Phys 631 2009
Richard A Lindgren
July 6 Monday
Room 203 9:00 AM - 10:50 AM

- Perspective and Expectations
- Material Covered
- Course Home Page on the Web http://people.virginia.edu/~ral5q/classes/phys631/summer09
  - Syllabus, Course Material, Resources, Homework and WebAssign, Quizzes, Final Grade
- Today's Lecture
  - Tooling up
  - Matter and Interactions
  - One Dimensional motion
  - Vectors/Math Review
  - Clickers

Perspective and Expectations

- This group is unique especially with your widely different physics backgrounds.
- I will be concerned with physics more than pedagogy. You all have better pedagogical techniques than I have.
- We want to expose you to as much physics as possible that we think you should know.
  - Concepts
  - Solve Physics problems
  - You will need resources
  - Your textbook. Other books listed on website
  - TA and each other
  - Google

Course Home Page

- http://people.virginia.edu/~ral5q/classes/phys631/summer09
    - Syllabus
    - Course Material
    - Resources
    - Homework
    - WebAssign
    - Quizzes
    - Final Grade

Final Grade

- Graded Homework (Summer + Fall) 25%
- Clicker Quizzes 5 %
- Three Quizzes 40%
- Final (End of September) 30%

Material Covered - Lectures

First Week

- One Dimensional Motion, Vectors and Math review
- Projectile Motion and
  Forces and Motion I
- Forces and Motion II
- Kinetic Energy and Work
- Potential Energy

Second Week

- Center of mass and Linear Momentum
- Quiz 1
- Rotation and Angular Momentum
- Equilibrium, Elasticity, and Fluids
- Gravitation

Material

Third Week

- Oscillations and pendulums
- Quiz 2
- Waves
- Waves II and Sound
- Temperature, Heat, and First Law of Thermo

Fourth Week

- Kinetic Theory, Entropy, and Second Law of Thermo
- Quiz 3

Todays Lecture

- Tooling up
- Matter and Interactions
- One Dimensional Motion
  - constant speed
  - constant acceleration
- Math Review

- Scales from the very small to the very large
- Fundamental quantities, units, and unit conversions
- Scientific notation
- Accuracy and significant figures
- Dimensional analysis
- Estimating
- Scalars and vectors

The system of units we will use is the

Standard International (SI) system;

the units of the fundamental quantities are:

- Length – meter
- Mass – kilogram
- Time – second

Fundamental Physical Quantities and Their Units

Unit prefixes for powers of 10, used in the SI system:

Prefix	Symbol	Multiple
Exa†	E	10^{18}
Peta†	P	10^{15}
Tera	T	10^{12}
Giga	G	10^{9}
Mega	M	10^{6}
Kilo	k	10^{3}
Hecto†	h	10^{2}
Deka†	da	10^{1}

Prefix	Symbol	Multiple
Deci†	d	10^{-1}
Centi	c	10^{-2}
Milli	m	10^{-3}
Micro	μ	10^{-6}
Nano	n	10^{-9}
Pico	p	10^{-12}
Femto†	f	10^{-15}
Atto†	a	10^{-18}

Accuracy and Significant Figures

The number of significant figures represents the accuracy with which a number is known.

Terminal zeroes after a decimal point are significant figures:

2.00 has 3 significant figures

2 has 1 significant figure.

Webassign, homework answer is correct if it is within 1% of the right answer. If the answer is 100, then correct answers would be equal to and between 99 and 101.

Accuracy and Significant Figures

If numbers are written in scientific notation, it is clear how many significant figures there are:

$6. \times 10^{24}$ has one

6.1×10^{24} has two

6.14×10^{24} has three

…and so on.

Calculators typically show many more digits than are significant. It is important to know which are accurate and which are meaningless.

In Webassign, homework answer is correct if it is within 1% of the right answer.

Scientific Notation

Scientific notation: use powers of 10 for numbers that are not between 1 and 10 (or, often, between 0.1 and 100):

When multiplying numbers together, you add the exponents algebraically.

$$(2\times 10^{4})(3\times 10^{6}) = 6\times 10^{4+6} = 6\times 10^{10}$$

When dividing numbers, you subtract the exponents algebraically.

$$\frac{2\times 10^{4}}{4\times 10^{6}} = 0.5\times 10^{4-6} = 0.5\times 10^{-2} = 5.\times 10^{-3}$$

Example

$$\frac{7.5 \times 10^{-3}}{2.5 \times 10^{-4}} = \frac{7.5}{2.5} \times 10^{-3} \times 10^{+4} = 3.0 \times 10 = 30$$

1-4 Dimensional Analysis

The dimension of a quantity is the particular combination that characterizes it (the brackets indicate that we are talking about dimensions):

$[v] = [L]/[T]$

Note that we are not specifying units here – velocity could be measured in meters per second, miles per hour, inches per year, or whatever.

Force=ma $[F]=[M][L]/[T^2]$

Estimates or Guesstimates– How a Little Reasoning Goes a Long Way

Estimates are very helpful in understanding what the solution to a particular problem might be.

Generally an order of magnitude is enough – is it 10, 100, or 1000?

Final quantity is only as accurate as the least well estimated quantity in it

Guesstimates

1) How many golf balls would it take to circle the equator?

- You need the diameter of the golf ball which is about 2 inches or 4 cm.
  - 2.5 cm = 1 inch
- And you need the circumference of the earth. ????
- Divide the circumference by the diameter to get the number of golf balls
- How to estimate the circumference of the earth?
- US is 3000 miles wide because it takes a jet at 500 mph to fly from NY to LA in 6 hours
- It is also 3 times zones wide and there are 24 time zones across the world and therefore the circumference is 3000 x 24/3=24,000 miles or 40,000 km

The number of golf balls is N

$$N = 4 \times 10^4\ km \times \frac{10^3 m}{km} \times \frac{10^2 cm}{m} \times \frac{1}{4 cm} = 10^9 \text{golf balls}$$

2) If we gave every family on earth a house and a yard, what per cent of the earth's surface would they occupy?

3)How many cells are there in the human body?

4) What is the total amount of human blood in the world?

Matter: What do we mean by it?

Solids, Liquids and Gases

Electrons orbiting the nucleus make up the atoms and the atoms make up the solids, liquids and gases.

For example the surface of a solid might look something like this.

Solids

- STM images of a surface through silicon
- Atoms are arranged in a crystalline array or 3D solid
- Note defects in the lower image

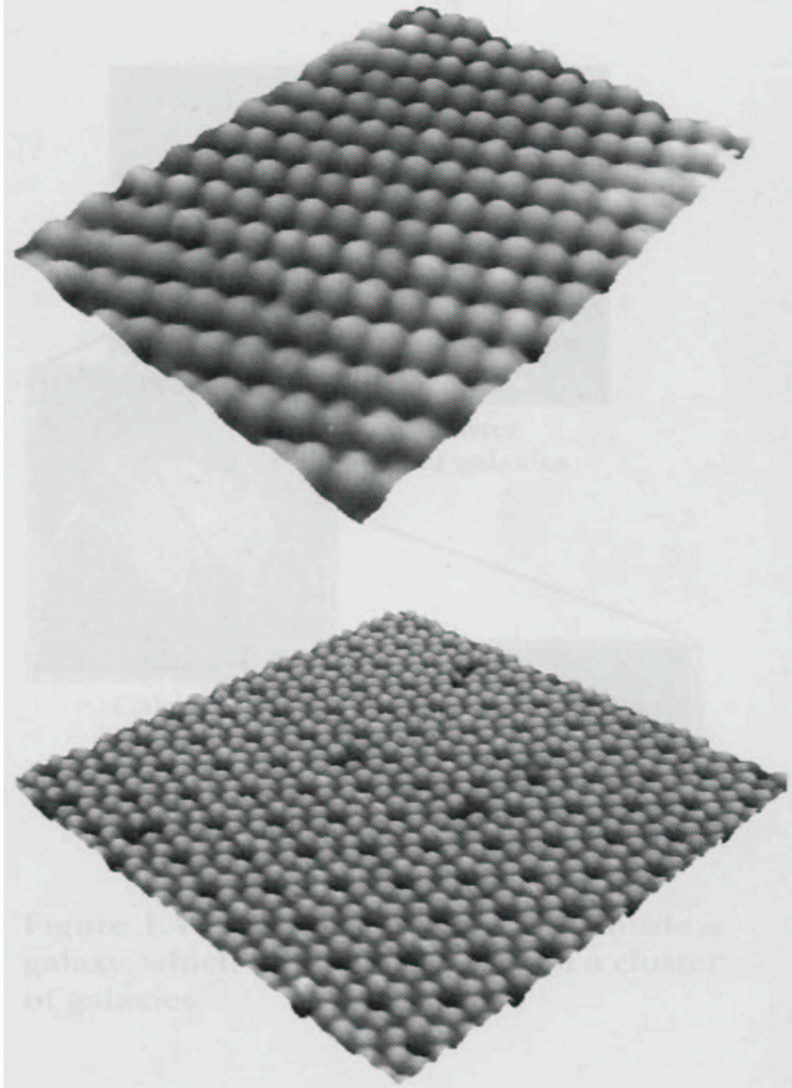


Figure 1.3 Two different surfaces of a crystal of pure silicon. The images were made with a scanning tunneling microscope.

Ordinary matter on
earth is made up of
tiny **Atoms**

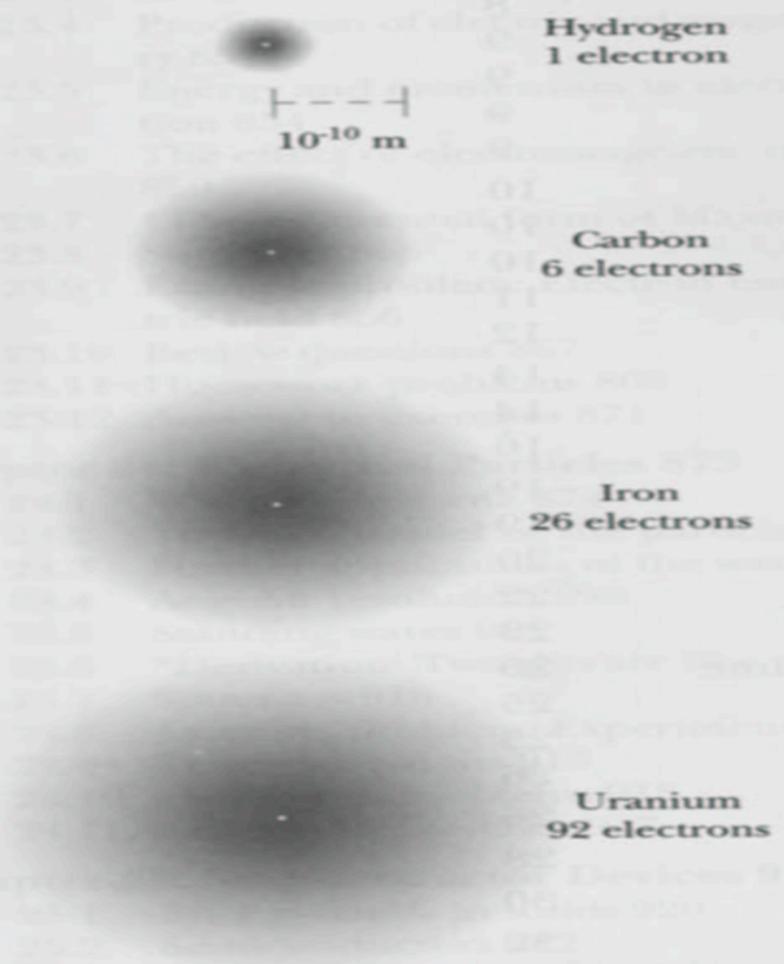


Figure 1.1 Atoms of hydrogen, carbon, iron, and uranium. The white dot shows the location of the nucleus. On this scale, however, the nucleus would be much too small to see.

In this course we will mostly work with the gravitational force (Action at a distance) and contact interactions or forces.

Projectile motion - gravity

Book resting on a table. What forces act on the book?

Friction

When a bowling ball strikes a pin and knocks it over,
we call it a contact interaction.

Also keep in mind that we assume in many instances that we are dealing with rigid bodies.

Interactions (Action at a distance)

Four types four types of such interactions also called fundamental interactions.

- Strong - inside the nucleus of the atom
- Electromagnetic - between charged particles
  - Electric
  - Magnetic
- Weak - involves the neutrino
- **Gravitational – Man - Earth**
  Earth - Sun

Strength : Strong > electromagnetic > weak > gravitational

Principles

- Conservation of Energy
- Conservation of Momentum
- Newton's Laws
- Principle of Relativity
  - Laws of physics work the same for an observer in uniform motion as for an observer at rest.

- Mathematics

Predict the future

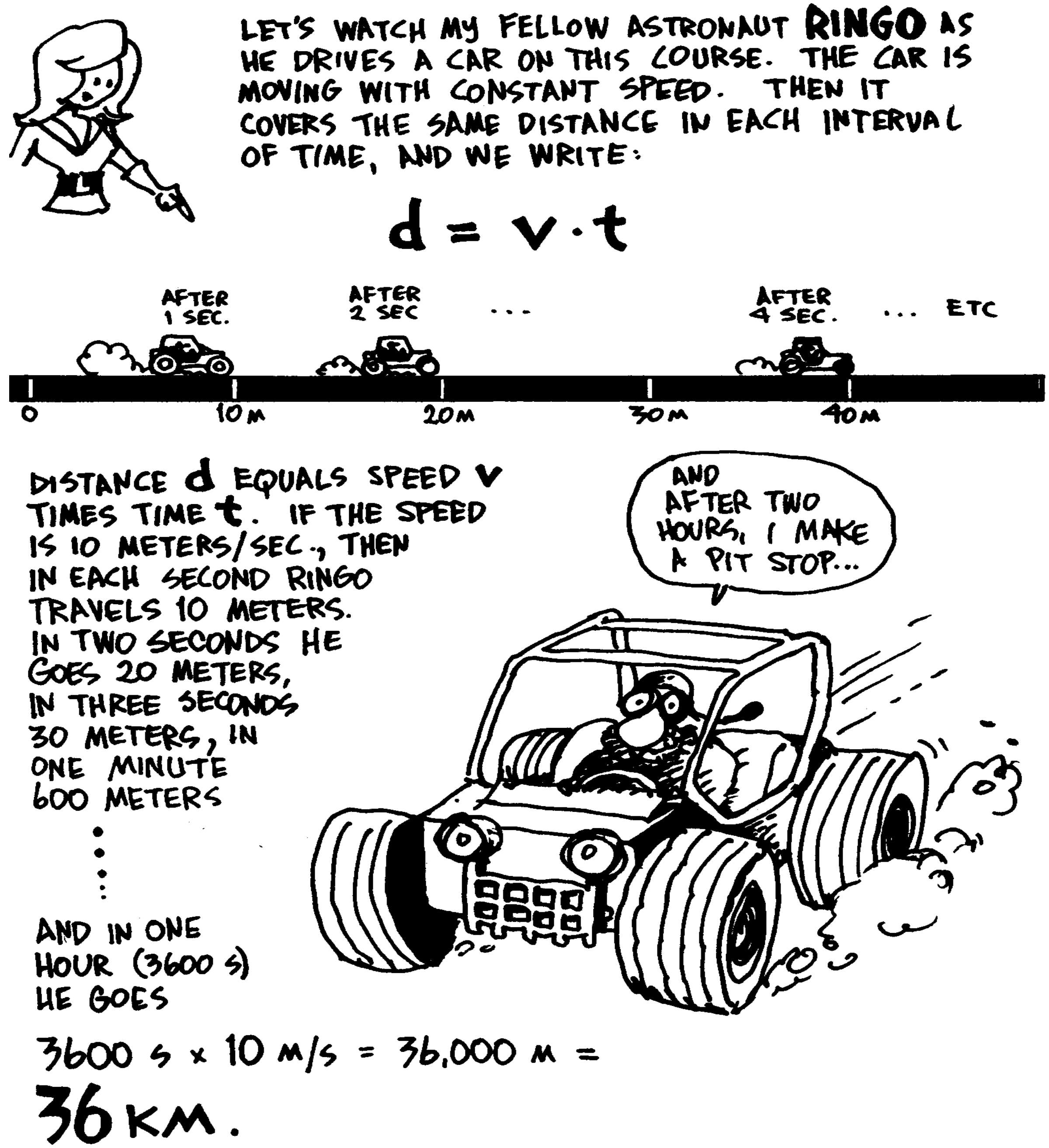
Start with Motion in One Dimensional

Need to define a few parameters:

- Time
- Position
- Velocity
- Acceleration

Chapter 2

Motion Along a Straight Line
One dimensional motion

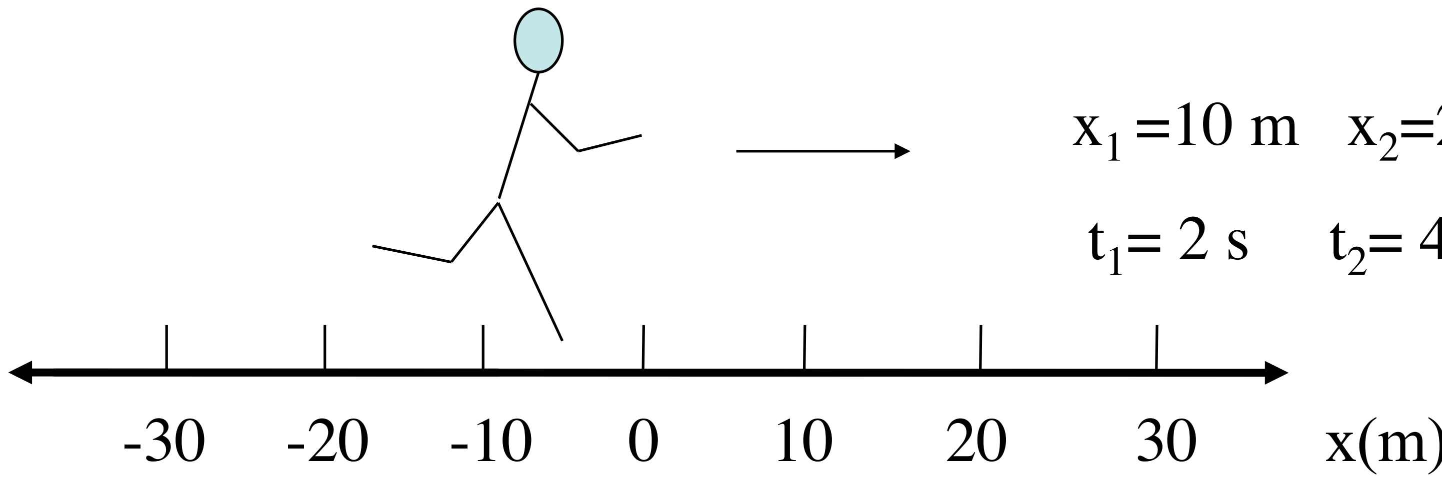


TYPICAL SPEEDS

Motion	v(mph)	v(m/s)	v/c
Light	669,600,000	300,000,000	1
Earth around sun	66,600	29,600	10^{-4}
Moon around Earth	2300	1000	$3*10^{-6}$
Jet fighter	2200	980	$3*10^{-6}$
Sound in air	750	334	10^{-6}
Commercial airliner	600	267	10^{-6}
Cheetah	62	28	10^{-7}
Falcon diving	82	37	10^{-7}
Olympic 100m dash	22	10	$3*10^{-8}$
Flying bee	12	5	10^{-8}
Walking ant	0.03	0.01	$3*10^{-11}$
Swimming sperm	0.0001	0.000045	10^{-13}

Nonrelativistic speeds

A person running along a straight line at some velocity.



$x_1 = 10$ m $\quad x_2 = 20$ m

$t_1 = 2$ s $\quad t_2 = 4$ s

<u>Average velocity</u>

velocity = (distance traveled)/time

$$v = (x_2 - x_1) / (t_2 - t_1)$$

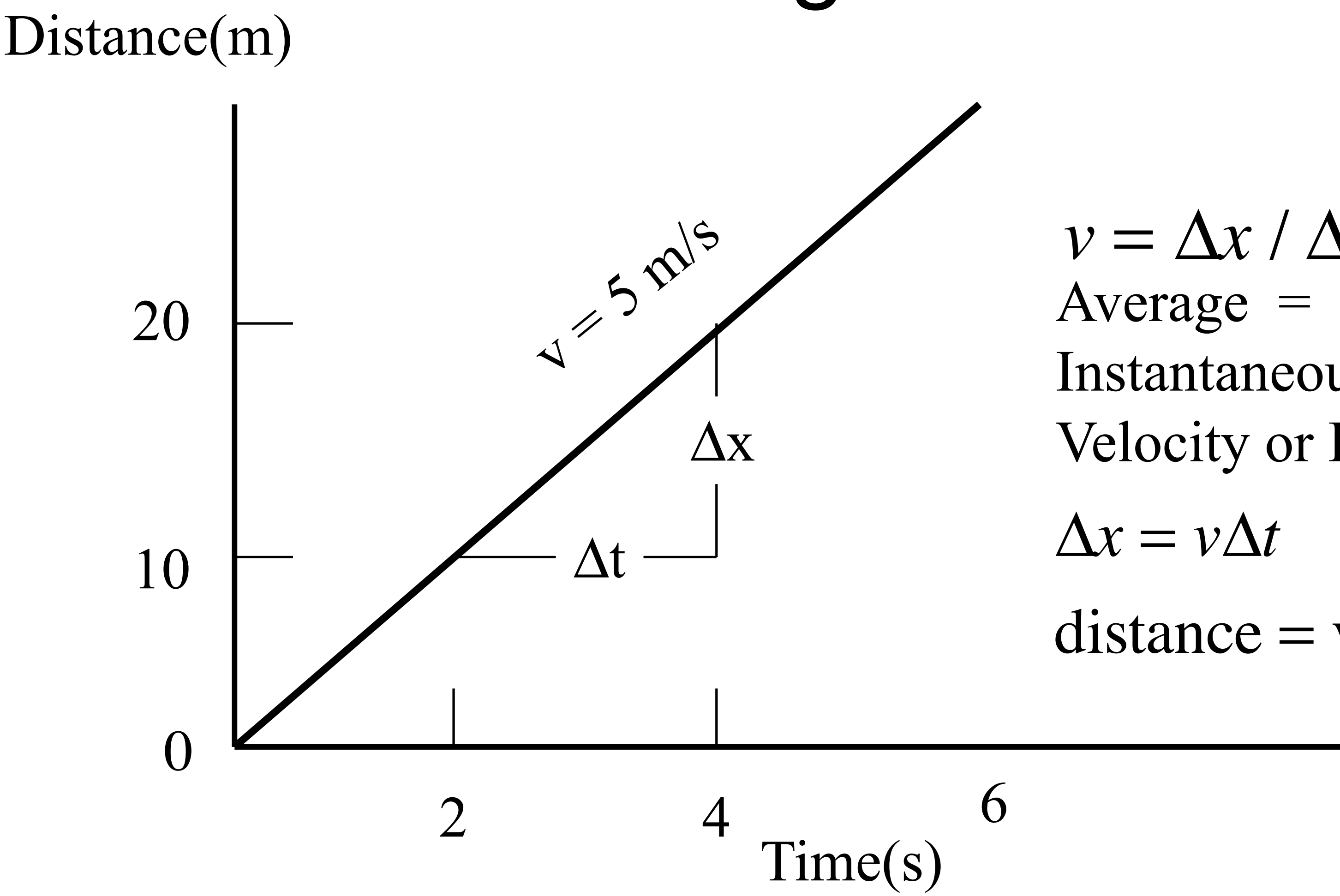
$$v = \Delta x / \Delta t$$

$$x_2 - x_1 = 20 - 10 = 10m$$

$$t_2 - t_1 = 4 - 2 = 2s$$

$$v = 10m / 2s = 5m / s$$

Distance-time graph for running in a straight line



$v = \Delta x \,/\, \Delta t$

Average = Instantaneous Velocity or Exact

$\Delta x = v\Delta t$

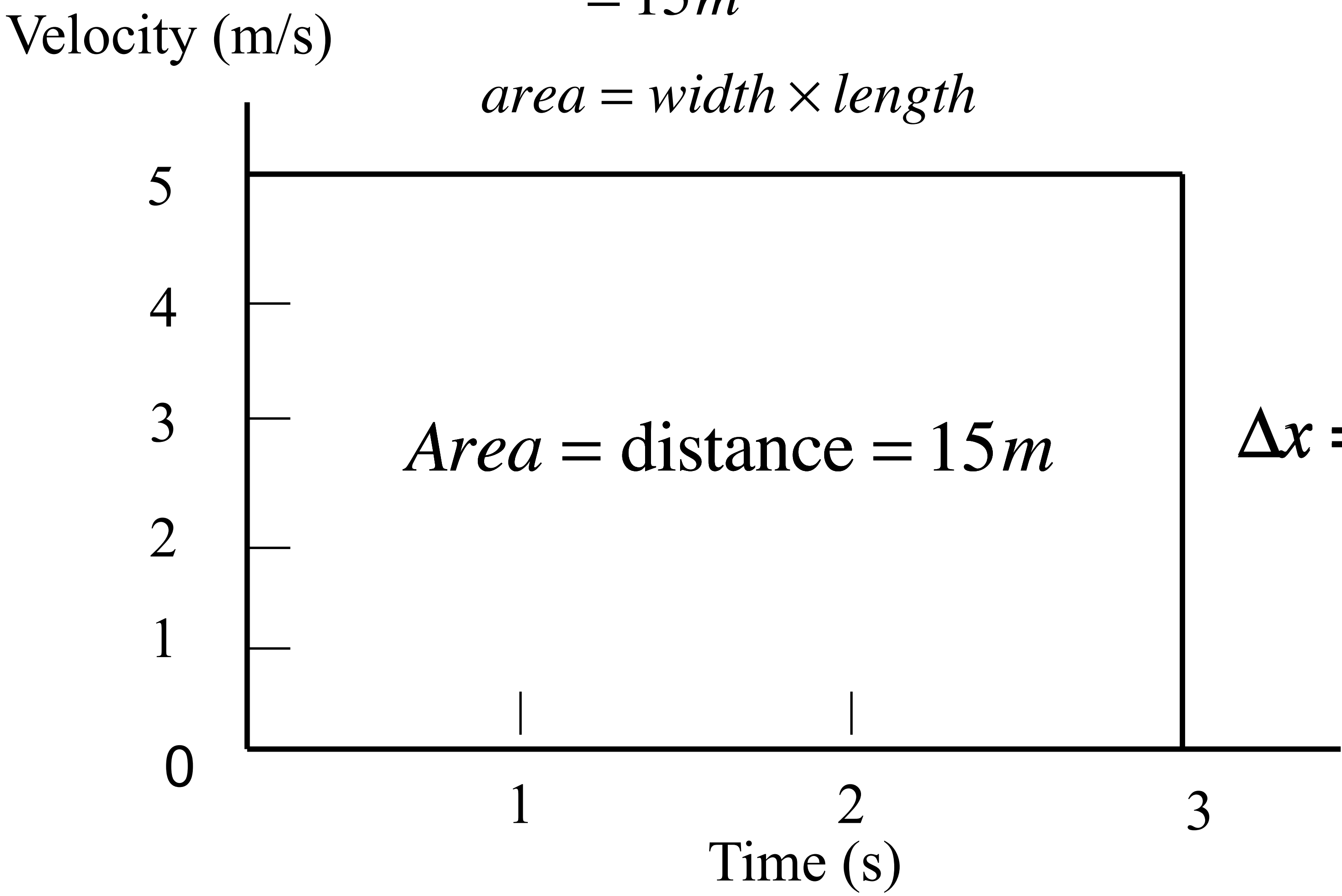
$\text{distance} = \text{velocity} \times \textit{time}$

Constant velocity-time graph

$$\text{distance} = velocity \times time = 5m / s \times 3s$$

$$= 15m$$

$$area = width \times length$$

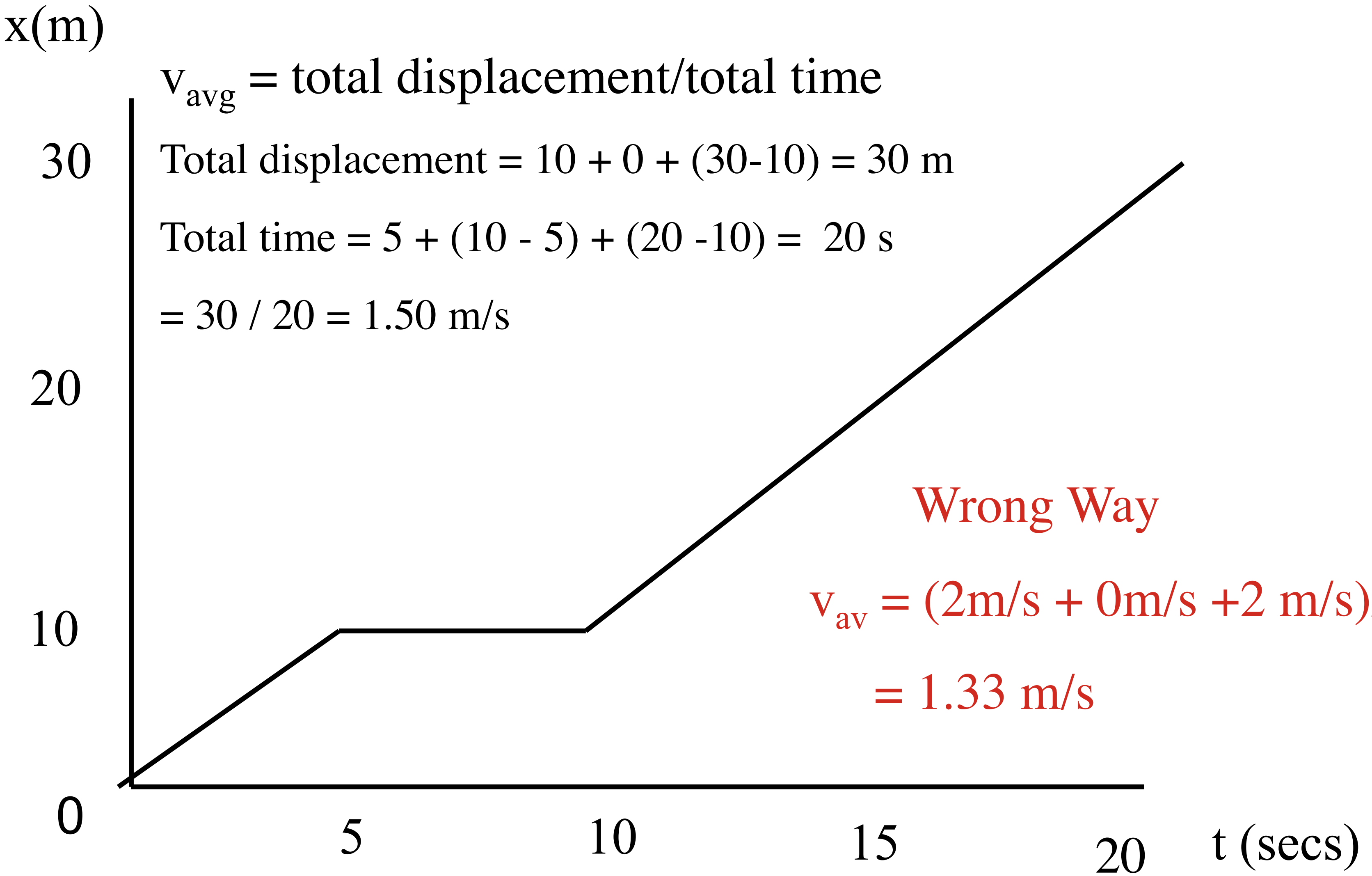


$$\Delta x = v\Delta t$$

What is meant by v_{avg}

Suppose I run for 5 s at a velocity of 2 m/s, then I rest for 5 s, and then I run for 10 s at a velocity of 2 m/s. What is my average velocity over the 20 s?

Distance-time graph for changing V
What is meant by v_{avg}



v_{avg} = total displacement/total time

Total displacement = 10 + 0 + (30-10) = 30 m

Total time = 5 + (10 - 5) + (20 -10) = 20 s

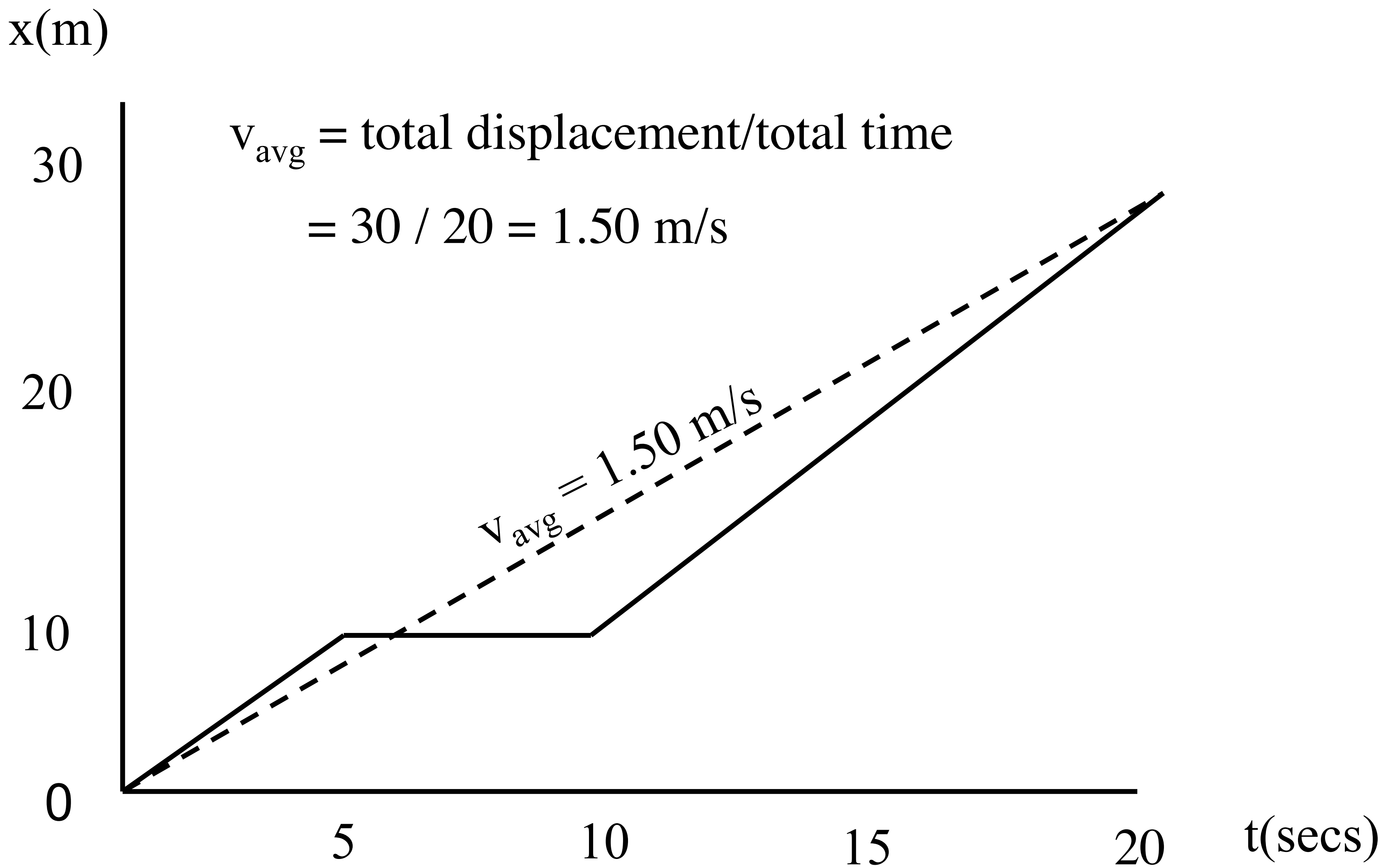
= 30 / 20 = 1.50 m/s

Wrong Way

v_{av} = (2m/s + 0m/s +2 m/s)/3

= 1.33 m/s

Distance-time graph for changing V



What is the difference between average velocity and average speed?

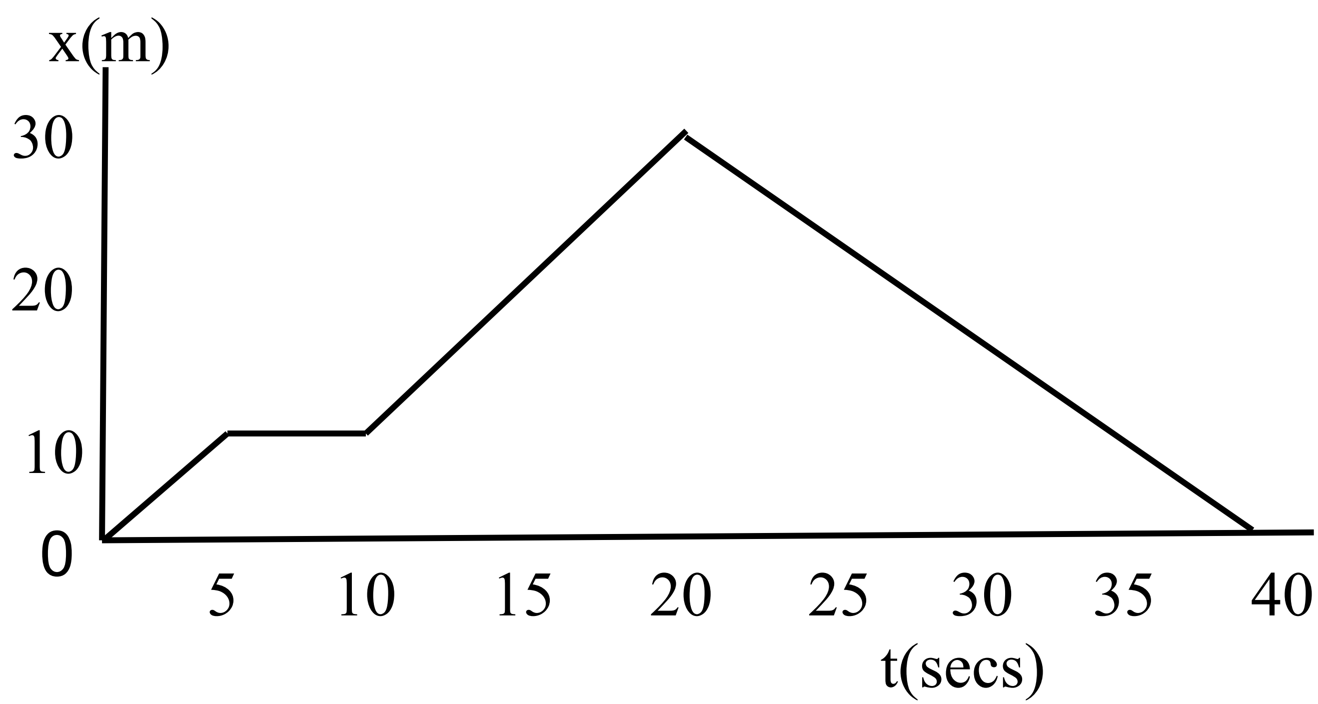
Suppose I run for 5 s at a velocity of 2 m/s, then I rest for 5 s, and then I run for 10 s at a velocity of 2 m/s. Now I run for 20 s at - 2m/s or backwards. What is my average velocity and speed over the entire 40 s?

Average velocity and average speed are not always the same.

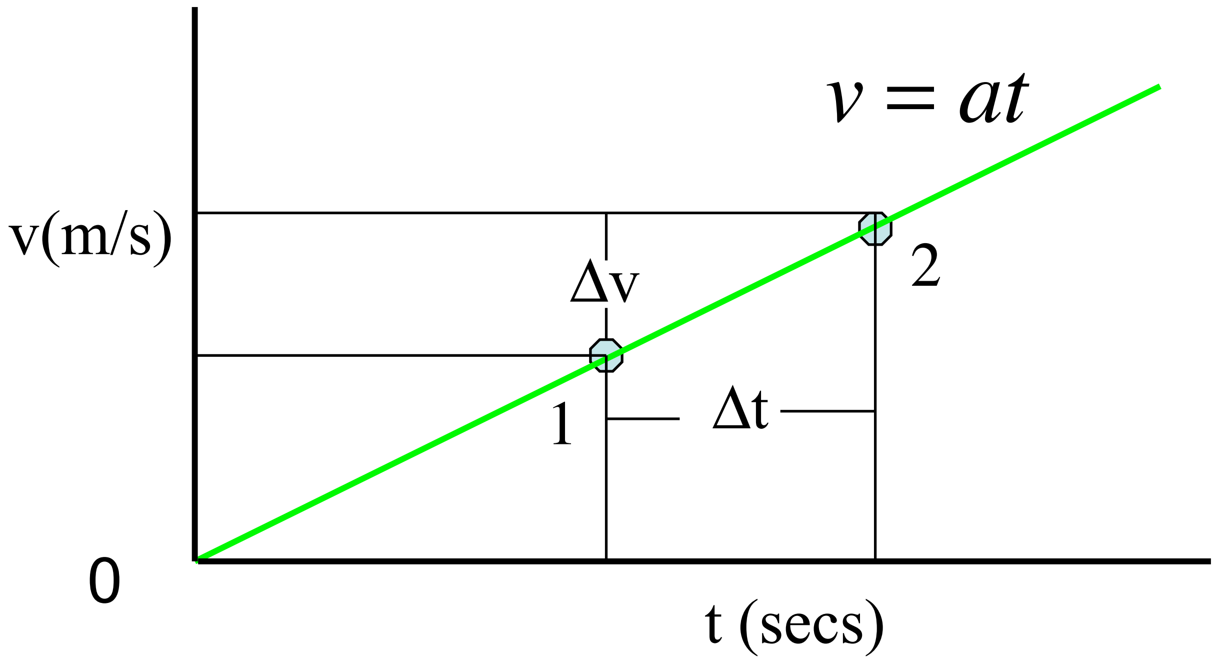
Average velocity $= v_{avg}$ = total displacement/total time

$= (30 - 30) / 40 = 0$ m/s

Average speed $= s_{avg} = (30 + 30)/40 = 1.5$ m/s



Here is the velocity-time graph for uniform acceleration.



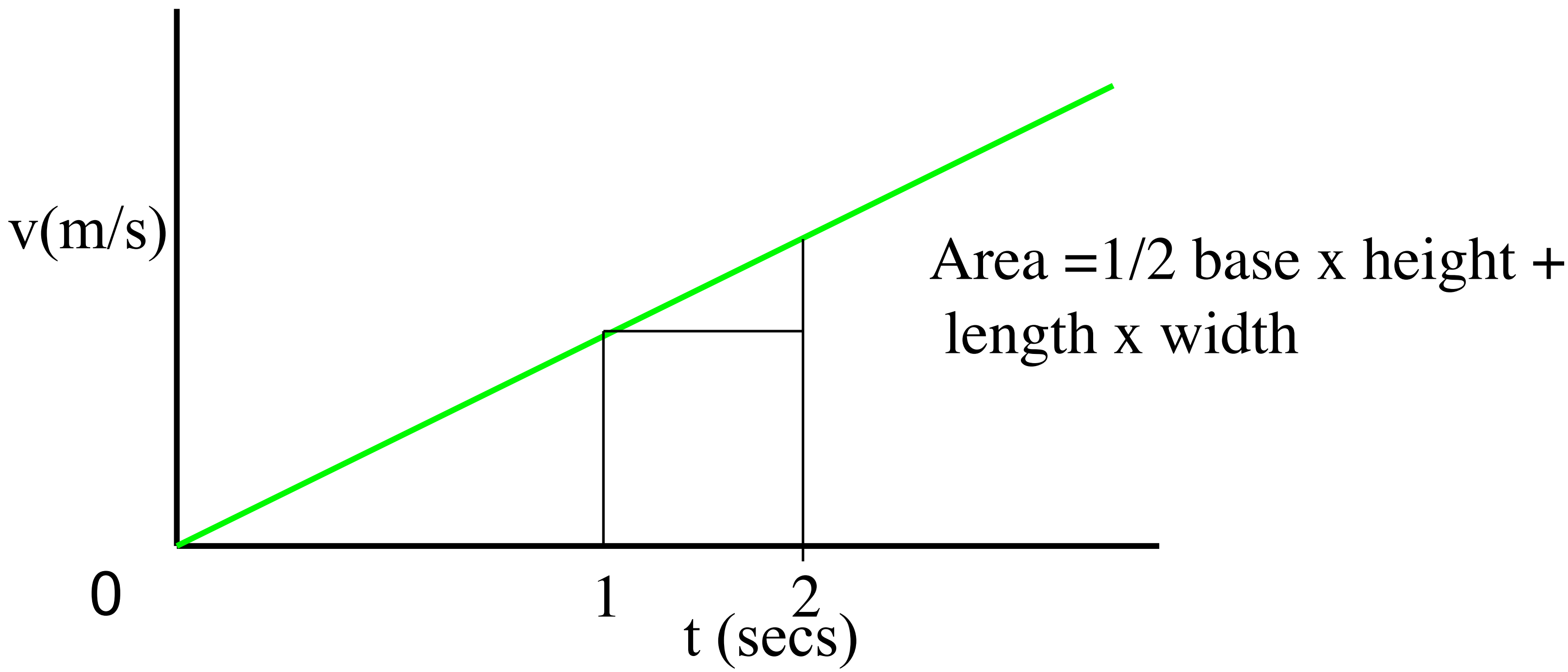
$$a = velocity\,/\,time = (v_2 - v_1)\,/\,(t_2 - t_1) = \Delta v\,/\,\Delta t$$

= average acceleration

= slope of graph

Units of a are (m/s^2 in mks system of units)

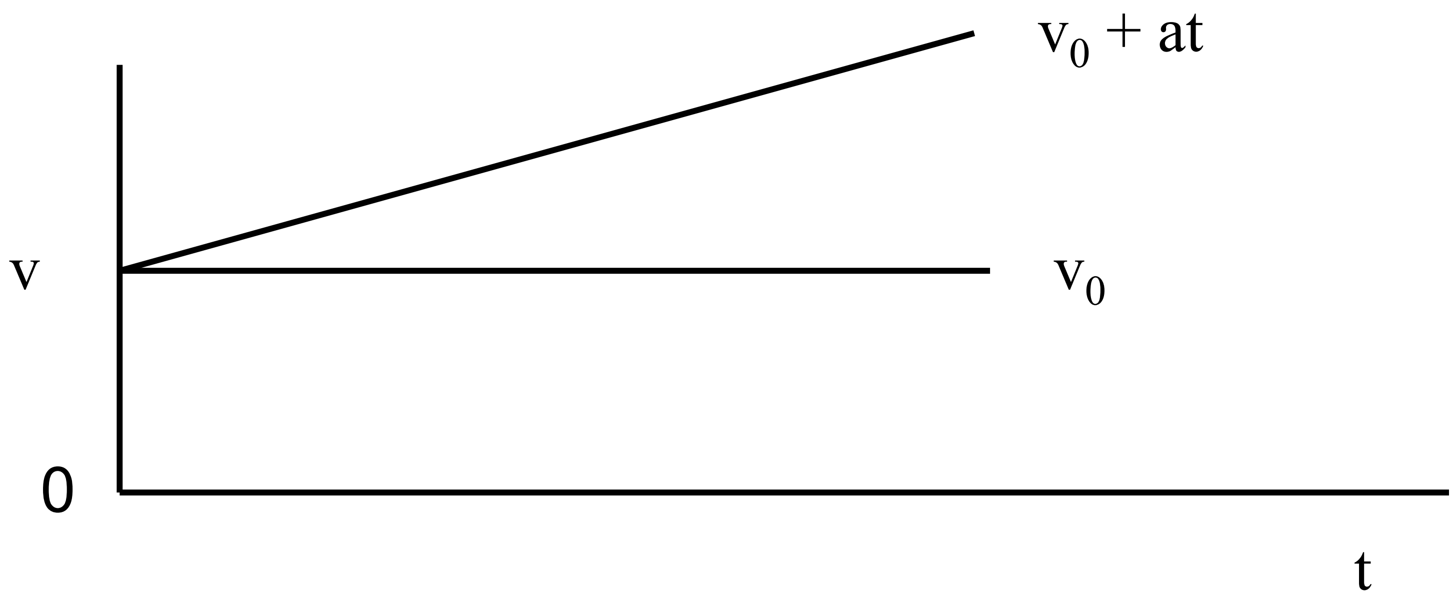
How far does an object move from point 1 to point 2?
It is equal to the total area under the green line in between points 1 and 2.



$$Area = \frac{1}{2}(t_2 - t_1)(v_2 - v_1) + v_1(t_2 - t_1) = \frac{1}{2}(v_2 + v_1)(t_2 - t_1)$$

$$x = v_{avg} \times t$$

NON-ZERO INITIAL SPEED



$$v = v_0 + at$$

$$x = v_{av}t = \frac{1}{2}(v_0 + v)t$$

$$x = \frac{1}{2}(v_0 + v_0 + at)t = v_0 t + \frac{1}{2}at^2$$

Summary of Equations in 1D (constant acceleration)

$$v = v_0 + at$$

$$v_{avg} = \frac{1}{2}(v_0 + v)$$

$$x = v_{av} t$$

$$x = v_0 t + \frac{1}{2} a t^2$$

$$v^2 = v_0^2 + 2ax$$

Under what conditions do these apply?

Same Equations with initial velocity = 0

$$v = at$$

$$v_{avg} = \frac{1}{2}v$$

$$x = v_{avg}t$$

$$t = \frac{x}{v_{avg}}$$

$$x = \frac{1}{2}at^2$$

$$v^2 = 2ax$$

$$t = \sqrt{\frac{2x}{a}}$$

Lets look at a numerical example and then a demo.

Galileo’s Result (1564)

Dropping things from rest

Galileo’s experiments produced a surprising Result.

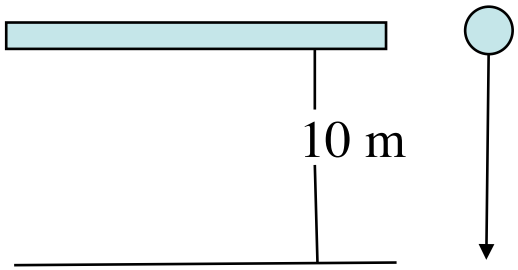
All objects fall with the same acceleration Regardless of mass and shape.

$G = 9.8\ m/s^2$ or $32\ ft/s^2$

Neglecting air resistance.

Free Fall Example

Find the time it takes for a free-fall drop from 10 m height.
Take the downward direction as positive displacement. Use two methods.



Method 1

$$x = v_{avg} t$$

$$t = 10 / v_{avg}$$

Find v_{avg}

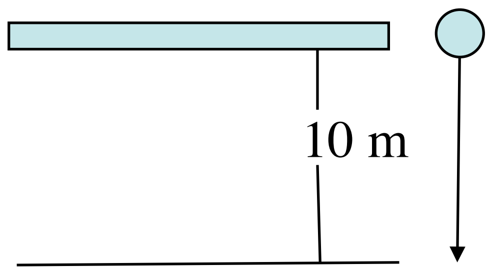
Method 2

$$x = \tfrac{1}{2} a t^2$$

$$t = \sqrt{\frac{2x}{a}}$$

Free Fall Example

Find the time it takes for a free-fall drop from 10 m height. Take the downward direction as positive displacement. Use two methods.



Method 1

$$t = 10 / v_{avg}$$

$$v_{avg} = (0 + v_f) / 2$$

Find v_f

$$v_f^2 = 0^2 + 2gx$$

$$v_f = \sqrt{2 * 9.8 * 10}$$

$$v_f = 14 m / s$$

$$v_{avg} = \frac{1}{2} v_f$$

$$v_{avg} = \frac{14 m / s}{2} = 7 m / s$$

$$t = \frac{10 m}{7 m / s}$$

$$t = 1.43 s$$

Find the time it takes for a free-fall drop from 10 m height. Take the downward direction as positive displacement. Use two methods.

Method 2

$$t = \sqrt{\frac{2x}{a}}$$

$$x = \tfrac{1}{2} g t^2$$

$$t = \sqrt{2x / g}$$

$$t = \sqrt{2 * 10 / 9.8}$$

$$t = 1.43 \text{ s}$$

Demos (Motion in one dimension)

- Free fall acceleration of weights equally spaced on a string 50 cm apart.

  - Times of the first three are:

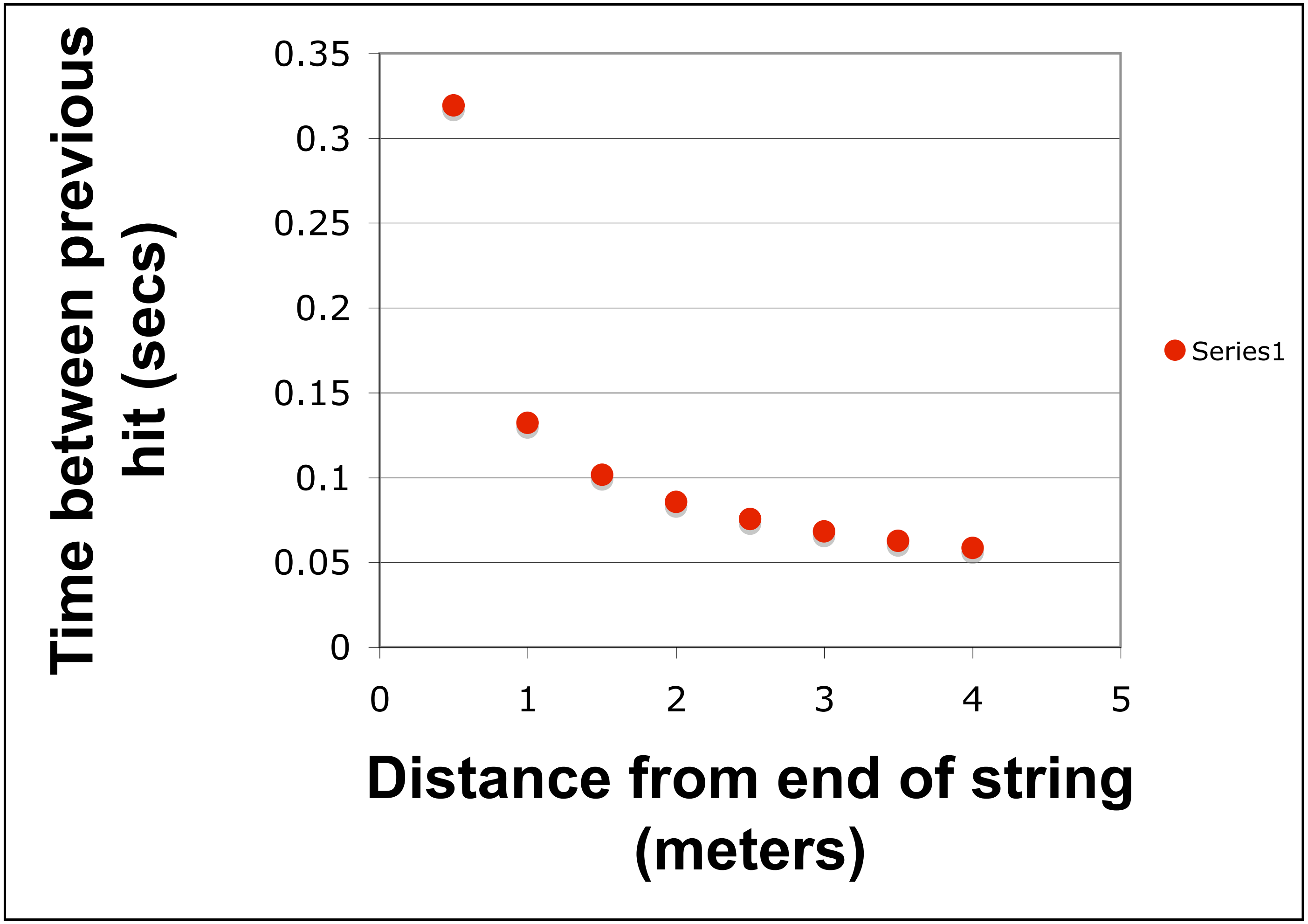
$$x = \tfrac{1}{2} g t^2$$
$$t = \sqrt{2x / g}$$
$$t = \sqrt{2 * 0.5 / 9.8}$$
$$t = 0.32 \text{ s}$$

$$x = \tfrac{1}{2} g t^2$$
$$t = \sqrt{2x / g}$$
$$t = \sqrt{2 * 1.0 / 9.8}$$
$$t = 0.45 \text{ s}$$

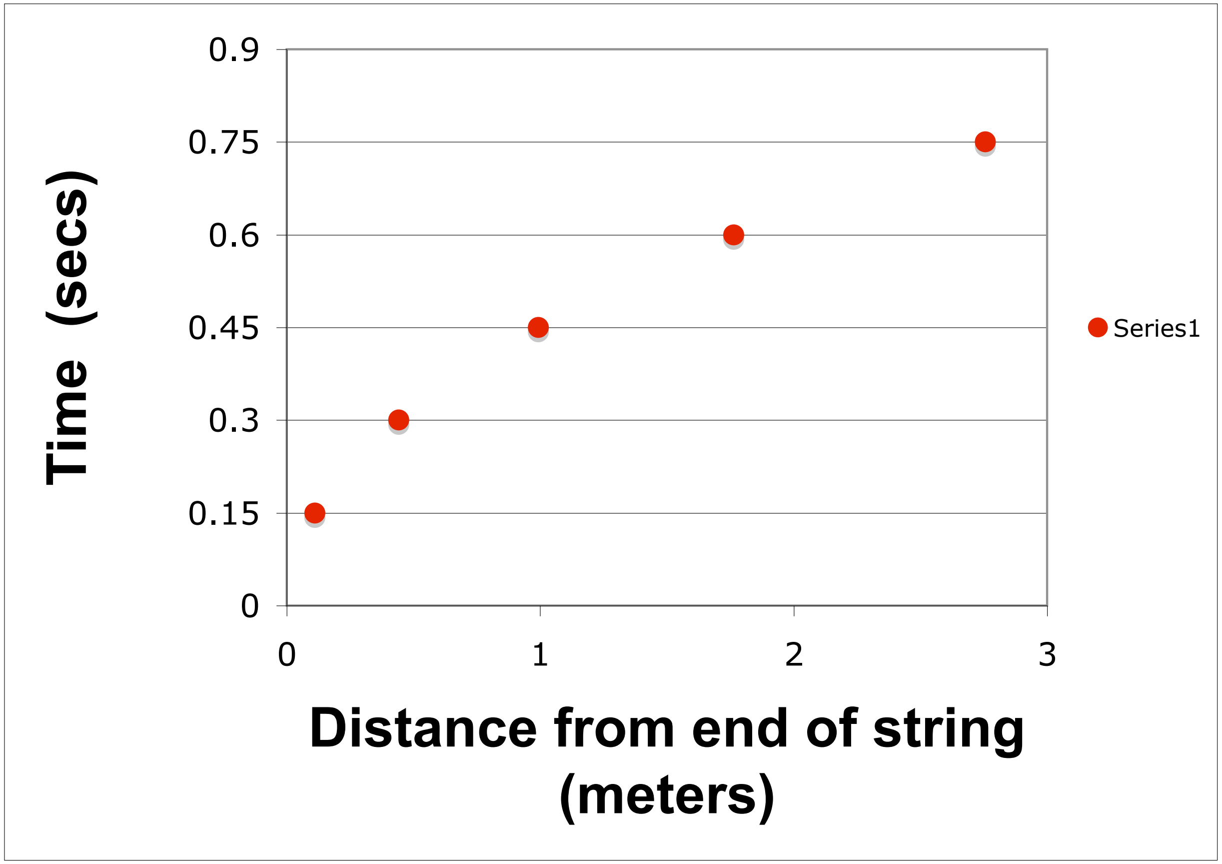
$$x = \tfrac{1}{2} g t^2$$
$$t = \sqrt{2x / g}$$
$$t = \sqrt{2 * 1.5 / 9.8}$$
$$t = 0.55 \text{ s}$$

  - Time between hits = 0.32, 0.13, 0.10, ……

Time between hits vrs distance



Weights spaced apart in increasing distances such that they hit at equal successive time intervals



stone #	Time	Distance
1	0.15	0.11
2	0.3	0.44
3	0.45	0.99
4	0.6	1.76
5	0.75	2.76

$$\text{distance} = \frac{1}{2} g(\text{time})^2$$

How far does a train go when it starts from rest and uniformly increases its speed to 120 m/s in 1 min?

$t = 0$ $\qquad\qquad\qquad$ $v = at$

$v = 0$

$$v_{avg} = \frac{(0 + 120m / s)}{2} = 60m / s$$

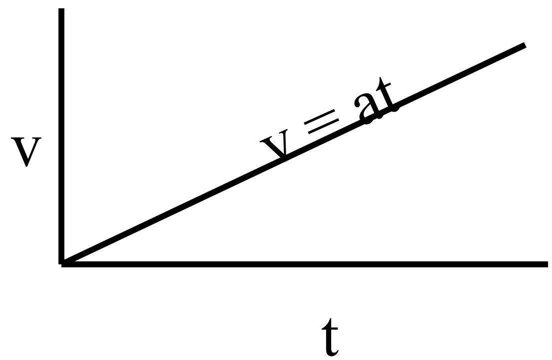
$$x = v_{avg}t = (60m / s) * (60s) = 3600m$$

How far does a train go when it starts from rest and uniformly increases its speed to v m/s in time t?

$t = 0$

$v = 0$

$v = at$



Find v_{avg}

$$v_{avg} = \frac{1}{2}(v_0 + v) = \frac{1}{2}(0 + at) = \frac{1}{2}at$$

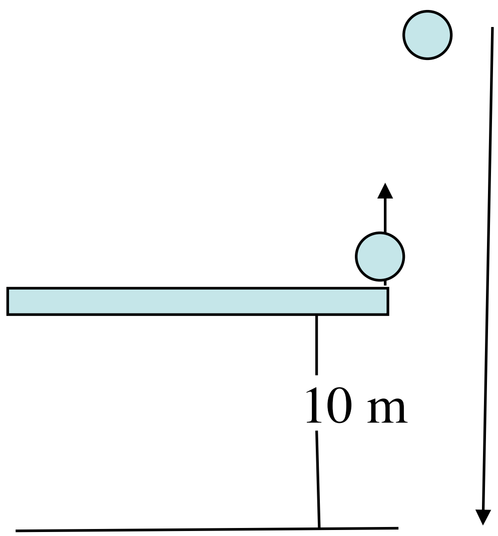
$$x = v_{avg}t = \frac{1}{2}at * t$$

$$x = \frac{1}{2}at^2$$

$$x = \frac{1}{2}(2m/s^2)(60^2 s) = 3600m$$

Non-zero Initial Velocity Example

Find the time t it takes for a platform diver 10 m high to hit the water if he takes off vertically with a speed of - 4 m/s and the speed v with which the diver strikes the water.



Choose positive down

$$v^2 = v_0^2 + 2ax$$

$$v = \sqrt{(-4)^2 + (2)(9.8)(10)}$$

$$v = 14.6 m / s$$

$$v_{avg} = \frac{(-4 + 14.6)}{2} = 5.3 m / s$$

$$t = \frac{10 m}{v_{avg}} = \frac{10 m}{5.3 m / s} = 1.89 s$$

Math Review

- Algebra -
  - Solving simultaneous equations
  - Cramers Rule
  - Quadratic equation
- Trigonometry and geometry
  - sin, cos, and tan, Pythagorean Theorem,
  - straight line, circle, parabola, ellipse
- Vectors
  - Unit vectors
  - Adding, subtracting, finding components
  - Dot product
  - Cross product
- Derivatives
- Integrals

http://people.virginia.edu/~ral5q/classes/phys631/summer07/math-practice.html

Simultaneous Equations

$$2x + 5y = -11$$
$$x - 4y = 14$$

FIND X AND Y

$$x = 14 + 4y$$
$$2(14 + 4y) + 5y = -11$$
$$28 + 8y + 5y = -11$$
$$13y = -39$$
$$y = -3$$
$$x = 14 + 4(-3) = 2$$

Cramer's Rule

$$\boxed{\begin{aligned} a_1x + b_1y &= c_1 \\ a_2x + b_2y &= c_2 \end{aligned}}$$

$$\begin{aligned} 2x + 5y &= -11 \\ x - 4y &= 14 \end{aligned}$$

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{c_1b_2 - c_2b_1}{a_1b_2 - a_2b_1}$$

$$= \frac{(-11)(-4) - (14)(5)}{(2)(-4) - (1)(5)} = \frac{44 - 70}{-8 - 5} = \frac{-26}{-13} = 2$$

$$y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}$$

$$= \frac{(2)(14) - (1)(-11)}{(2)(-4) - (1)(5)} = \frac{28 + 11}{-8 - 5} = \frac{39}{-13} = -3$$

Quadratic Formula

EQUATION:

$$ax^2 + bx + c = 0$$

SOLVE FOR X:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

SEE EXAMPLE NEXT PAGE

Example

$$2x^2 + x - 1 = 0$$

$$a = 2$$

$$b = 1$$

$$c = -1$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(2)(-1)}}{2(2)}$$

$$x = \frac{-1 \pm \sqrt{9}}{4} = \frac{-1 \pm 3}{4}$$

$$x_- = \frac{-1 - 3}{4} = -1$$

$$x_+ = \frac{-1 + 3}{4} = \frac{1}{2}$$

Derivation

$$ax^2 + bx + c = 0$$

$$x^2 + (\frac{b}{a})x + (\frac{c}{a}) = 0$$

Complete the Square

$$\left[x + (\frac{b}{2a})\right]^2 - (\frac{b}{2a})^2 + (\frac{c}{a}) = 0$$

$$\left[x + (\frac{b}{2a})\right]^2 = -(\frac{c}{a}) + (\frac{b^2}{4a^2})$$

$$(2ax + b)^2 = 4a^2\left[-(\frac{c}{a}) + (\frac{b^2}{4a^2})\right]$$

$$(2ax + b)^2 = b^2 - 4ac$$

$$2ax + b = \pm\sqrt{b^2 - 4ac}$$

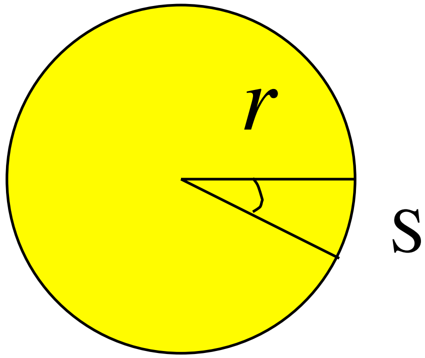
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Arc Length and Radians

$r = radius$

$D = diameter$

$C = circumfrance$



$$2r = D$$

$$\frac{C}{D} = \pi = 3.14159$$

$$\frac{C}{2r} = \pi$$

$$C = 2\pi r$$

$$\frac{C}{2\pi} = r$$

$$\frac{C}{2\pi} = \frac{S}{\theta} = r$$

$$S = r\theta$$

θ is measured in radians

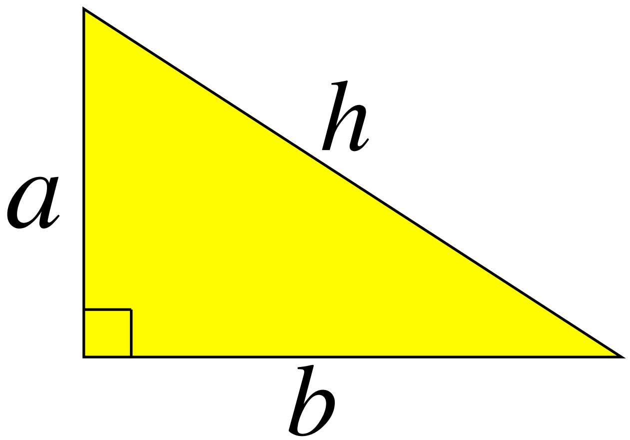
$$\theta = 2\pi$$

$$S = r2\pi = C$$

$$2\pi rad = 360^o$$

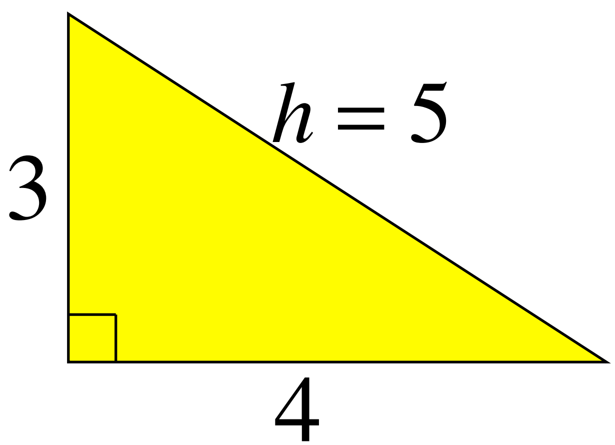
$$1rad = \frac{360^o}{2\pi} = 57.3 \,{}^{\deg}\!/_{rad}$$

Pythagorean Theorem



$$h^2 = a^2 + b^2$$

EXAMPLE

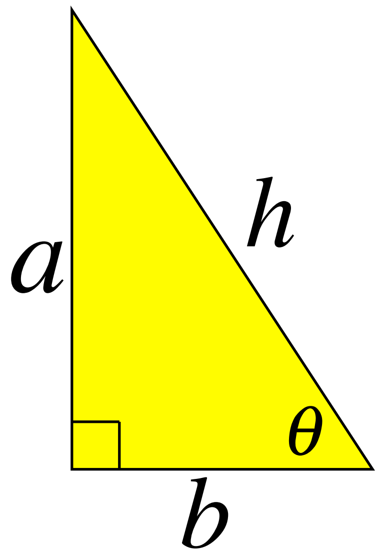


$$h^2 = 3^2 + 4^2$$

$$h = \sqrt{9+16} = \sqrt{25}$$

$$h = 5$$

Trigonometry



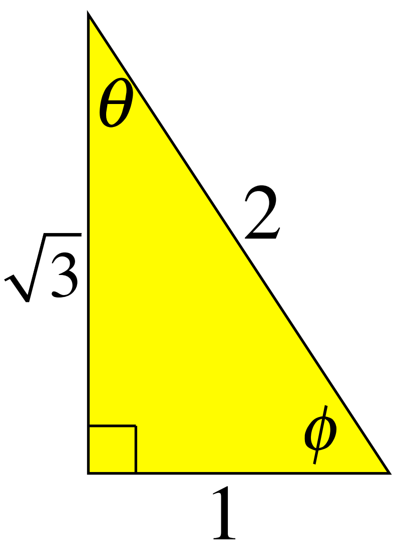
$$\sin\theta = \frac{opp}{hyp} = \frac{a}{h}$$

$$\cos\theta = \frac{adj}{hyp} = \frac{b}{h}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{opp}{adj} = \frac{a}{b}$$

$$\sin^2\theta + \cos^2\theta = 1$$

EXAMPLE



$$\sin\theta = \frac{1}{2}, \theta = 30^o$$

$$\cos\theta = \frac{\sqrt{3}}{2}, \theta = 30^o$$

$$\sin\phi = \frac{\sqrt{3}}{2}, \phi = 60^o$$

$$\cos\phi = \frac{1}{2}, \phi = 60^o$$

$$\text{degrees} = \text{radians} \times \frac{180}{\pi}$$

Small Angle Approximation

Small-angle approximation is a useful simplification of the laws of trigonometry which is only approximately true for finite angles.

FOR $\theta \leq 10^o$

$$\Downarrow$$

$$\sin\theta \simeq \theta$$

EXAMPLE

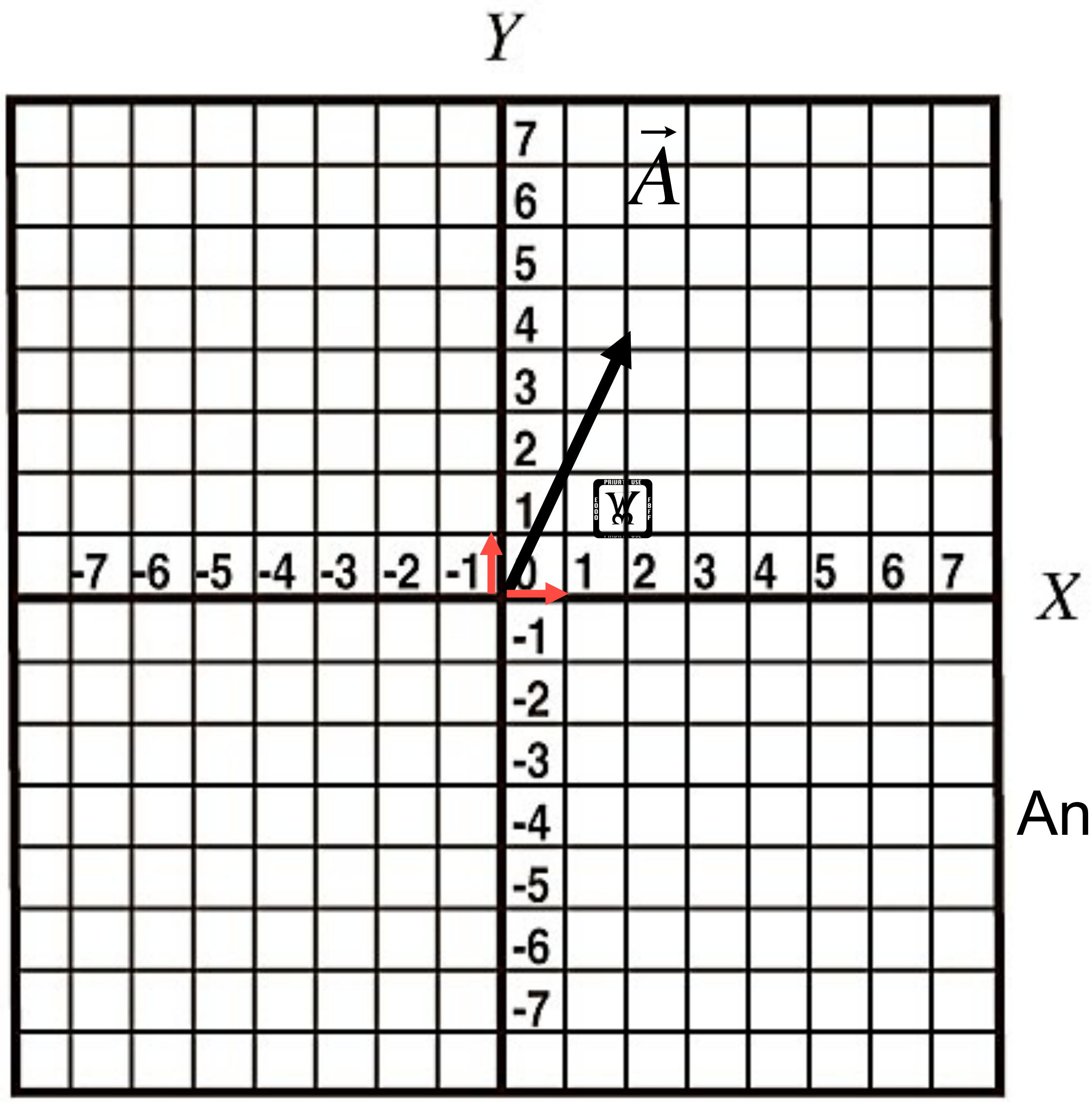
$$\sin(10^o) = 0.173648178$$

$$10^o = 0.174532925 \text{ radians}$$

Vectors and Unit Vectors

- Representation of a vector : has magnitude and direction
  - i and j unit vectors (Note the hat on the unit vector)
  - angle and magnitude
  - x and y components
- Example of vectors
- Addition and subtraction
- Scalar or dot product

Vectors



$$\vec{A} = \left\langle 2\hat{i} + 4\hat{j} \right\rangle$$

Red arrows are the i and j unit vectors.

Magnitude =

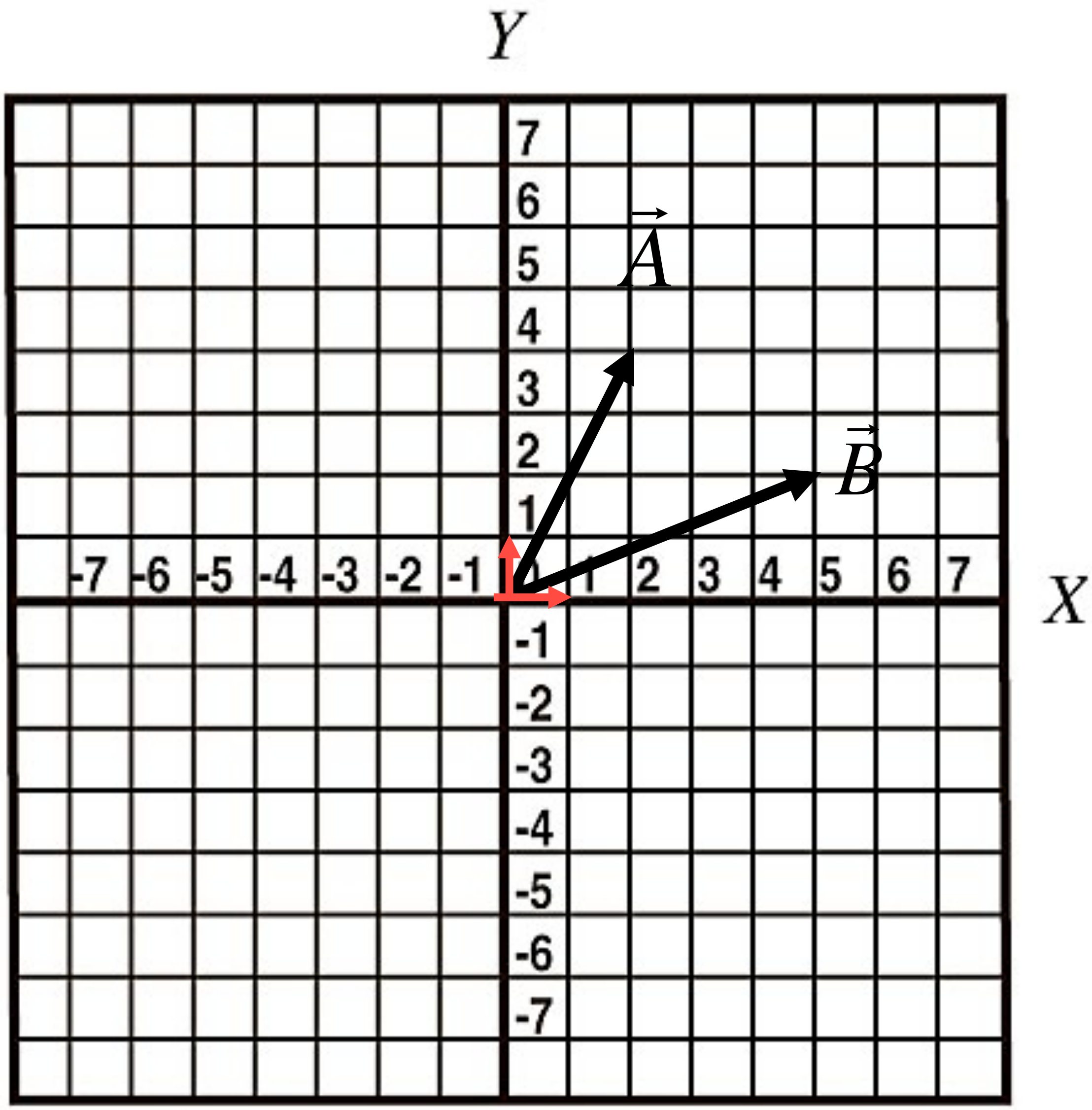
$$|A| = \sqrt{2^2 + 4^2} = \sqrt{20} = 4.47$$

Angle between A and x axis =

$$\tan\theta = y / x = 4 / 2 = 2$$

$$\theta = 63.4\,\mathrm{deg}$$

Adding Two Vectors

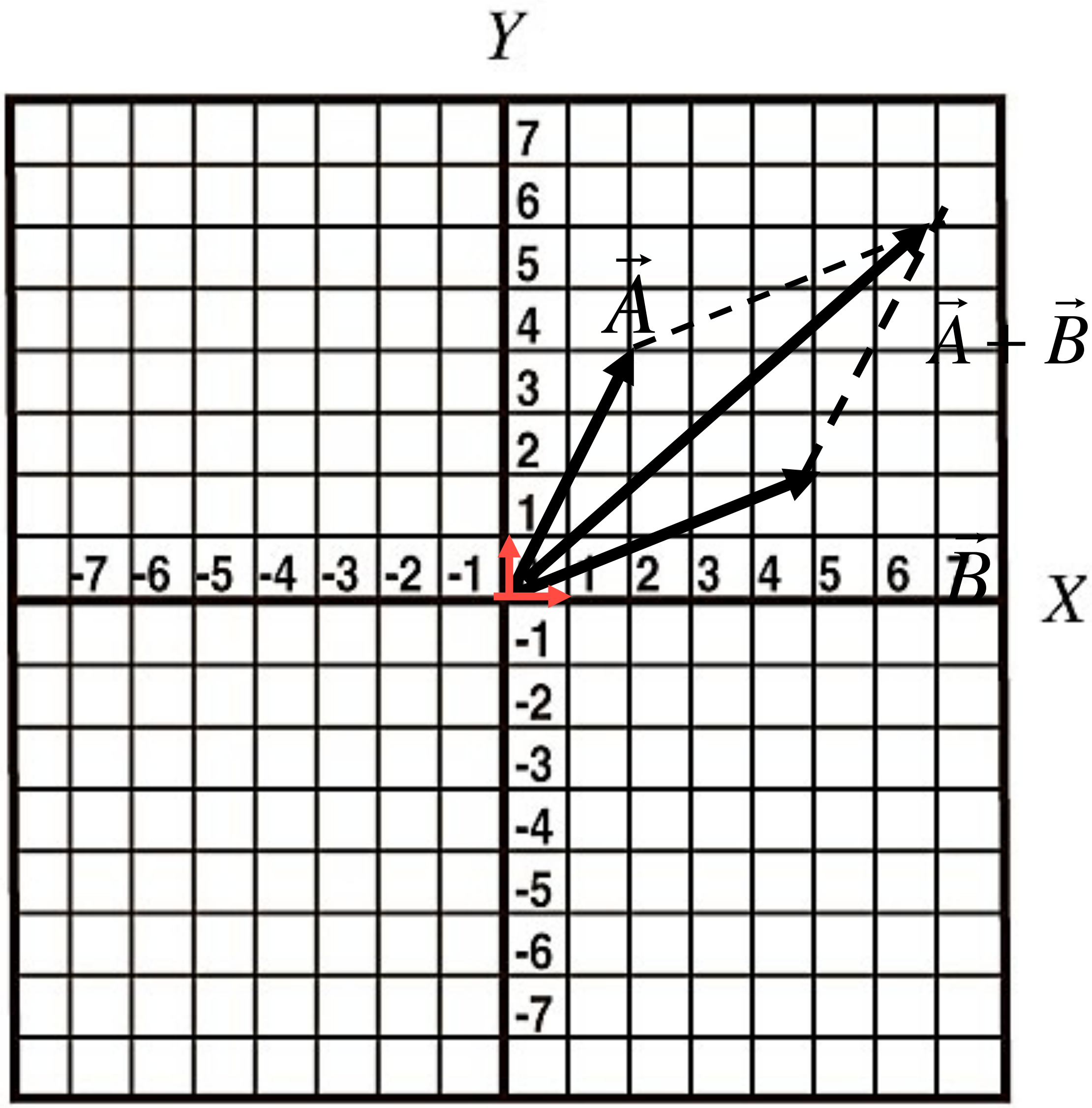


$$\vec{A} = \left\langle 2\hat{i} + 4\hat{j} \right\rangle$$

$$\vec{B} = \left\langle 5\hat{i} + 2j \right\rangle$$

Create a
Parallelogram with
The two vectors
You wish you add.

Adding Two Vectors



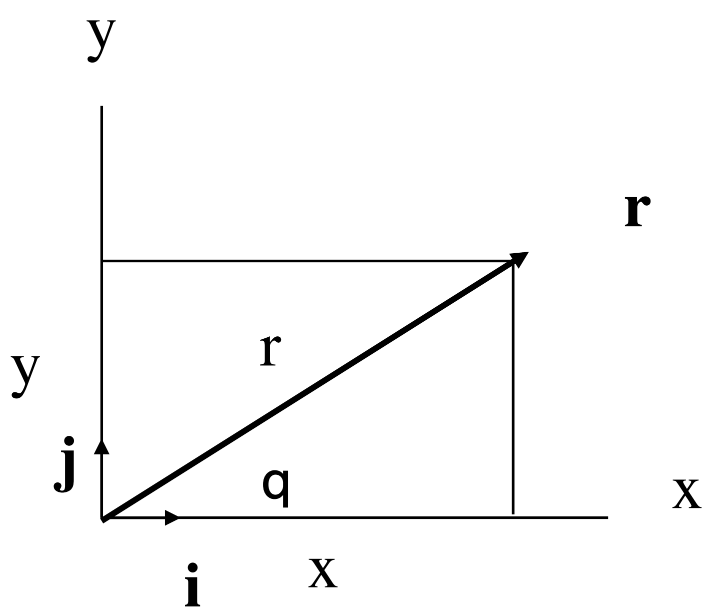
$$\vec{A} = \langle 2\hat{i} + 4\hat{j} \rangle$$

$$\vec{B} = \langle 5\hat{i} + 2j \rangle$$

$$\vec{A} + \vec{B} = \langle 7\hat{i} + 6\hat{j} \rangle$$

Note you add x and y components

Vector components in terms of sine and cosine



$$\cos\theta = {}^{x}\!/_{r} \qquad x = r\cos\theta$$

$$\sin\theta = {}^{y}\!/_{r} \qquad y = r\sin\theta$$

$$r = x\hat{i} + y\hat{j}$$

$$r = (r\cos\theta)\hat{i} + (r\sin\theta)\hat{j}$$

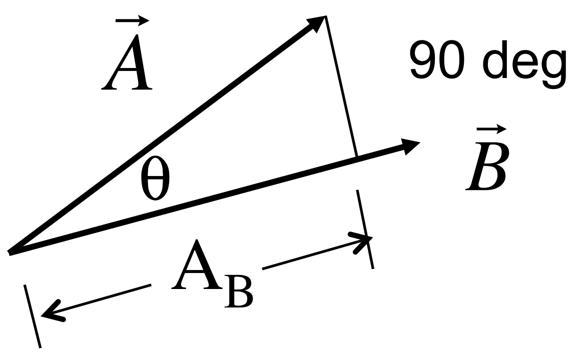
$$\tan\theta = y / x$$

Scalar product = $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y$

$$\vec{A} = \left\langle 2\hat{i} + 4\hat{j} \right\rangle$$

$$\vec{B} = \left\langle 5\hat{i} + 2j \right\rangle$$

$$\vec{A} \cdot \vec{B} = (2)(5) + (4)(2) = 18$$

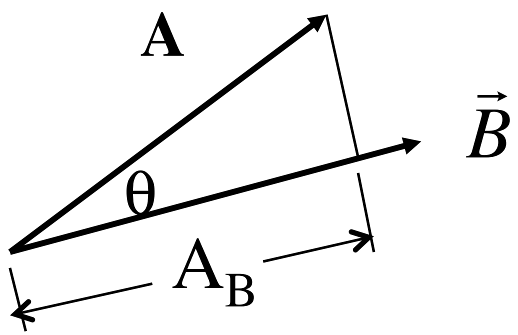


Also $\vec{A} \cdot \vec{B} = |A||B|\cos\theta$

$$\cos\theta = \frac{18}{\sqrt{20}\sqrt{29}} = 0.748$$

$$\theta = 41.63\,\mathrm{deg}$$

A_B is the perpendicular projection of A on B. Important later.



$$\vec{A} = \left\langle 2\hat{i} + 4\hat{j} \right\rangle$$

$$\vec{B} = \left\langle 5\hat{i} + 2j \right\rangle$$

$$\vec{A} \cdot \vec{B} = (2)(5) + (4)(2) = 18$$

$$A_B = \frac{\vec{A} \cdot \vec{B}}{B}$$

$$A_B = \frac{18}{\sqrt{29}} = 3.34$$

Example using definition of Work

$\vec{\text{A}}$=$\vec{\text{F}}$

$\vec{\text{B}}$=$\vec{\text{d}}$

Work=$\vec{\text{F}}$•$\vec{\text{d}}$

Also

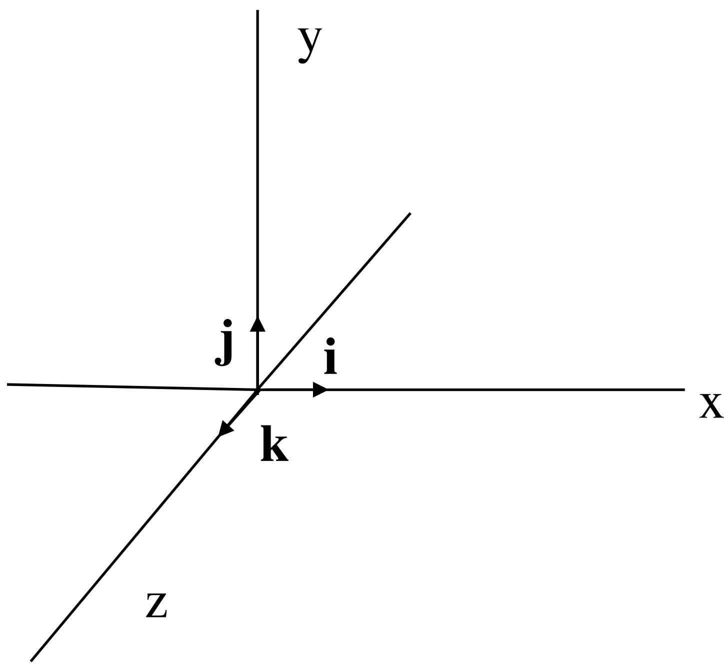
$$A_B = \left| A \right| \cos\theta$$

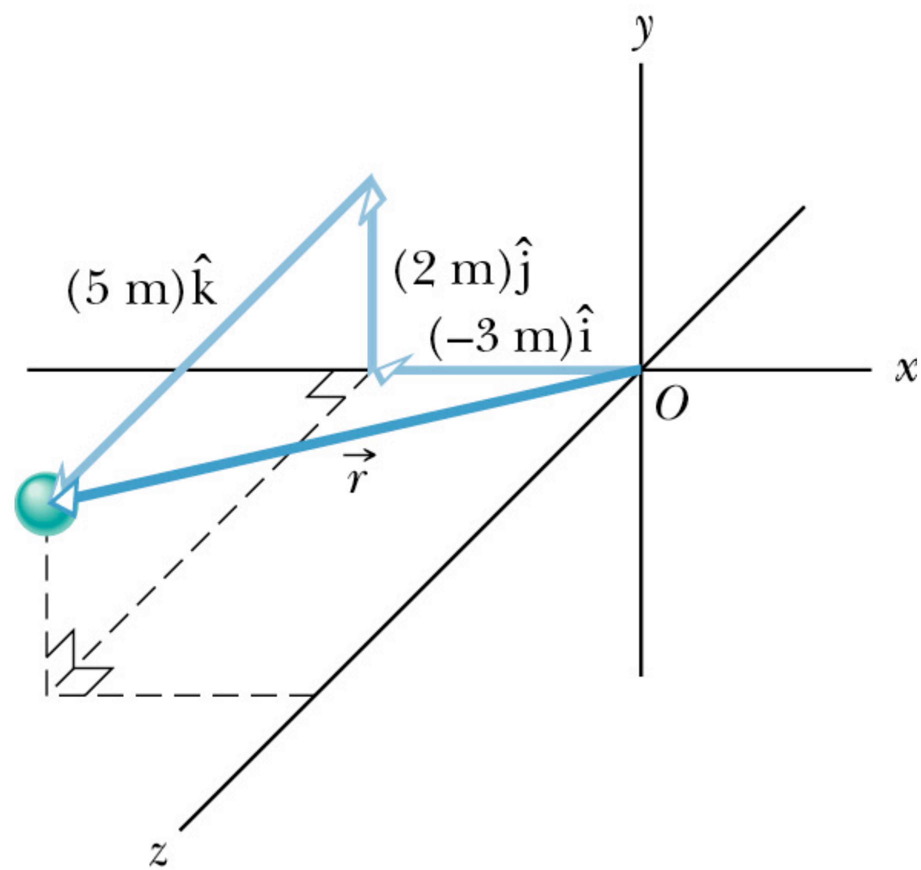
$$A_B = \sqrt{20}(0.748)$$

$$A_B = (4.472)(0.748) = 3.34$$

Vectors in 3 Dimensions

For a Right Handed 3D-Coordinate Systems





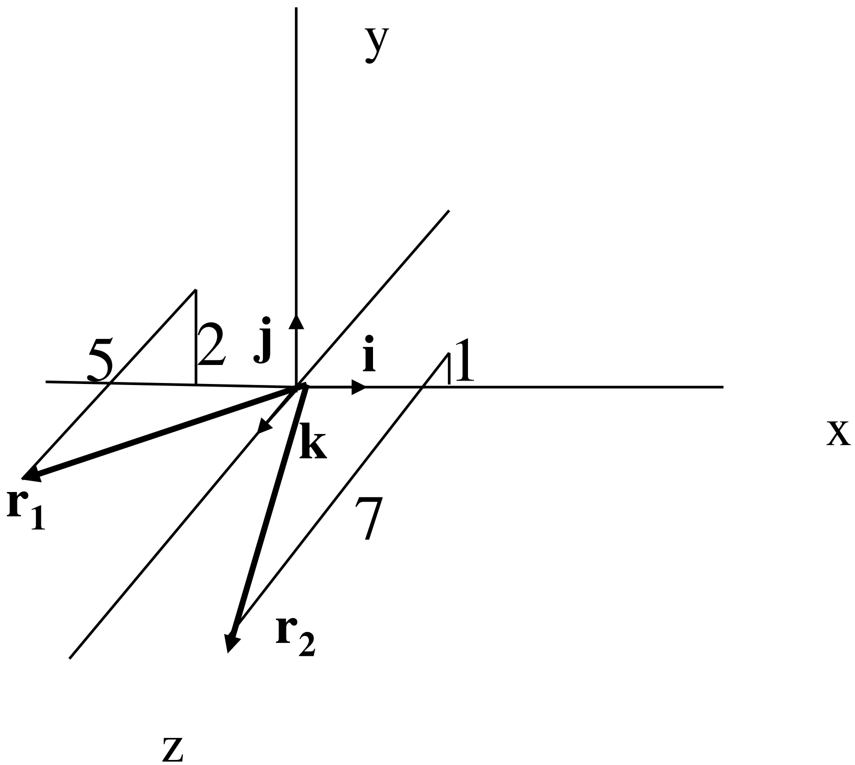
$$\hat{i} \times \hat{j} = \hat{k}$$

Right handed rule. Also called cross product

$$\vec{r} = -3\hat{i} + 2\hat{j} + 5\hat{k}$$

Magnitude of $|\vec{r}| = \sqrt{3^2 + 2^2 + 5^2}$

Suppose we have two vectors in 3D and we want to add them



$$r_1 = -3\hat{i} + 2\hat{j} + 5\hat{k}$$

$$r_2 = 4\hat{i} + 1\hat{j} + 7\hat{k}$$

Adding vectors

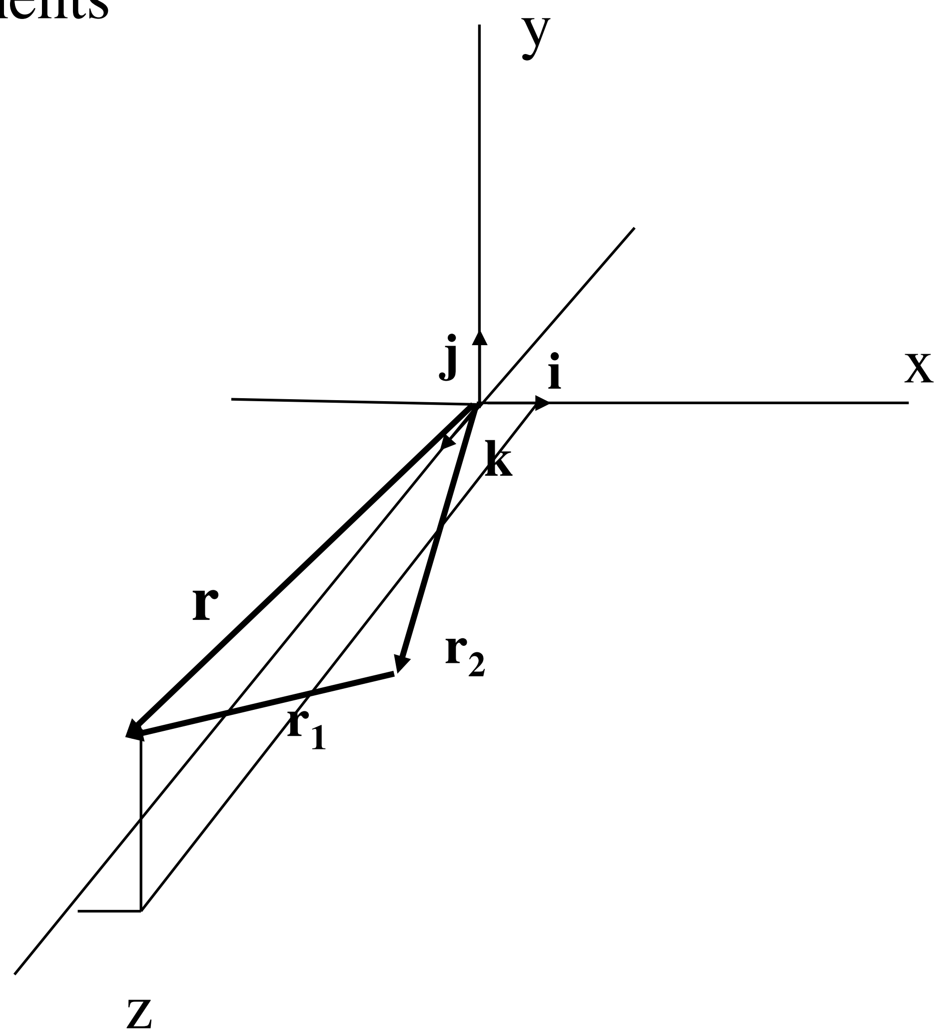
Now add all 3 components

$$\vec{r} = \vec{r}_1 + \vec{r}_2$$

$$\vec{r}_1 = -3\hat{i} + 2\hat{j} + 5\hat{k}$$

$$\vec{r}_2 = 4\hat{i} + 1\hat{j} + 7\hat{k}$$

$$\vec{r} = 1\hat{i} + 3\hat{j} + 12\hat{k}$$



Scalar product = $\vec{r}_1 \bullet \vec{r}_2$

The dot product is important in the of discussion of work.

$Work = \vec{F} \cdot \vec{d}$ Work = Scalar product

$$\vec{r}_1 = -3\hat{i} + 2\hat{j} + 5\hat{k}$$

$$\vec{r}_2 = 4\hat{i} + 1\hat{j} + 7\hat{k}$$

$$\vec{r}_1 \bullet \vec{r}_2 = r_{1x}r_{2x} + r_{1y}r_{2y} + r_{1z}r_{2z}$$

$$\vec{r}_1 \bullet \vec{r}_2 = (-3)(4) + (2)(1) + (5)(7) = 25$$

Cross Product = $\vec{A} \times \vec{B}$

$$\vec{A} = -3\hat{i} + 2\hat{j} + 5\hat{k}$$

$$\vec{B} = 4\hat{i} + 1\hat{j} + 7\hat{k}$$

Let $\vec{C} = \vec{A} \times \vec{B}$

$$\vec{C} = (A_y B_z - B_y A_z)\hat{i} - (A_z B_x - B_z A_x)\hat{j} + (A_x B_y - B_x A_y)\hat{k}$$

$$= [(2 \times 7) - (1 \times 5)]\hat{i} - [(5 \times 4) - (7 \times -3)]\hat{j} + [(-3 \times 1) - (4 \times -3)]\hat{k}$$

$$= (14\text{-}5)\hat{i} - (20 + 21)\hat{j} + (-3 + 12)\hat{k}$$

$$= 9\hat{i} - 41\hat{j} - 9\hat{k}$$

Examples of Cross products

$\vec{T} = \vec{r} \times \vec{F}$ = Torque on a body due to a force acting on the body causing it to rotate

$\vec{F} = q(\vec{v} \times \vec{B})$ = Force on a charge q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$

See your textbook Chapter 3 for more information on vectors and cross and dot products

Differential Calculus

Definition of Velocity when it is smoothly changing

Define the instantaneous velocity

Recall

$$v = \frac{(x_2 - x_1)}{(t_2 - t_1)} = \frac{\Delta x}{\Delta t} \qquad \text{(average)}$$

$$v = \lim \frac{\Delta x}{\Delta t} \text{ as } \Delta t \longrightarrow 0 = dx/dt \quad \text{(instantaneous)}$$

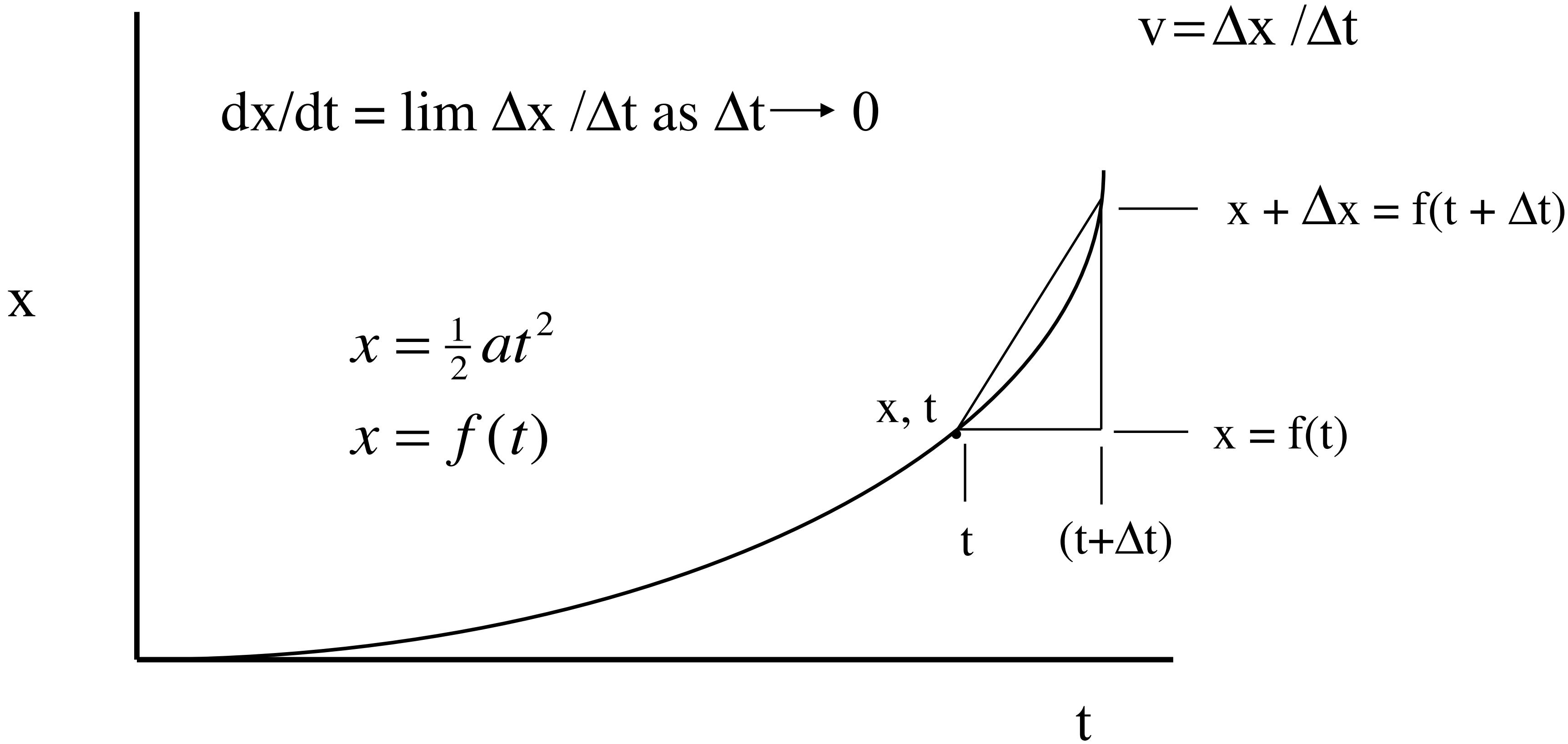
Example

$$x = \tfrac{1}{2} a t^2$$

$$x = f(t)$$

What is $\frac{dx}{dt}$?

DISTANCE-TIME GRAPH FOR UNIFORM ACCELERATION



Differential Calculus: an example of a derivative

$$x = \tfrac{1}{2} a t^2 \qquad\qquad f(t) = \tfrac{1}{2} a t^2$$

$$x = f(t) \qquad\qquad f(t+\Delta t) = \tfrac{1}{2} a (t+\Delta t)^2$$

$$= \tfrac{1}{2} a (t^2 + 2t\Delta t + (\Delta t)^2)$$

dx/dt = lim Δx /Δt as Δt $\longrightarrow$ 0

$$= \frac{f(t+\Delta t) - f(t)}{\Delta t} = \frac{\tfrac{1}{2} a (t^2 + 2t\Delta t + (\Delta t)^2) - \tfrac{1}{2} a t^2}{\Delta t}$$

$$= \frac{\tfrac{1}{2} a (2t\Delta t + (\Delta t)^2)}{\Delta t} = \tfrac{1}{2} a (2t + \Delta t) \underset{\Delta t \to 0}{\longrightarrow} at$$

$$\frac{dx}{dt} = at$$

velocity in the x direction

Problem 4-7 The position of an electron is given by the following displacement vector $\vec{r} = 3t\hat{i} - 4t^2\hat{j} + 2\hat{k}$, where t is in secs and **r** is in m.

What is the electron's velocity vector **v**(t)?

$$\vec{v} = \frac{d\vec{r}}{dt} = 3\hat{i} - 8t\hat{j}$$

What is the electron's velocity vector and components at t= 2 s?

$$\vec{v} = \frac{dr}{dt} = 3\hat{i} - 16\hat{j}$$

$$v_x = 3m/s$$

$$v_y = -16m/s$$

What is the magnitude of the velocity or speed?

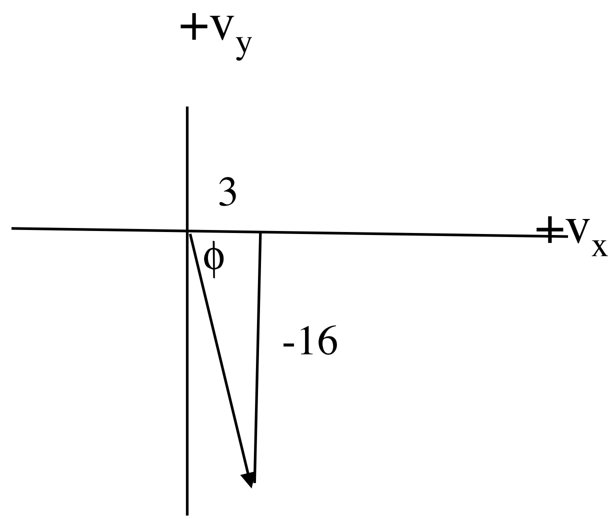
$$v = \sqrt{3^2 + 16^2} = 16.28m/s$$

What is the angle relative to the positive direction of the x axis?

What is the angle relative to the positive direction of the x axis?

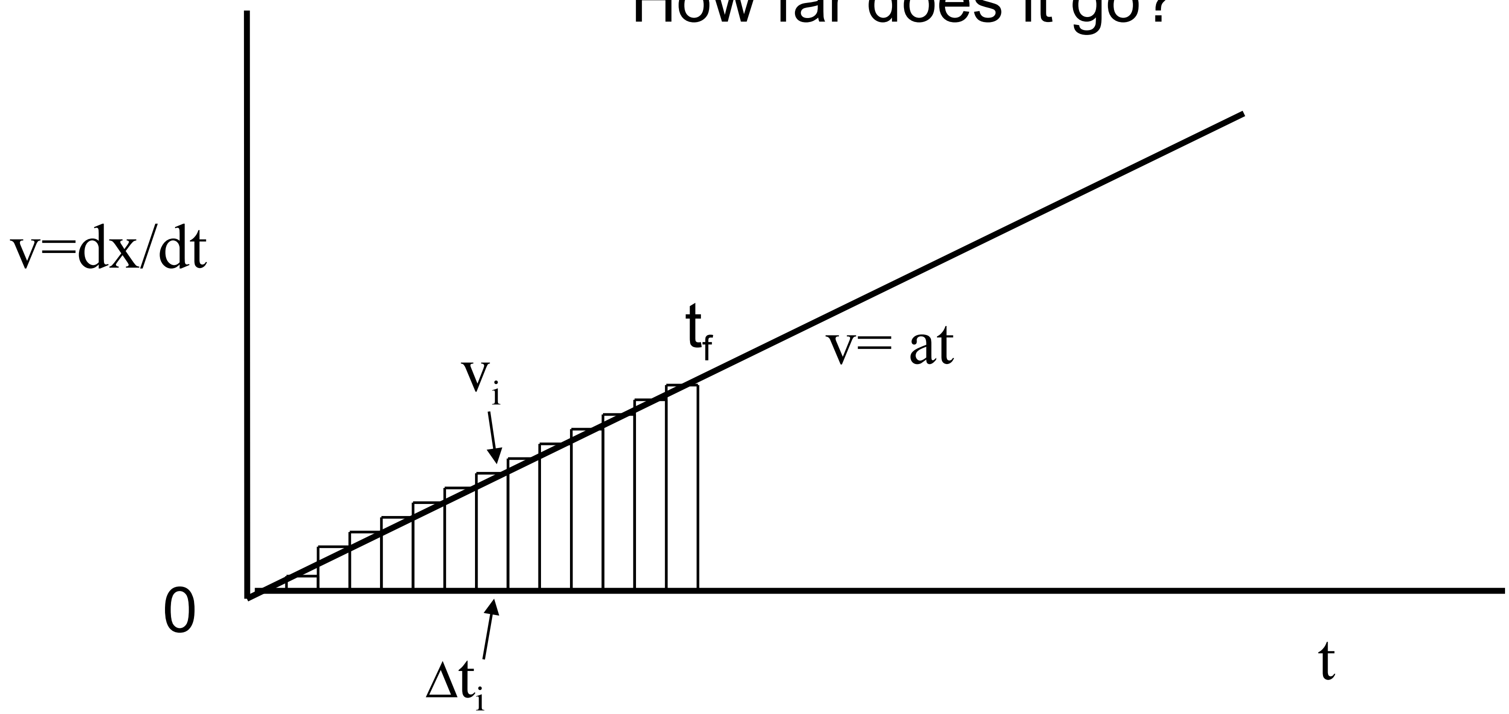
$$v_x = 3m/s$$

$$v_y = -16m/s$$



$$\phi = \tan^{-1}\left(\frac{-16}{3}\right) = \tan^{-1}(-5.33) = -79.3\deg$$

Integral Calculus
How far does it go?



$$x = \sum_{i=1}^{N} \Delta x_i = \sum_{i=1}^{N} v_i \Delta t_i = \sum_{i=1}^{N} a t_i \Delta t_i \qquad N = \frac{t_f}{\Delta t_i}$$

Distance equals area under speed graph regardless of its shape

Area = x = 1/2(base)(height) = 1/2(t)(at) = 1/2at^2

Integration:anti-derivative

$$\sum_{i=1}^{N} at_i \Delta t_i = \int_0^{t_f} atdt \quad where\ \Delta \mathrm{t_i} \rightarrow 0 \text{ and } \mathrm{N} \rightarrow \infty$$

$$\int_0^{t_f} atdt = \frac{1}{2} at^2 \Big|_0^{\mathrm{t_f}} = \frac{1}{2} a\ (\mathrm{t_f^2} - 0) = \frac{1}{2} a\ \mathrm{t_f}^2$$

$$x = \tfrac{1}{2} at^2$$

Vector Questions

1. Consider three vectors:

$$\vec{A} = 3\hat{i} + 0\hat{j}$$
$$\vec{B} = 2\sqrt{3}\hat{i} + 2\hat{j}$$
$$\vec{C} = -5\hat{i} + 5\sqrt{3}\hat{j}$$

a. Draw the three vectors.
b. What is the length or magnitude of $\vec{A}$, $\vec{B}$ and $\vec{C}$?
c. What is the angle between $\vec{A}$ and $\vec{C}$, $\vec{A}$ and $\vec{B}$, $\vec{B}$ and $\vec{C}$?

2. Consider three vectors:

$$\vec{A} = 4\hat{i} + 6\hat{j} - 2\hat{k}$$
$$\vec{B} = 2\hat{i} + 7\hat{j} - 1\hat{k}$$
$$\vec{C} = 0\hat{i} + 3\hat{j} + 5\hat{k}$$

a. What is the length or magnitude of $\vec{A}$, also written as $|\vec{A}|$?
b. Write the expression for 2 $\vec{A}$.
c. What is $\vec{A} + \vec{B}$?
d. What is $\vec{C} - \vec{A}$?
e. What is $\vec{C} \times \vec{A}$?
f. What is the magnitude of $\vec{C} \times \vec{A}$?
g. What is $\vec{B} \cdot \vec{C}$?
h. What is the angle between $\vec{A}$ and $\vec{C}$?
i. Does $\vec{B} \cdot \vec{C}$ equal $\vec{C} \cdot \vec{B}$?
j. How is $\vec{C} \times \vec{A}$ and $\vec{A} \times \vec{C}$ related**?**
k. Give an example of the use of dot product in Physics and explain.
l. Give an example of the use of cross product in Physics and explain.
m. Imagine that the vector $\vec{A}$ is a force whose units are given in Newtons. Imagin
$\vec{B}$ is a radius vector through which the force acts in meters. What is the value o
torque $(\vec{\tau} = \vec{r} \times \vec{F})$, in this case?
n. Now imagine that $\vec{A}$ continues to be a force vector and $\vec{C}$ is a displacement vec
units are meters. What is the work done in applying force $\vec{A}$ through a displace
o. What is the vector sum of a vector $\vec{D}$ given by 40 m, 30 degrees and a vector
12 m, 225 degrees? Use the method of resolving vectors into their component
adding the components

Three Important Rules of Differentiation

Power Rule

$$y = cx^n$$

$$dy / dx = ncx^{n-1}$$

$$y = 30x^5$$

$$\frac{dy}{dx} = 5(30)x^4 = 150x^4$$

Product Rule

$$y(x) = f(x)g(x)$$

$$\frac{dy}{dx} = \frac{df}{dx}g(x) + f(x)\frac{dg}{dx}$$

$$y = 3x^2(\ln x)$$

$$\frac{dy}{dx} = 2(3)x(\ln x) + 3x^2(\frac{1}{x}) = 6x\ln x + 3x$$

$$\frac{dy}{dx} = 3x(2\ln x + 1)$$

Chain Rule

$$y(x) = y(g(x))$$

$$\frac{dy}{dx} = \frac{dy}{dg}\frac{dg}{dx}$$

$$y = (5x^2 - 1)^3 = g^3 \text{ where g=5x}^2 - 1$$

$$\frac{dy}{dx} = 3g^2\frac{dg}{dx} = 3(5x^2 - 1)^2(10x)$$

$$\frac{dy}{dx} = 30x(5x^2 - 1)^2$$

Differentiation Practice

QUESTION: Differentiate the following values with respect to x, t, or z. And let a and b be constants.

1. $y = x^n$
2. $y = x^5$
3. $y = a$
4. $y = \dfrac{x^3}{3} + \dfrac{3}{x^3}$
5. $y = (5ax^2)(12x^{-3})$
6. $y = \sin x$
7. $y = a\cos x$
8. $y = f(x)g(x)$
9. $y = x^3 \sin x$
10. $y = f(g(x))$
11. $y = \sin ax$
12. $y = e^x$
13. $y = -e^{x^2+a}$
14. $y = \ln x$
15. $y = \dfrac{x+1}{x^2}$
16. $y = x\ln x - x$
17. $y = z^{32}$
18. $y = 2t^3 - 21t^2 + 60t - 10$
19. $y = \dfrac{x}{\ln x}$

Some Derivatives

$$y = x^5$$

$$y = x^n$$

$$y = a$$

$$y = (5ax^2)(12x^{-3})$$

$$y = \sin x$$

$$y = \frac{x+1}{x^2}$$

Integration Practice

1. $\int x^n dx$
2. $\int dx$
3. $\int x dx$
4. $\int a dx$
5. $\int (ay^3 \pm by^2) dy$
6. $\int z^{-3} dz$
7. $\int \frac{c}{r^2} dr$
8. $\int \frac{1}{x} dx$
9. $\int e^{ax} dx$
10. $\int (v_0 + at) dt$
11. $\int \cos\theta d\theta$
12. $\int \sin bt dt$
13. $\int_1^3 x^2 dx$
14. $\int f\left(\frac{dg}{dx}\right) dx$, where f and g are both the functions of x.
15. $\int x \cos x dx$
16. $\int x \sin x dx$
17. $\int \sin^2 ax dx$
18. $\int \frac{dx}{\sqrt{x^2 \pm a^2}}$

Some integrals

$$\int x^n \, dx$$

$$\int dx$$

$$\int a \, dx$$

$$\int (v_0 + at) \, dt$$

$$\int (ay^3 \pm by^2) dy$$

$$\int x \, dx$$

$$\int \cos\theta \, d\theta$$

Misconceptions - Clickers

Instructions for using Clickers in 203 while Eluminatye is on.

1)Minimize Elluminate Window
2)Load Thumb drive
3)Click on "My Computer"
4)Click on "Udisk"
5)Click on MycourseMac on Udisk
6)Click on I-clicker
7)Click on Start
8)Click on

ConcepTest 2.1 Walking the Dog

You and your dog go for a walk to the park. On the way, your dog takes many side trips to chase squirrels or examine fire hydrants. When you arrive at the park, do you and your dog have the same displacement?

1) yes
2) no

ConcepTest 2.1 Walking the Dog

You and your dog go for a walk to the park. On the way, your dog takes many side trips to chase squirrels or examine fire hydrants. When you arrive at the park, do you and your dog have the same displacement?

1) yes
2) no

Yes, you have the same displacement. Since you and your dog had the same initial position and the same final position, then you have (by definition) the same displacement.

Follow-up: Have you and your dog traveled the same distance?

ConcepTest 2.6b Cruising Along II

You drive 4 miles at 30 mi/hr and then another 4 miles at 50 mi/hr. What is your average speed for the whole 8-mile trip?

1) more than 40 mi/hr
2) equal to 40 mi/hr
3) less than 40 mi/hr

ConcepTest 2.6b Cruising Along II

You drive 4 miles at 30 mi/hr and then another 4 miles at 50 mi/hr. What is your average speed for the whole 8-mile trip?

1) more than 40 mi/hr
2) equal to 40 mi/hr
3) less than 40 mi/hr

It is not 40 mi/hr! Remember that the average speed is distance/time. Since it takes longer to cover 4 miles at the slower speed, you are actually moving at 30 mi/hr for a longer period of time! Therefore, your average speed is closer to 30 mi/hr than it is to 50 mi/hr.

Follow-up: How much further would you have to drive at 50 mi/hr in order to get back your average speed of 40 mi/hr?

ConcepTest 2.8a Acceleration I

If the velocity of a car is non-zero (*$v \neq 0$*), can the acceleration of the car be zero?

1) **Yes**
2) **No**
3) **Depends on the velocity**

ConcepTest 2.8a Acceleration I

If the velocity of a car is non-zero (*v≠0*), can the acceleration of the car be zero?

1) Yes
2) No
3) Depends on the velocity

Sure it can! An object moving with *constant* velocity has a non-zero velocity, but it has zero acceleration since the velocity is not changing.

ConcepTest 2.8b Acceleration II

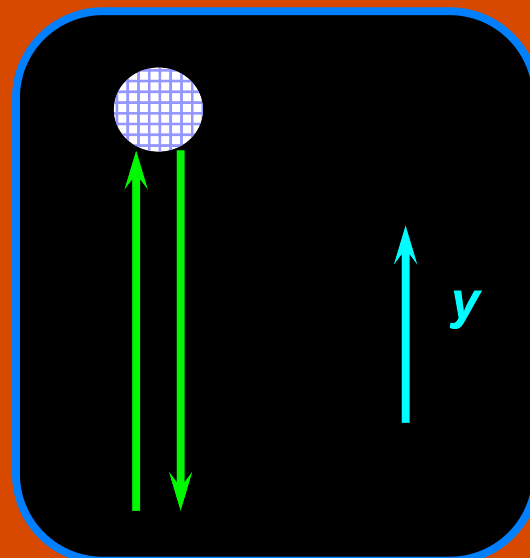
When throwing a ball straight up, which of the following is true about its velocity *v* and its acceleration *a* at the highest point in its path?

1) both *v* = *0* and *a* = *0*
2) *v* ¹ *0*, but *a* = *0*
3) *v* = *0*, but *a* ¹ *0*
4) both *v* ¹ *0* and *a* ¹ *0*
5) not really sure

ConcepTest 2.8b Acceleration II

When throwing a ball straight up, which of the following is true about its velocity *v* and its acceleration *a* at the highest point in its path?

1) both $v = 0$ and $a = 0$
2) $v \neq 0$, but $a = 0$
3) $v = 0$, but $a \neq 0$
4) both $v \neq 0$ and $a \neq 0$
5) not really sure



At the top, clearly $v = 0$ because the ball has momentarily stopped. But the velocity of the ball is changing, so its acceleration is definitely not zero! Otherwise it would remain at rest!!

Follow-up: …and the value of *a* is…?

ConcepTest 2.11 Two Balls in the Air

A ball is thrown straight upward with some initial speed. When it reaches the top of its flight (at a height *h*), a second ball is thrown straight upward with the same initial speed. Where will the balls cross paths?

1) at height h
2) above height h/2
3) at height h/2
4) below height h/2 but above 0
5) at height 0

ConcepTest 2.11 Two Balls in the Air

A ball is thrown straight upward with some initial speed. When it reaches the top of its flight (at a height *h*), a second ball is thrown straight upward with the same initial speed. Where will the balls cross paths?

1) at height h
2) above height h/2
3) at height h/2
4) below height h/2 but above 0
5) at height 0

The first ball starts at the top with no initial speed. The second ball starts at the bottom with a large initial speed. Since the balls travel the same time until they meet, the second ball will cover more distance in that time, which will carry it over the halfway point before the first ball can reach it.

Follow-up: How could you calculate where they meet?

ConcepTest 3.1b Vectors II

Given that A + B = C, and that $|A|^2 + |B|^2 = |C|^2$, how are vectors A and B oriented with respect to each other?

1) they are perpendicular to each other
2) they are parallel and in the same direction
3) they are parallel but in the opposite direction
4) they are at 45° to each other
5) they can be at any angle to each other

ConcepTest 3.1b Vectors II

Given that **A** + **B** = **C**, and that $|A|^2 + |B|^2 = |C|^2$, how are vectors **A** and **B** oriented with respect to each other?

1) they are perpendicular to each other

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4) they are at 45° to each other

5) they can be at any angle to each other

Note that the magnitudes of the vectors satisfy the Pythagorean Theorem. This suggests that they form a right triangle, with vector **C** as the hypotenuse. Thus, **A** and **B** are the legs of the right triangle and are therefore perpendicular.

ConcepTest 3.1c Vectors III

Given that A + B = C, and that |A| + |B| = |C| , how are vectors A and B oriented with respect to each other?

1) they are perpendicular to each other
2) they are parallel and in the same direction
3) they are parallel but in the opposite direction
4) they are at 45° to each other
5) they can be at any angle to each other

ConcepTest 3.1c Vectors III

Given that **A** + **B** = **C**, and that |**A**| + |**B**| = |**C**|, how are vectors **A** and **B** oriented with respect to each other?

1) they are perpendicular to each other
2) they are parallel and in the same direction
3) they are parallel but in the opposite direction
4) they are at 45° to each other
5) they can be at any angle to each other

The only time vector magnitudes will simply add together is when the direction does not have to be taken into account (i.e., the direction is the same for both vectors). In that case, there is no angle between them to worry about, so vectors **A** and **B** must be pointing in the same direction.