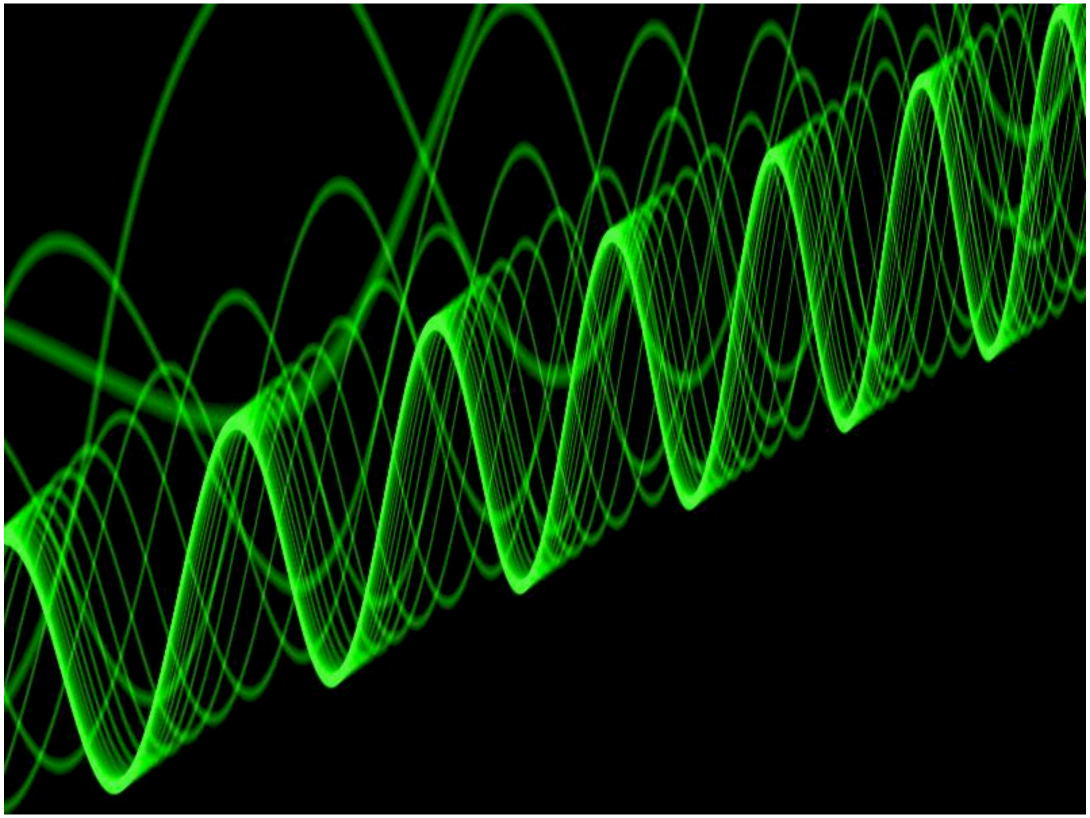
Chapter 15
Oscillations



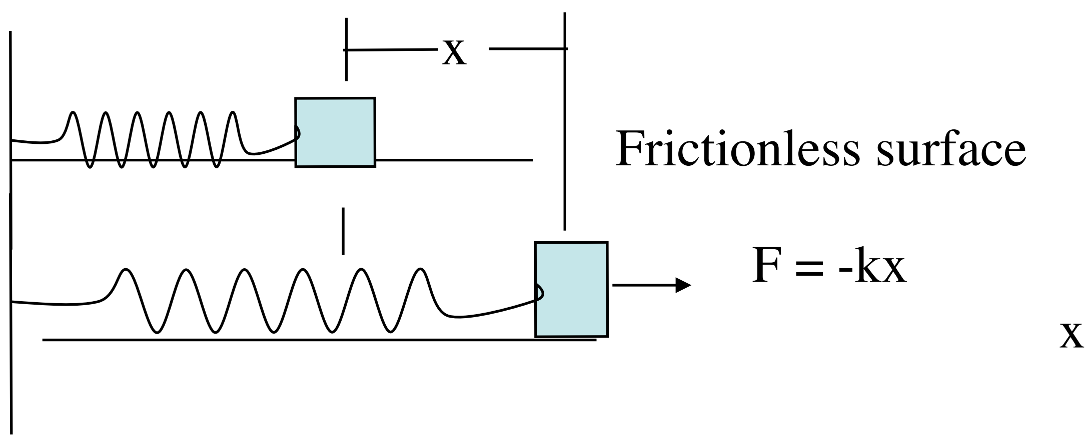
Summary

- Simple harmonic motion
- Hook's Law
- Energy

$$F = -kx$$

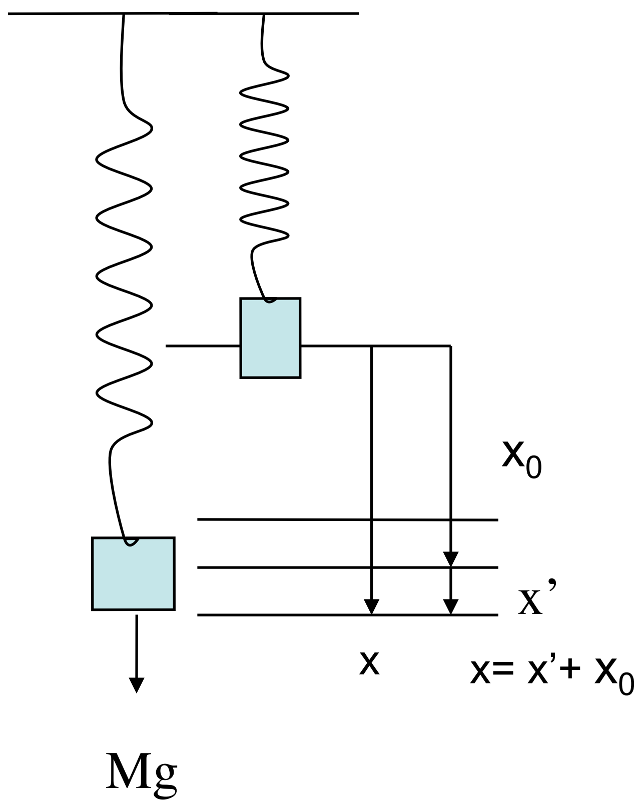
- Pendulums: Simple. Physical, Meter stick

Simple Picture of an Oscillation



SHM in vertical position

mg=kx



$$ma = -kx + mg$$

$$x = x' + mg / k$$

$$ma = -k(x' + mg / k) + mg$$

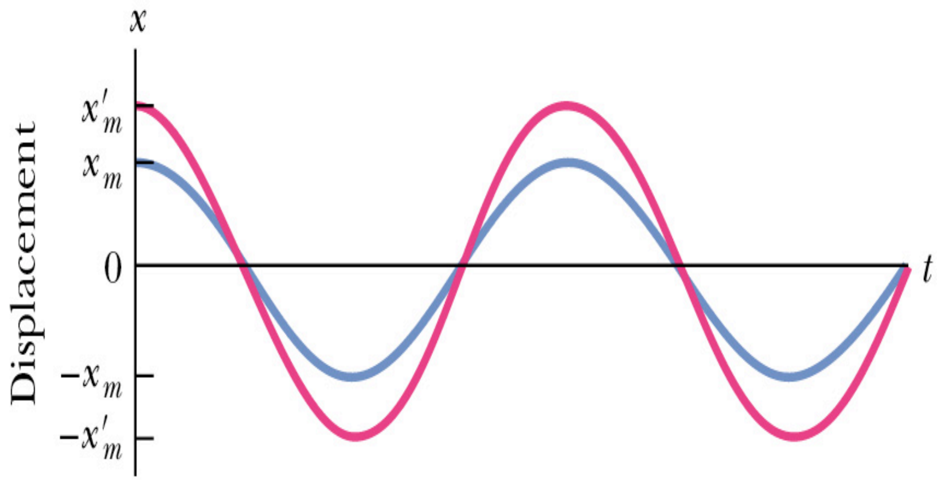
$$ma = -kx'$$

Equilibrium position with x_0=mg/k

Definitions

$$x(t) = x_m \cos(\omega t + \phi)$$

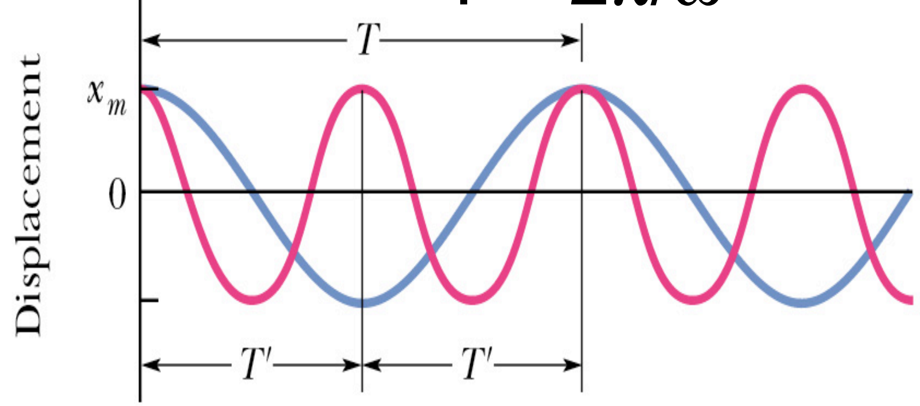
- Displacemnt at time t: $x(t)$
- Amplitude: x_m
- Phase: $(\omega t + \phi)$
- Angular frequency: ω
- Time: t
- Phase constant or phase angle: ϕ



(a)

$T = 2T'$

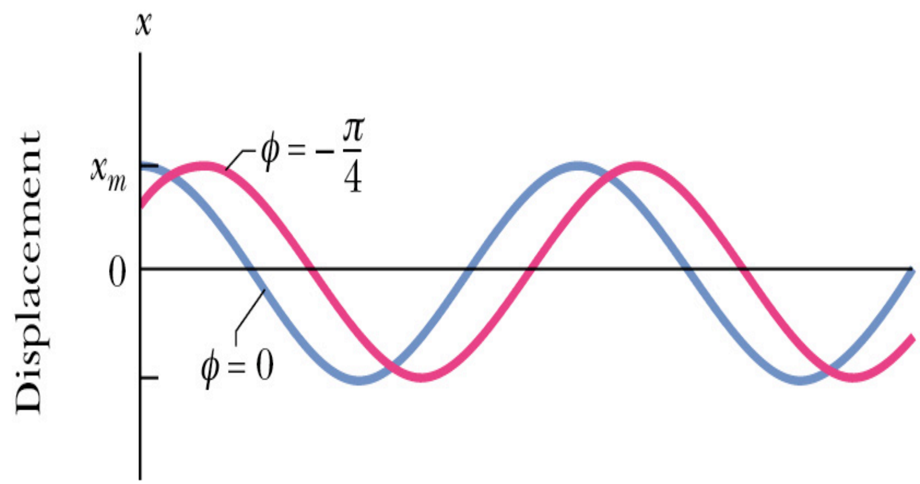
$T = 2\pi/\omega$



(b)

$$x = x_m \cos(\omega t)$$

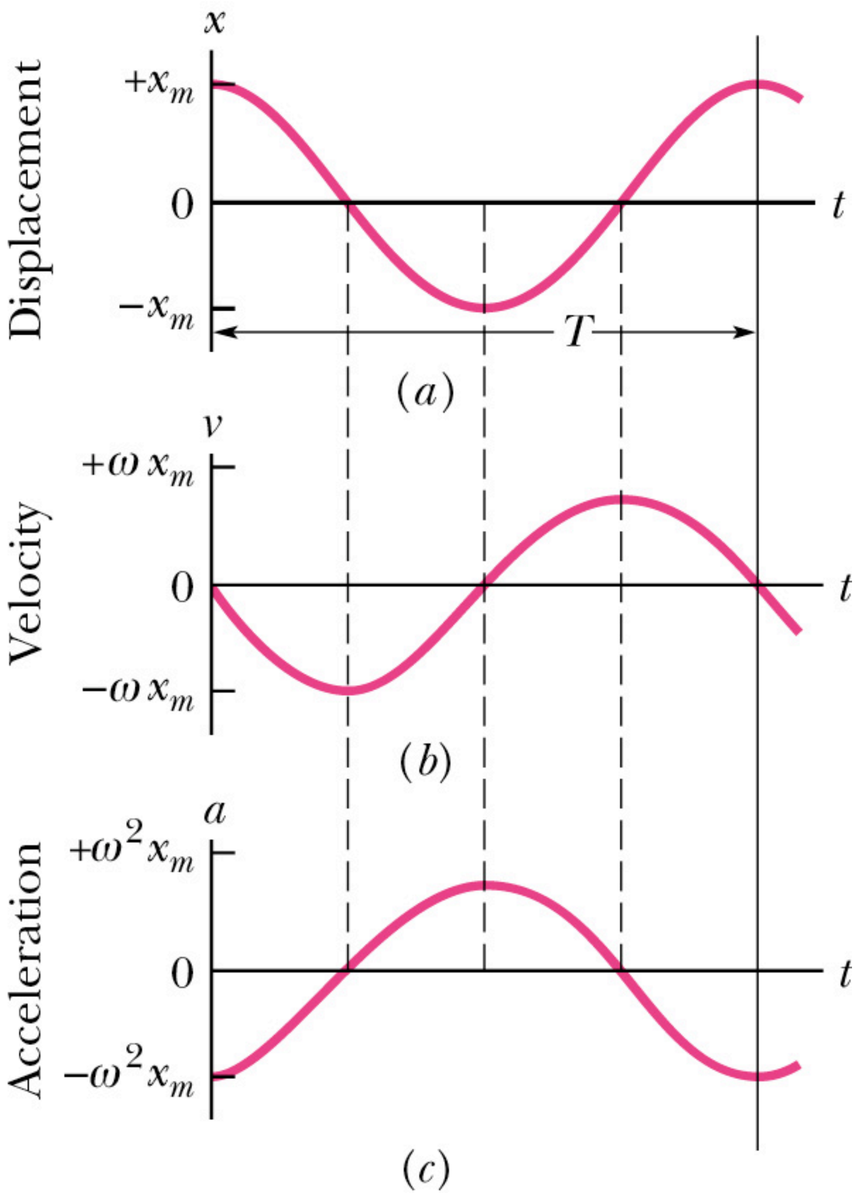
$$x = x_m \cos(\omega t - \pi/4)$$



We say the pink curve lags the blue one by 45 degrees

(c)

Relationships among x,v, and a and time dependence



$$x = x_m \cos(\omega t)$$

$$v = \frac{dx}{dt}$$

$$v = -x_m \omega \sin(\omega t)$$

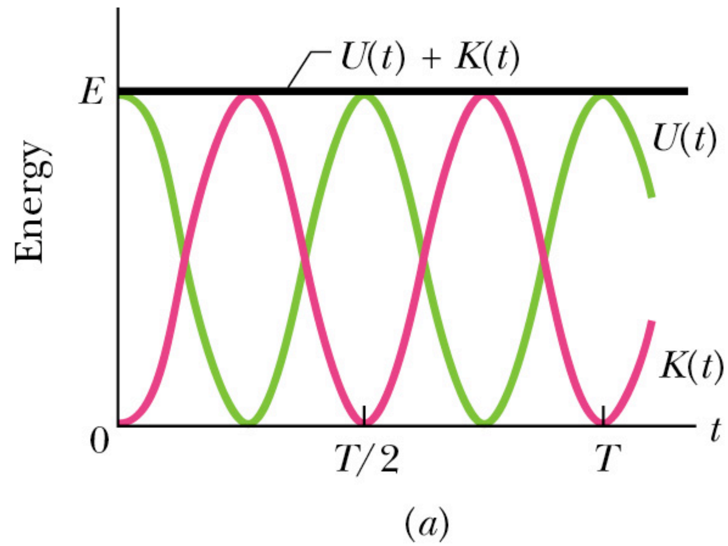
$$v_m = x_m \omega$$

$$a = \frac{dv}{dt}$$

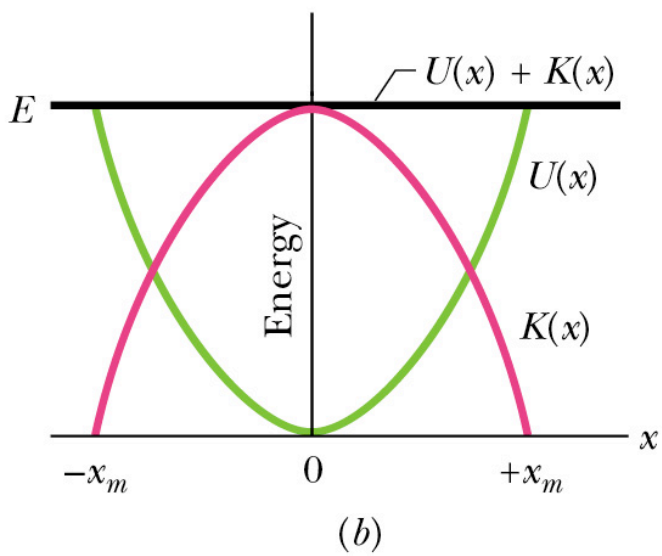
$$a = -x_m \omega^2 \cos(\omega t)$$

$$a_m = x_m \omega^2$$

Energy as a function of t and x



(a)



(b)

$$U = \frac{1}{2}kx^2 = \frac{1}{2}kx_m^2\cos^2(\omega t)$$

$$K = \frac{1}{2}m\omega^2 x_m^2 \sin^2(\omega t)$$

$$k = m\omega^2$$

$$E_m = U + K$$

$$E_m = \frac{1}{2}kx_m^2\cos^2(\omega t) + \frac{1}{2}kx_m^2\sin^2(\omega t)$$

$$E_m = \frac{1}{2}kx_m^2$$

$$U = \frac{1}{2}kx_m^2$$

$$K = E_m - \frac{1}{2}kx^2$$

Simple Harmonic Motion Summary

$$a = -\frac{k}{m}x$$

$$F = -kx = ma$$

$$x = x_m \cos(\omega t)$$

$$F = -kx_m \cos(\omega t) = -m\omega^2 x_m \cos(\omega t)$$

$$\frac{k}{m} = \omega^2$$

$$\omega = \sqrt{\frac{k}{m}}$$

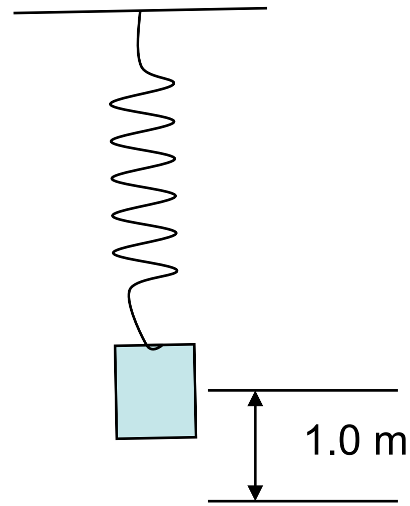
$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

Using Excel to solve numerically mass on a spring

$$F = -kx = \frac{\Delta p}{\Delta t}$$

$$\Delta p = -kx\Delta t$$

$$p_i = p_{i-1} - kx_{i-1}\Delta t$$



$$t = 0$$

$$v = 0$$

$$x_0 = 1.0\ m$$

$$\Delta t = 0.001 s$$

$$k = 10\ N/m$$

$$m = 0.5\ \text{kg}$$

)

$$x_i = x_{i-1} + \frac{1}{2}(v_i + v_{i-1})\Delta t$$

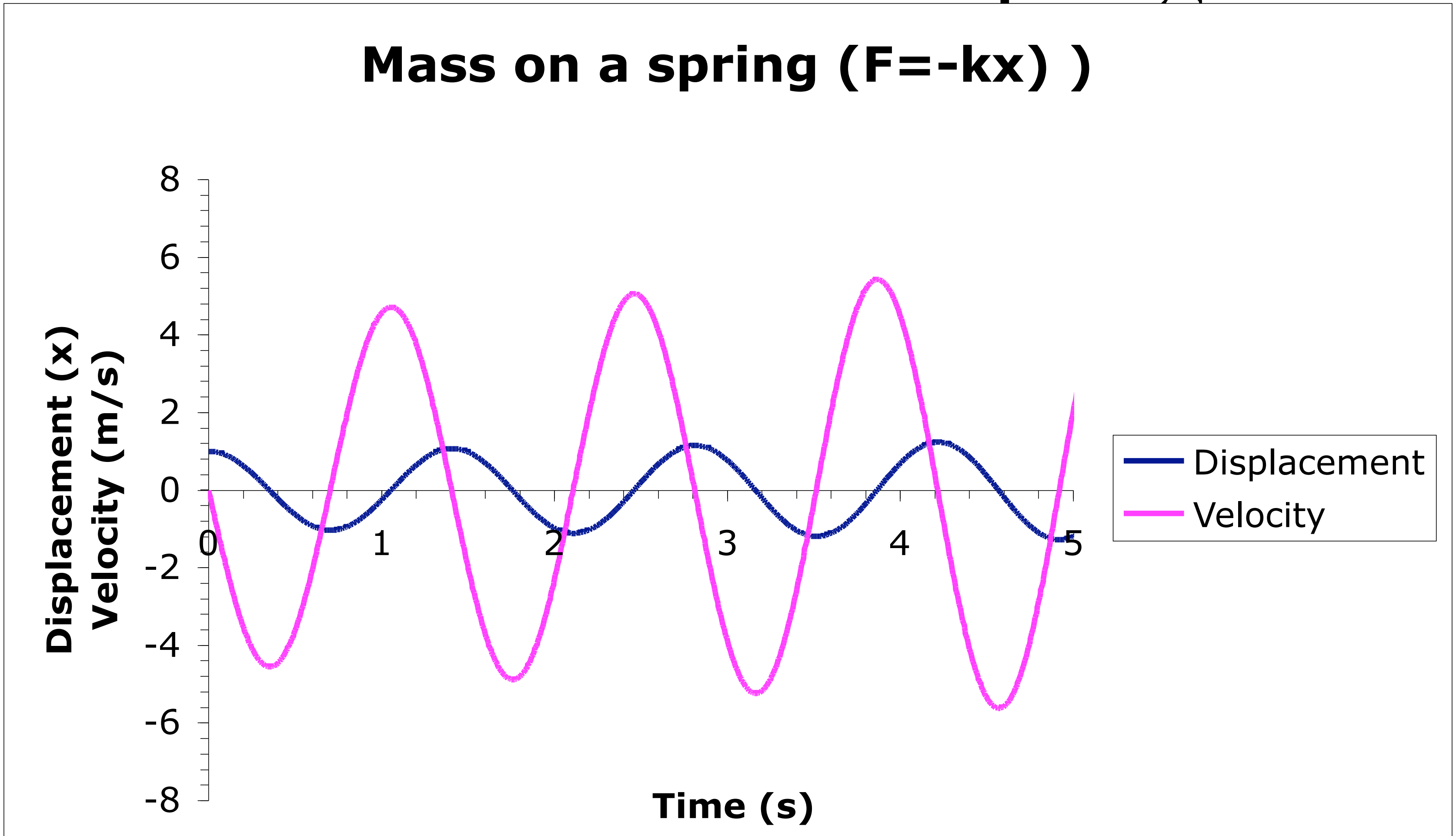
$$v_i = v_{i-1} - \frac{kx}{m}\Delta t$$

In Excel

$$C17 = C16 - ((k/m) * D16) * \Delta t$$

$$D17 = D16 + .5(C16 + C17) * \Delta t$$

Excel Mass on a spring



$x = x_m \cos(\omega t) = 1.0 \cos(4.47t)$ $\quad v = -x_m \omega \sin(\omega t) = -4.47 \sin(4.47t)$

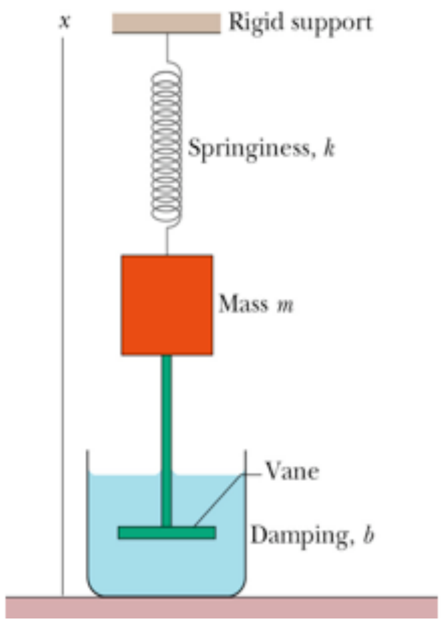
$\omega = 6.28(1/T)$

$T = 1.40$

$\omega = 4.47$

Damped Harmonic Motion Spread Sheet Problem
Due Friday July 17 9:00 AM

Figure 15-15 HRW



Same as Problem 59 Ch 15

t = time

x = displacement of mass

v = velocity of mass

a = acceleration of the mass

x_0 = initial displacement =0.12 m

k = spring constant=8.0 N/m

b = damping constant= 0.230 kg/s

m = mass of block=1.50 kg

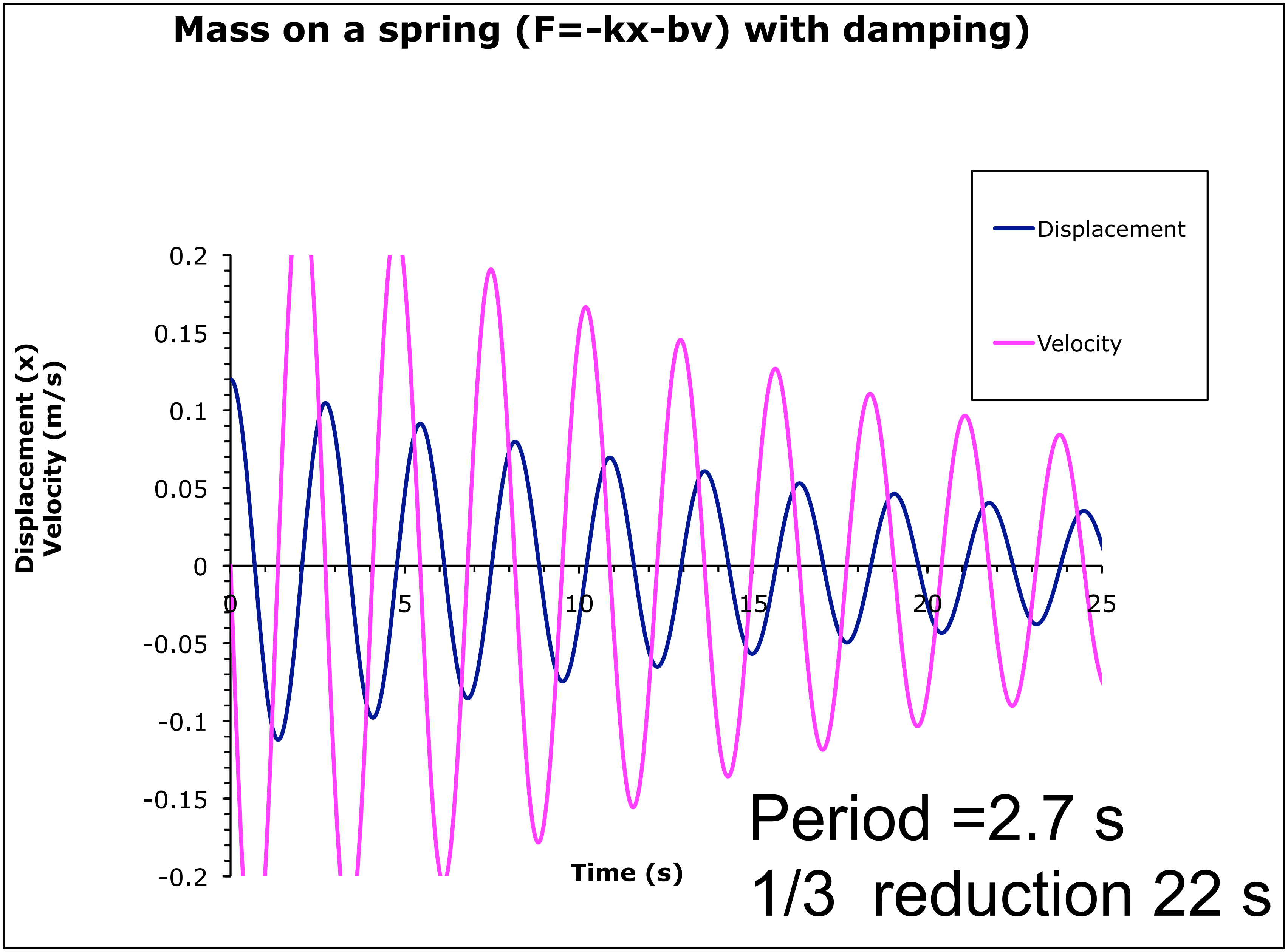
$$x(t) = x_m e^{-\frac{bt}{m}} \cos(\omega' t)$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \simeq \sqrt{\frac{8}{1.5}} = 2.3\ rad/s$$

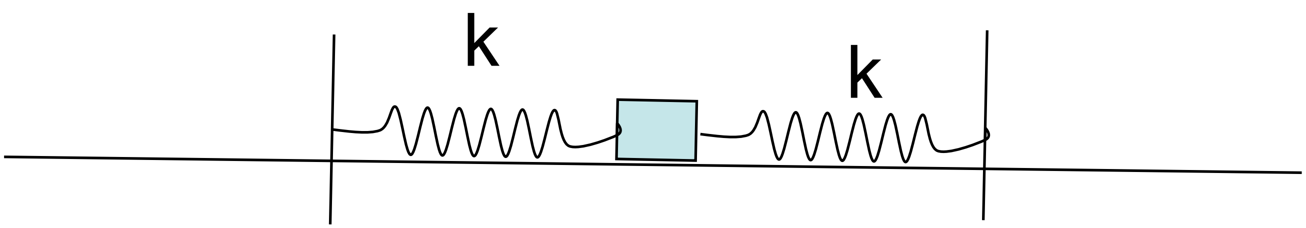
$$\omega' = 2\pi f$$

$$f = \frac{2.3}{6.28} = 0.37 \text{cycles per sec}$$

$$T = \frac{1}{f} = 2.7s$$


Mass on a spring (F=-kx-bv) with damping)
Displacement
Velocity
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
0
5
10
15
20
25
Displacement (x)
Velocity (m/s)
Time (s)
Period =2.7 s
1/3 reduction 22 s

What is the effective spring constant?

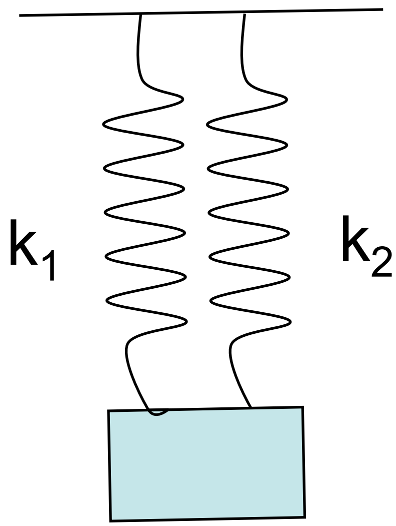


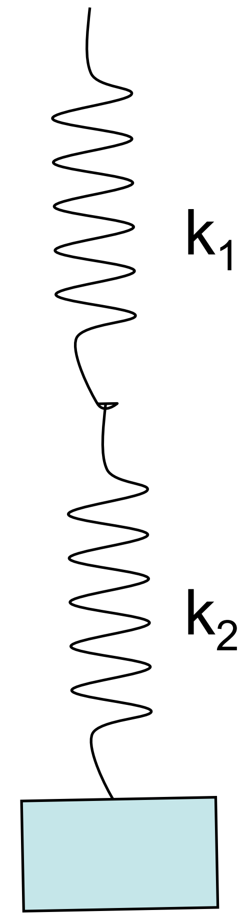
F=-kx-kx=-2kx

k_{eff}=2k

$$\omega = \sqrt{\frac{2k}{m}}$$

What is the effective spring constant.





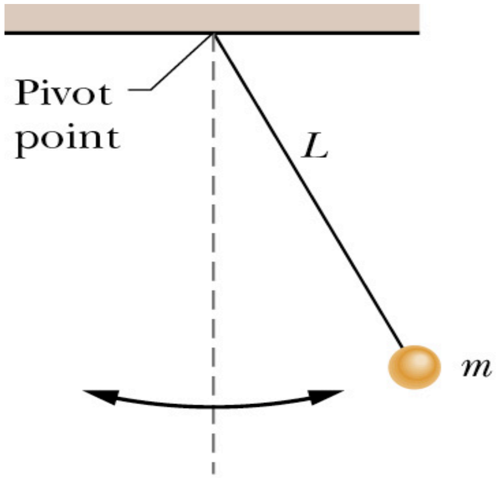
F= $-k_{eff}\,x = -\,k_1x - k_2x$

$k_{eff} = -\,k_1 - k_2$

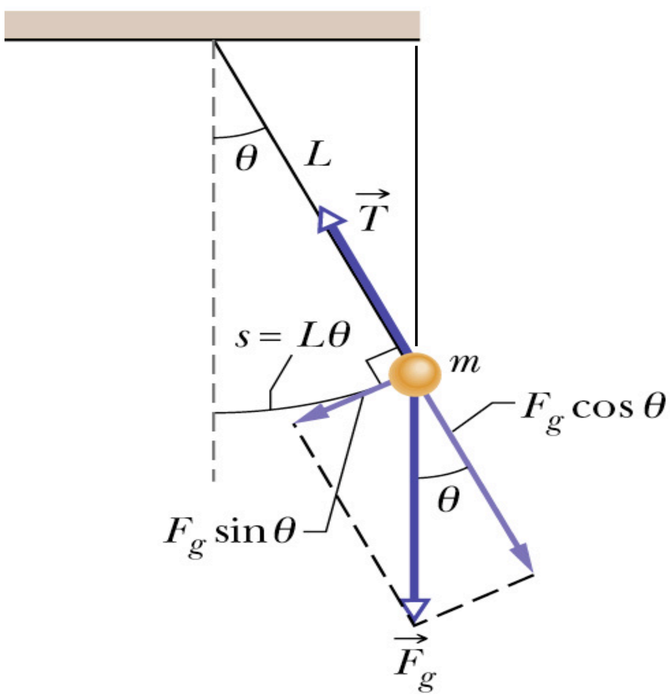
$$\omega = \sqrt{\frac{k_1 + k_2}{m}}$$

$$\omega = \sqrt{\frac{\frac{k_1k_2}{k_1 + k_2}}{m}}$$

Pendulum using rotational variables



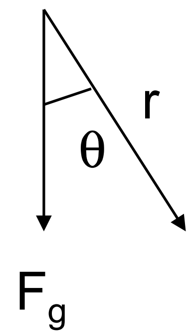
(a)



(b)

$$\tau = r \times F_g$$

$$\tau = r\sin\theta F_g = rF_\perp$$



L = r

$$\tau = -L\sin\theta mg$$ Negative sign- clockwise rotation

$$\tau = I\alpha$$

$$\alpha = -L(\frac{mg}{I})\sin\theta \qquad a = -\frac{k}{m}x$$

$$\alpha = -L(\frac{mg}{I})\theta \qquad a = -\frac{kx}{m}$$

$$\omega = \sqrt{\frac{Lmg}{I}} \qquad \omega = \sqrt{\frac{k}{m}}$$

For small angles

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

pendulum continued

$$\omega = \sqrt{\frac{Lmg}{I}}$$

$$I = mL^2$$ ← Moment of inertia for a ball on a massless string of length L

$$\omega = \sqrt{\frac{Lmg}{mL^2}}$$

$$\omega = \sqrt{\frac{g}{L}}$$

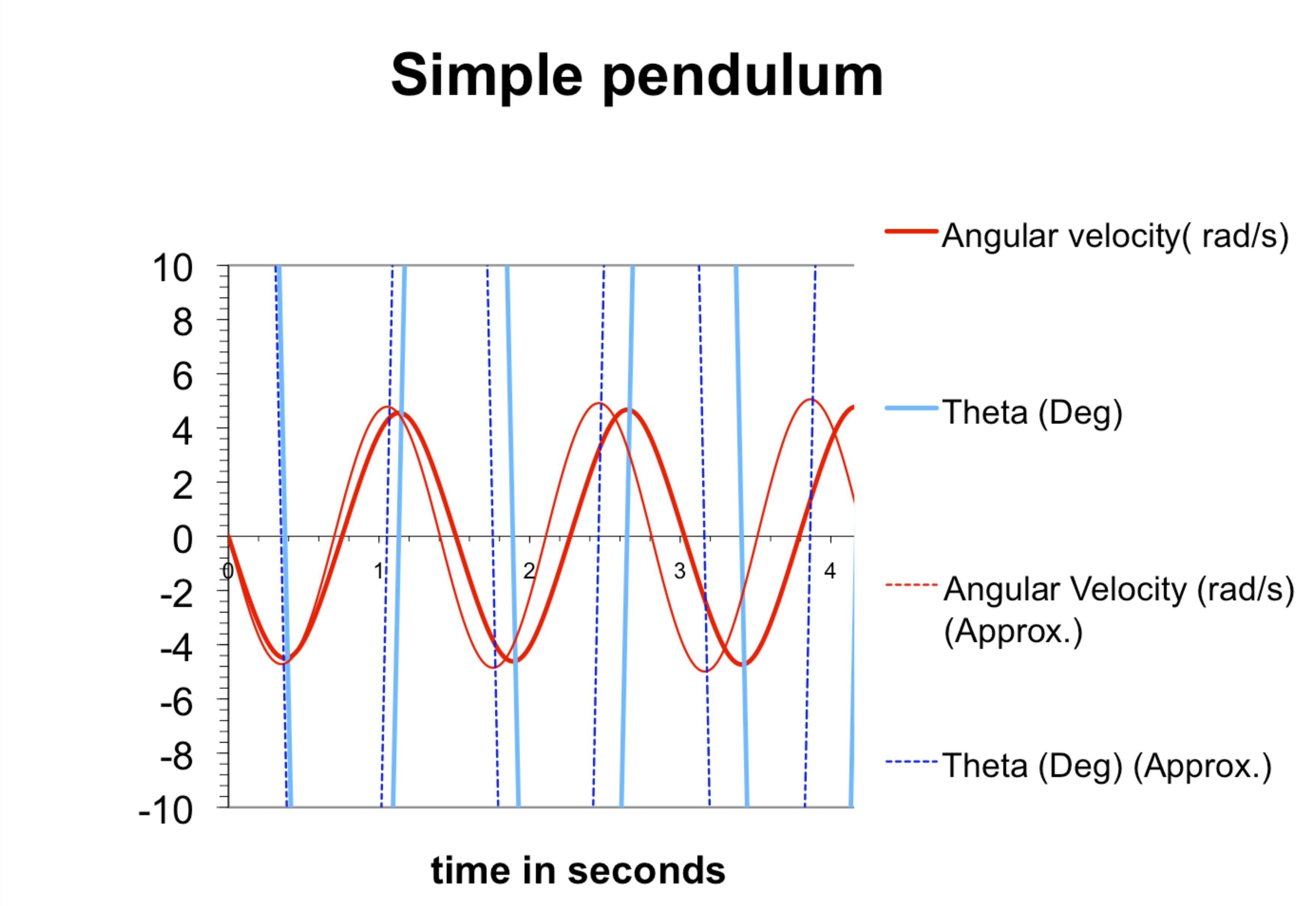
$$T = \frac{2\pi}{\omega}$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

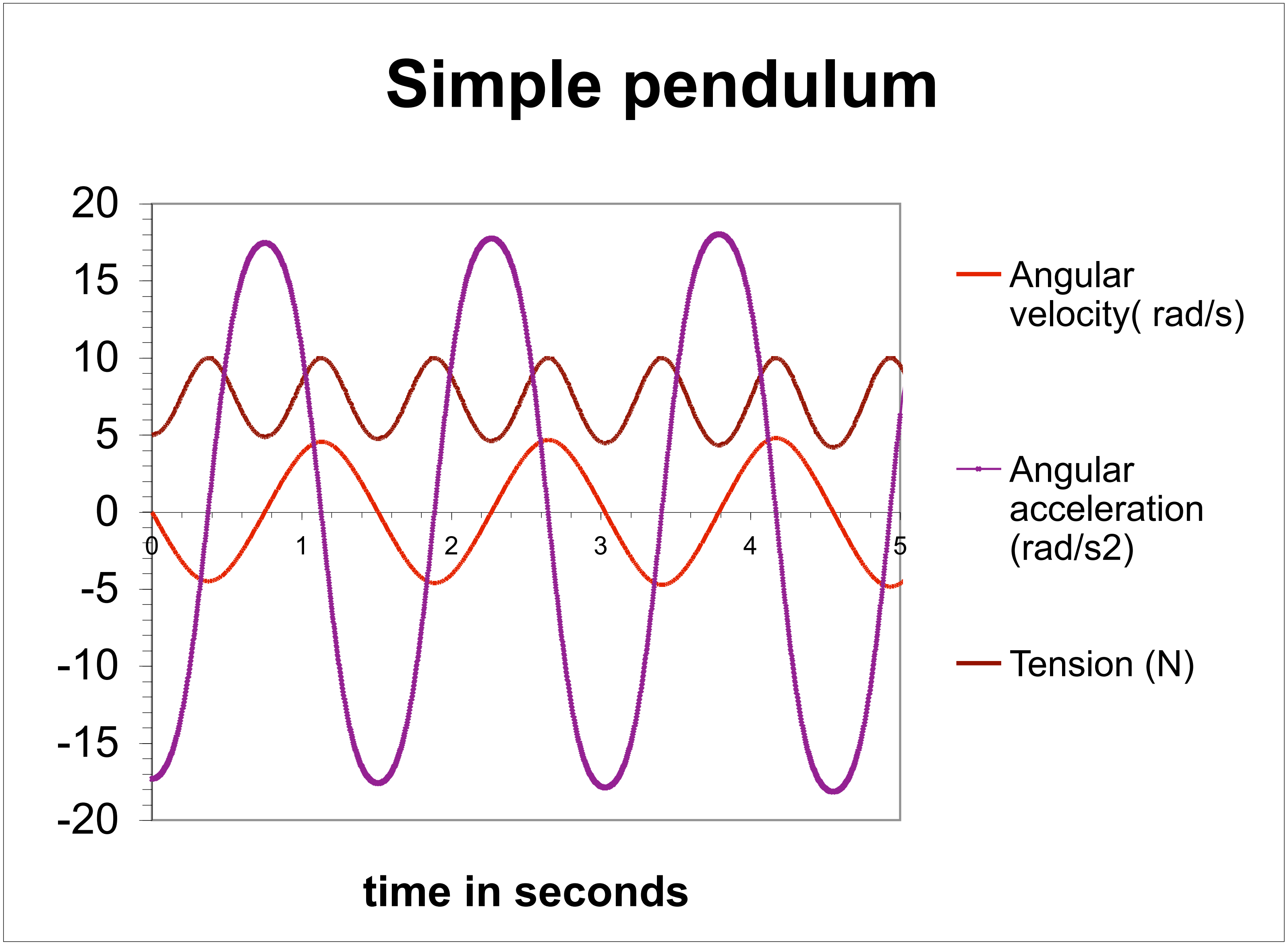
Pendulum

period	1.404251	
delta_t2	**0.004**	
g_2	**10**	
m_2=	**1**	mass
L=	**0.5**	length
w_init=	**0**	omega
theta_init	**60**	theta

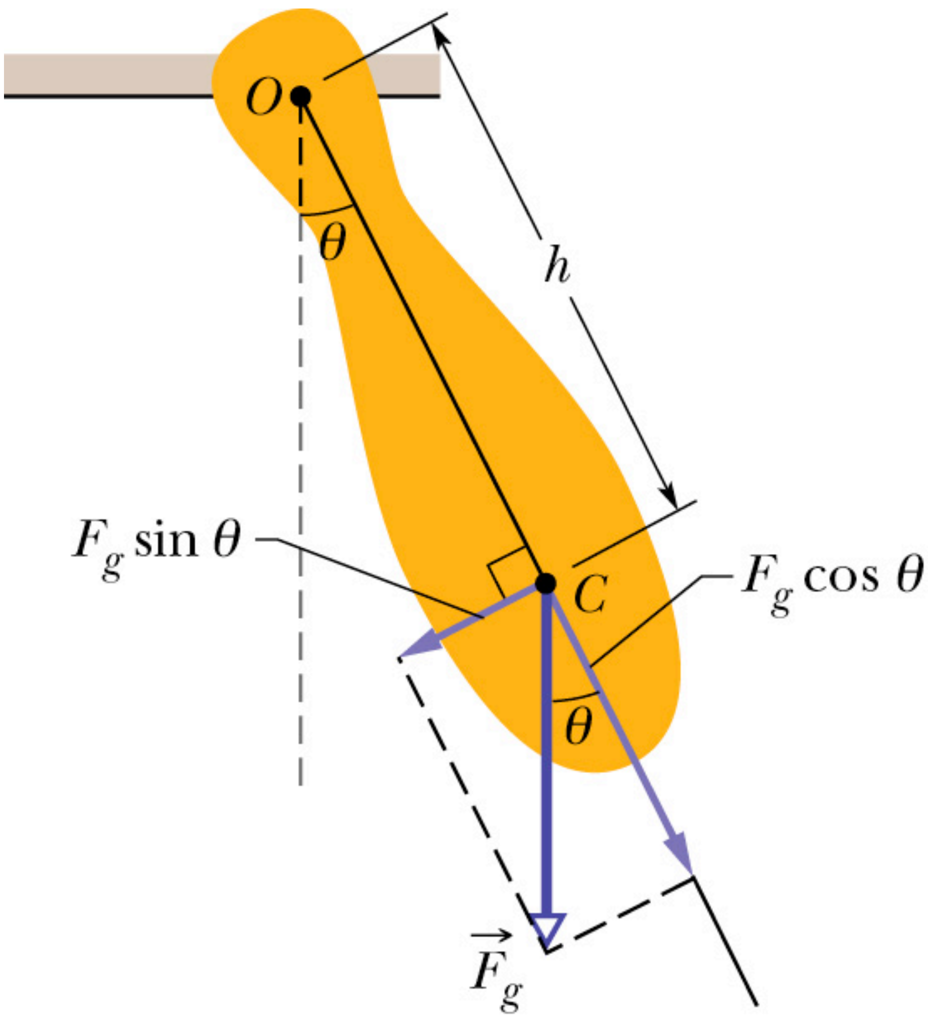
$$\text{Period}=2\pi\sqrt{\frac{L}{g}}=6.28\sqrt{\frac{0.5}{10}}=1.404$$



Shows the difference in results using the sin theta = theta approx.


Simple pendulum
20
15
10
5
0
-5
-10
-15
-20
0
1
2
3
4
5
time in seconds
Angular velocity(rad/s)
Angular acceleration (rad/s2)
Tension (N)

Physical Pendulum



$$\omega = \sqrt{\frac{hmg}{I}}$$

$$T = \frac{2\pi}{\omega}$$

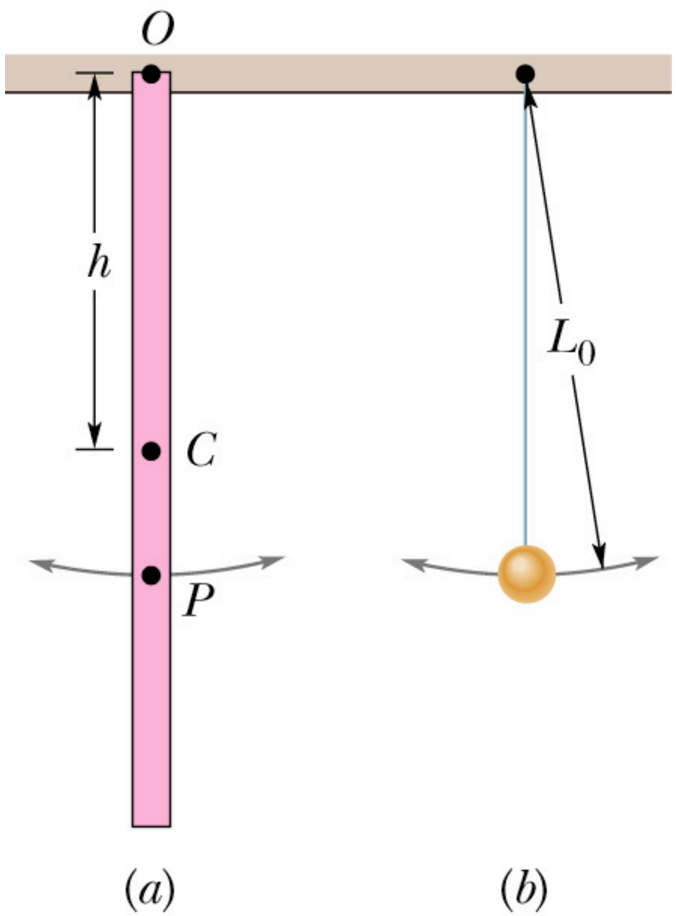
$$T = 2\pi\sqrt{\frac{I}{hmg}}$$

Replace L with h
Note h is the distance between the pivot point and the center of mass

Challenge

How do you find the point on a physical pendulum called the center of percussion or oscillation where the rotational motion is cancelled by the translational motion? If you suspend the pendulum from a point, then the distance to the CP is the same length as a simple pendulum to have the same period.

Meter stick pendulum



$$T = 2\pi\sqrt{\frac{I}{hmg}}$$

$$I = \frac{1}{3}mL^2$$

$$T = 2\pi\sqrt{\frac{\frac{1}{3}mL^2}{\frac{L}{2}mg}}$$

$$T = 2\pi\sqrt{\frac{2L}{3g}} = 2\pi\sqrt{\frac{L_0}{g}}$$

T=1.62 s for L =1 m and g =10 m/s^2

What is point CP again. It is where the rot. motion is canceled by the trans if you strike it at O. It is also the point where a simple pendulum of length OP would have the same period as the meter stick. L_0= 2/3L. Note CP = 1/2L -2/3L =1/6L.

Torsional pendulum

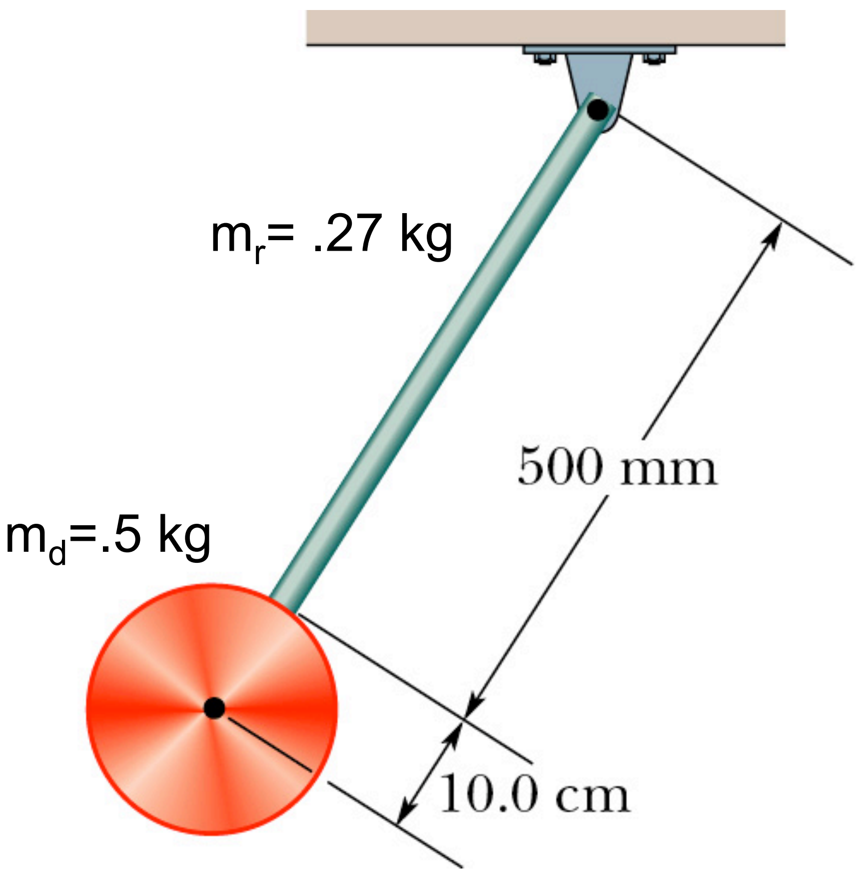
$t = -k\theta$ where k is called a torsional constant dependent on the wire properties

Used in watches

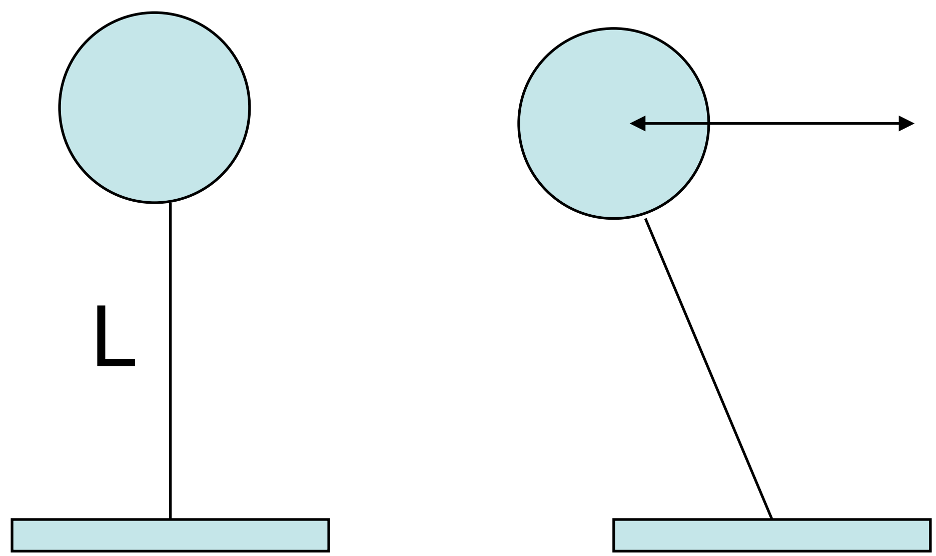
Used to measure gravity (Cavendish)

Used to measure Coulombs Law (MAPE next year)

a) Find the moment of inertia about the pivot point?
b) What is the distance between center of mass and pivot point?
c) Find the period of oscillation?

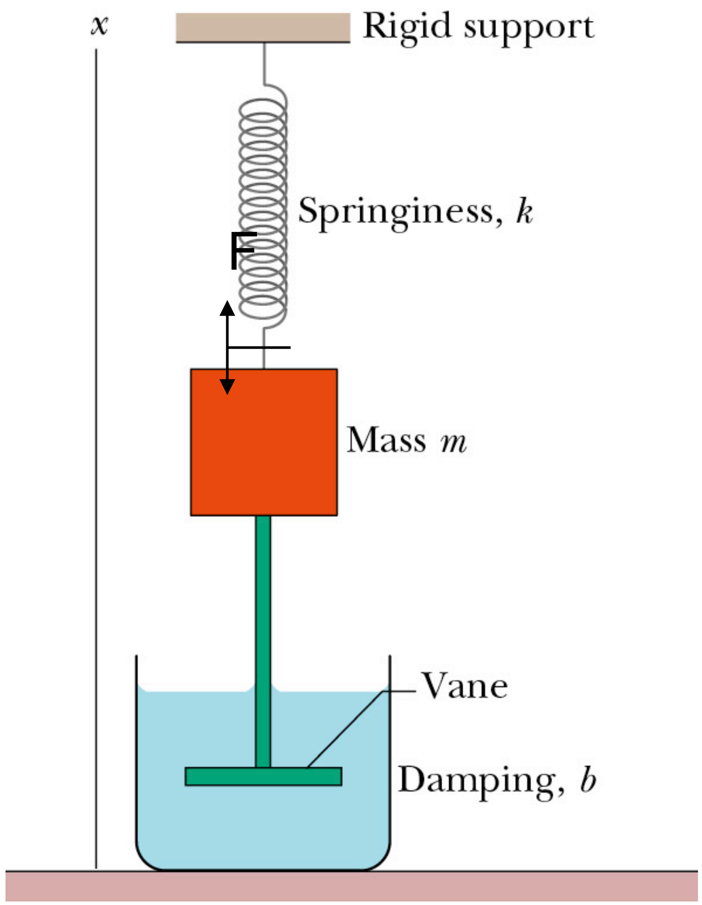


Find the period of a helium balloon whose buoyant force is 5% more than mg of the balloon?



Neglect Air Friction

Damped simple harmonic oscillator with applied force and Resonance



Attach the mass on the left to a motor that moves in a circle

Demo example with applied force and a dampening force. Air acts as the dampening force. The motor is the applied force.

$$F_0 \sin(\omega t) - by - kx = ma$$

$$m\frac{dp}{dt} = F_0 \sin(\omega t) - by - kx$$

$$v_i = v_{i-1} + \left(\frac{F_0}{m}\sin(\omega t_{i-1}) - \frac{b}{m}v_{i-1} - \frac{k}{m}x_{i-1}\right)\Delta t$$

ConcepTest 13.1a Harmonic Motion I

A mass on a spring in SHM has **amplitude *A*** and **period *T***. What is the **total distance traveled** by the mass after a time interval *T*?

1) 0
2) *A*/2
3) *A*
4) 2*A*
5) 4*A*

ConcepTest 13.3a Spring Combination I

A spring can be stretched a distance of 60 cm with an applied force of 1 *N*. If an identical spring is connected in parallel with the first spring, and both are pulled together, how much force will be required to stretch this parallel combination a distance of 60 cm?

1) 1/4 *N*
2) 1/2 *N*
3) 1 *N*
4) 2 *N*
5) 4 *N*

ConcepTest 13.5a Energy in SHM I

A mass oscillates in simple harmonic motion with amplitude A. If the mass is doubled, but the amplitude is not changed, what will happen to the total energy of the system?

1) **total energy will increase**
2) **total energy will not change**
3) **total energy will decrease**

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ConcepTest 13.9 Grandfather Clock

A grandfather clock has a weight at the bottom of the pendulum that can be moved up or down. If the clock is running slow, what should you do to adjust the time properly?

1) move the weight up
2) move the weight down
3) moving the weight will not matter
4) call the repair man

ConcepTest 13.1a Harmonic Motion I

A mass on a spring in SHM has amplitude *A* and period *T*. What is the total distance traveled by the mass after a time interval *T*?

1) 0
2) *A*/2
3) *A*
4) 2*A*
5) 4*A*

In the time interval *T* (the period), the mass goes through one complete oscillation back to the starting point. The distance it covers is: *A* + *A* + *A* + *A* (4*A*).

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1) 1/4 *N*
2) 1/2 *N*
3) 1 *N*
4) 2 *N*
5) 4 *N*

Each spring is still stretched 60 cm, so each spring requires 1 *N* of force. But since there are two springs, there must be a total of 2 *N* of force! Thus, the combination of two parallel springs behaves like a stronger spring!!

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1) **total energy will increase**
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3) **total energy will decrease**

The total energy is equal to the initial value of the elastic potential energy, which is $PE_s = 1/2\ kA^2$. This does not depend on mass, so a change in mass will not affect the energy of the system.

Follow-up: What happens if you double the amplitude?

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3) moving the weight will not matter
4) call the repair man

The period of the grandfather clock is too long, so we need to decrease the period (increase the frequency). To do this, the length must be decreased, so the adjustable weight should be moved up in order to shorten the pendulum length.

$$T = 2\pi\sqrt{L/g}$$