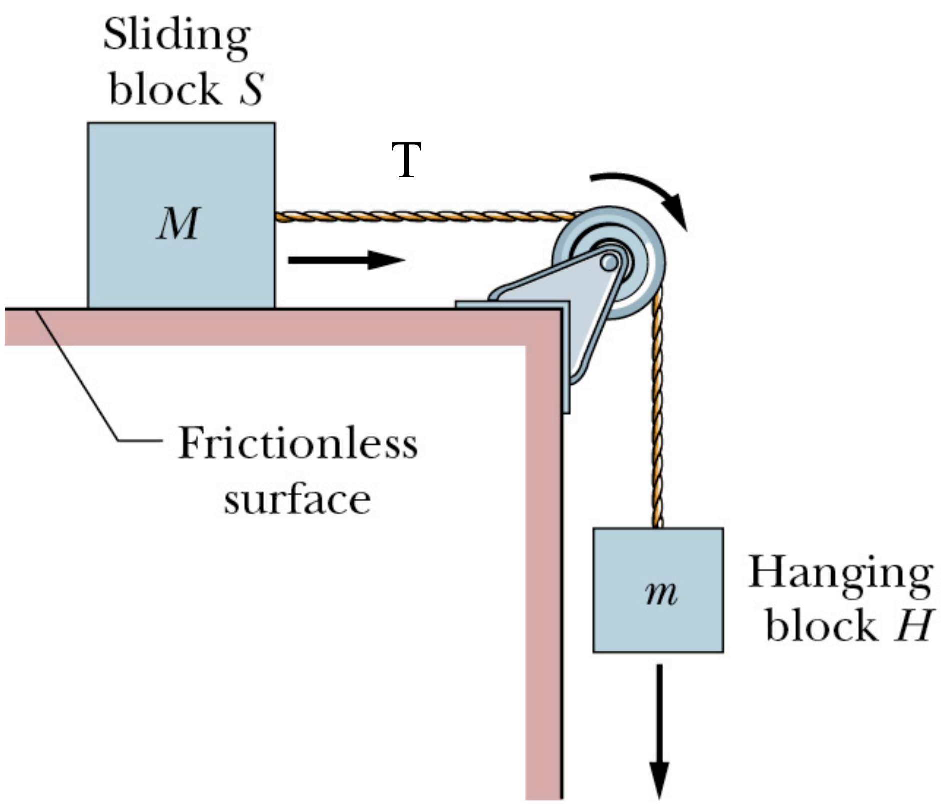
1. $\sum F_y = T - Mg = -Ma$

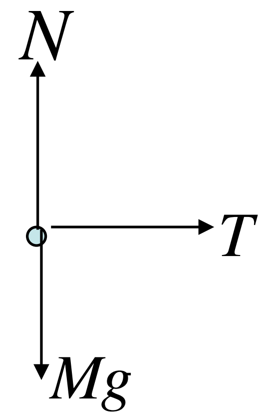
2. $\sum F_y = T - mg = ma$

$$a = \frac{M - m}{m + M} g$$

$$T = \frac{2mMg}{m + M}$$

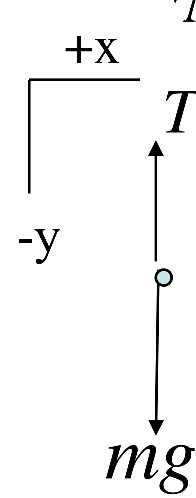
Free Body Diagram of an accelerating system:





$$\sum F_y = 0 = N - Mg$$

$$\sum F_x = T = Ma$$



$$\sum F_y = T - mg = -ma$$

$$Ma - mg = -ma$$

$$a(M + m) = mg$$

$$a = \frac{m}{M + m} g$$

$$T = \frac{M}{M + m} mg$$

Next time frictional forces and terminal velocity

- You need to know how to draw free body diagrams to solve problems.

ConcepTest 3.4a Firing Balls I

A small cart is rolling at constant velocity on a flat track. It fires a ball straight up into the air as it moves. After it is fired, what happens to the ball?

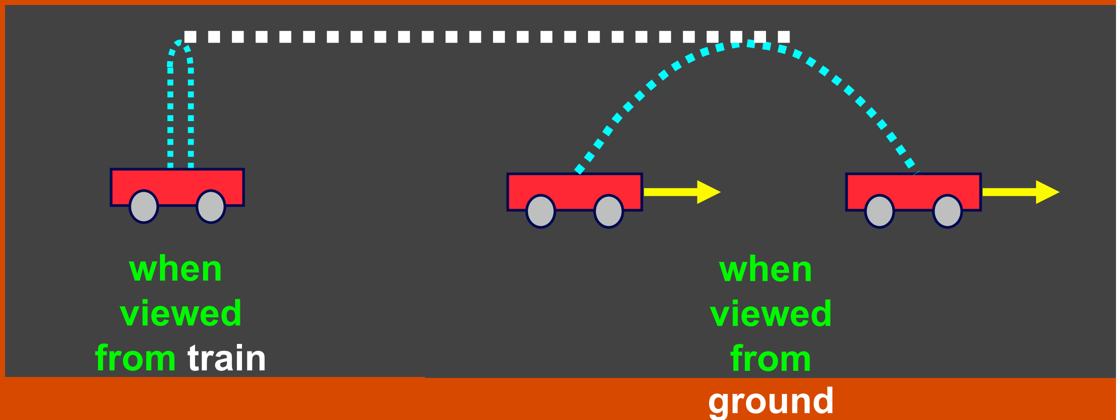
1) it depends on how fast the cart is moving
2) it falls behind the cart
3) it falls in front of the cart
4) it falls right back into the cart
5) it remains at rest

ConcepTest 3.4a Firing Balls I

A small cart is rolling at **constant** velocity on a flat track. It fires a ball straight up into the air as it moves. After it is fired, what happens to the ball?

1) it depends on how fast the cart is moving
2) it falls behind the cart
3) it falls in front of the cart
4) it falls right back into the cart
5) it remains at rest

In the frame of reference of the cart, the ball only has a vertical component of velocity. So it goes up and comes back down. To a ground observer, both the cart and the ball have the ***same horizontal velocity***, so the ball still returns into the cart.



ConcepTest 3.5 Dropping a Package

You drop a package from a plane flying at constant speed in a straight line. Without air resistance, the package will:

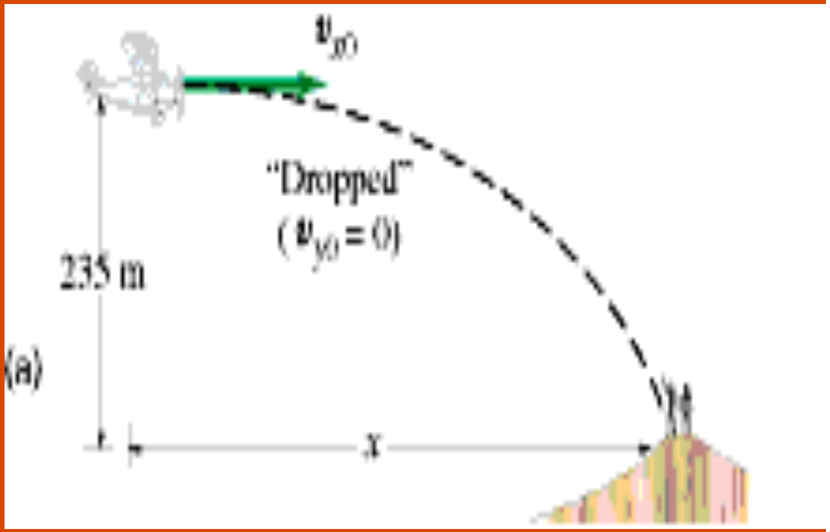
1) quickly lag behind the plane while falling
2) remain vertically under the plane while falling
3) move ahead of the plane while falling
4) not fall at all

ConcepTest 3.5 Dropping a Package

You drop a package from a plane flying at constant speed in a straight line. Without air resistance, the package will:

1) quickly lag behind the plane while falling
2) remain vertically under the plane while falling
3) move ahead of the plane while falling
4) not fall at all

Both the plane and the package have the ***same horizontal velocity*** at the moment of release. They will ***maintain*** this velocity in the ***x-direction***, so they stay aligned.



Follow-up: What would happen if air resistance is present?

ConcepTest 3.6a Dropping the Ball I

From the *same* height (and at the *same* time), one ball is dropped and another ball is fired horizontally. Which one will hit the ground first?

(1) the "dropped" ball

(2) the "fired" ball

(3) they both hit at the same time

(4) it depends on how hard the ball was fired

(5) it depends on the initial height

ConcepTest 3.6a Dropping the Ball I

From the *same* height (and at the *same* time), one ball is dropped and another ball is fired horizontally. Which one will hit the ground first?

(1) the "dropped" ball

(2) the "fired" ball

(3) they both hit at the same time

(4) it depends on how hard the ball was fired

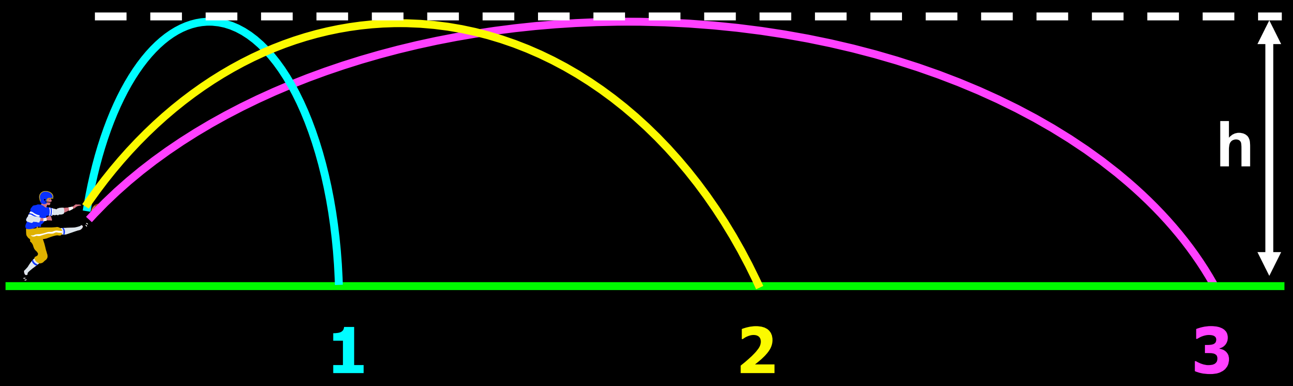
(5) it depends on the initial height

Both of the balls are falling vertically under the influence of gravity. They both fall from the same height. Therefore, they will hit the ground at the same time. The fact that one is moving horizontally is irrelevant – remember that the *x* and *y* motions are completely independent !!

Follow-up: Is that also true if there is air resistance?

ConcepTest 3.7a Punts I

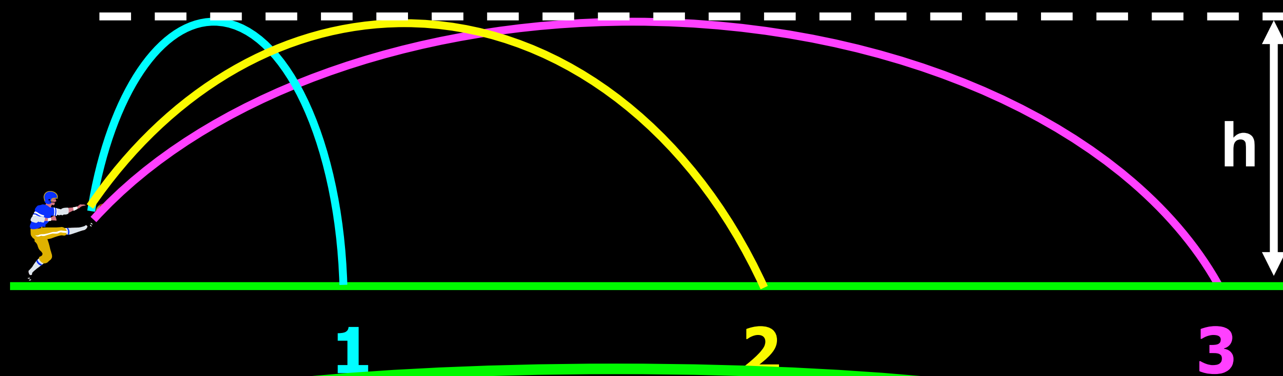
Which of the 3 punts has the longest hang time?



4) all have the same hang time

ConcepTest 3.7a Punts I

Which of the 3 punts has the longest hang time?



4) all have the same hang time

The time in the air is determined by the ***vertical motion*** !

Since all of the punts reach the **same height**, they all stay in the air for the **same time**.

Follow-up: Which one had the greater initial velocity?

ConcepTest 4.1a Newton's First Law I

A book is lying at rest on a table. The book will remain there at rest because:

1) there is a net force but the book has too much inertia
2) there are no forces acting on it at all
3) it does move, but too slowly to be seen
4) there is no net force on the book
5) there is a net force, but the book is too heavy to move

ConcepTest 4.1a Newton's First Law I

A book is lying at rest on a table. The book will remain there at rest because:

1) there is a net force but the book has too much inertia
2) there are no forces acting on it at all
3) it does move, but too slowly to be seen
4) there is no net force on the book
5) there is a net force, but the book is too heavy to move

There are forces acting on the book, but the only forces acting are in the *y*-direction. Gravity acts downward, but the table exerts an upward force that is equally strong, so the two forces cancel, **leaving no net force**.

ConcepTest 4.1c Newton's First Law III

You put your book on the bus seat next to you. When the bus stops suddenly, the book slides forward off the seat. Why?

1) a net force acted on it
2) no net force acted on it
3) it remained at rest
4) it did not move, but only seemed to
5) gravity briefly stopped acting on it

ConcepTest 4.1c Newton's First Law III

You put your book on the bus seat next to you. When the bus stops suddenly, the book slides forward off the seat. Why?

1) a net force acted on it
2) no net force acted on it
3) it remained at rest
4) it did not move, but only seemed to
5) gravity briefly stopped acting on it

The book was initially moving forward (since it was on a moving bus). When the bus stopped, the book **continued moving forward**, which was its **initial state of motion**, and therefore it slid forward off the seat.

Follow-up: What is the force that usually keeps the book on the seat?