

Probing Dense Nuclear Matter through Inclusive Electron Scattering

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Nuclear Matter

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Outline

- * Introduction
- * Inclusive Scattering
- * Can y -scaling tell us about SRCs
- * DIS and access to dense matter
- * An experiment at 11 GeV
- * Conclusion

Introduction

Inclusive scattering may provide an important contribution in the exploration of rare and exotic fluctuations that may allow an exposure the properties of dense nuclear matter.

Inclusive electron scattering from nuclei provides a rich, yet complicated mixture of physics that has yet to be fully exploited.

- Momentum distributions and the spectral function $P(k,E)$.
- Short Range Correlations and Multi-Nucleon Correlations
- Scaling (x, y, φ', ξ)
- Medium Modifications -- tests of EMC; 6-quark admixtures
- Duality
- Structure Function Q^2 dependence and Higher Twists

The inclusive nature of these studies make disentangling all the different pieces a challenge but experiments over a range of Q^2 and with different A will help.

DIS at $x > 1$ or studying Superfast Quarks

- In the nucleus we can have $0 < x < A$
- In the Bjorken limit, $x > 1$ DIS tells us the virtual photon scatters incoherently from quarks
- **Quarks can obtain** momenta $x > 1$ by abandoning confines of the nucleon
 - deconfinement, color conductivity, parton recombination multiquark configurations
 - correlations with a nucleon of high momentum (short range interaction)
- DIS at $x > 1$ is a filter that selects out those nuclear configurations in which the nucleon wave functions overlap. We are studying the dynamics of partons that have abandoned the confines of the nucleon.

$$\langle r_{NN} \rangle \approx 1.7 \text{ fm} \approx 2 \times r_n = 1.6 \text{ fm}$$

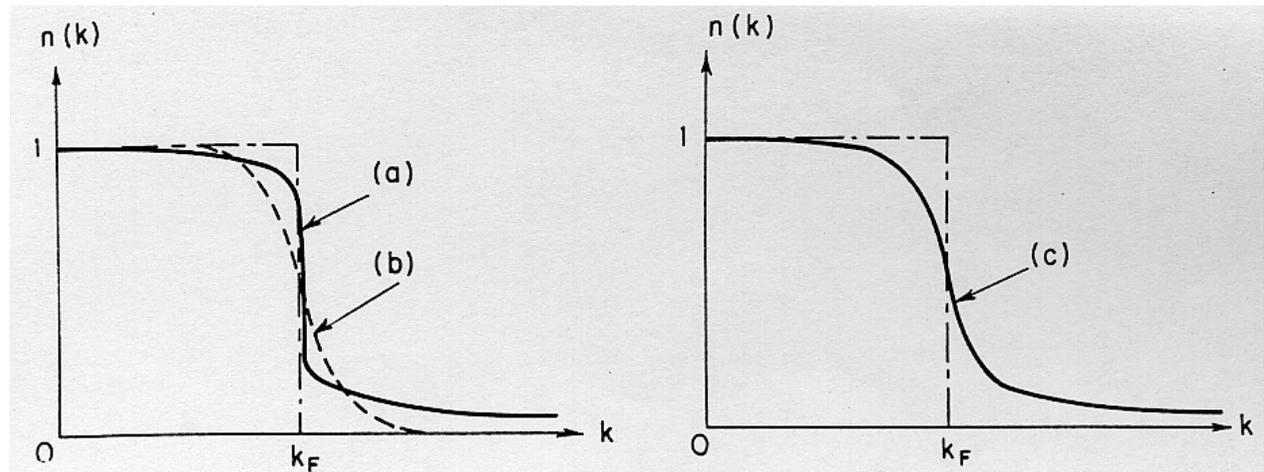
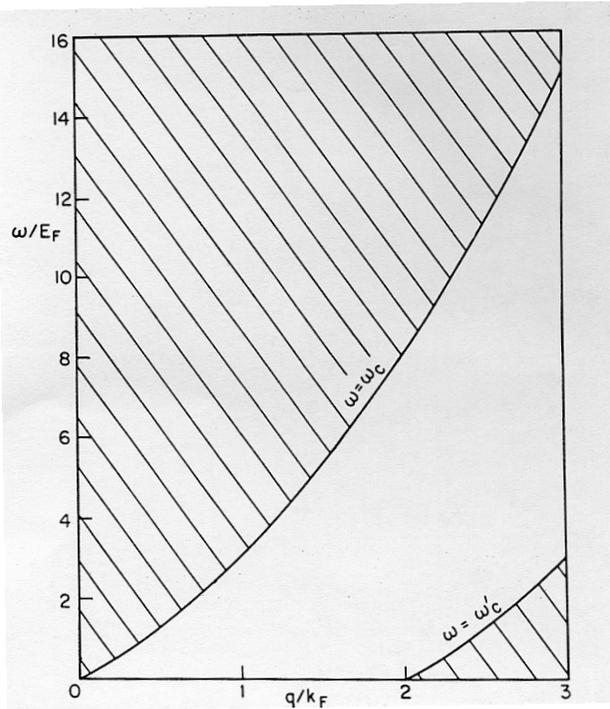
The probability that nucleons overlap is large and at $x > 1$ we are kinematically selecting those configurations.

Correlations and Inclusive Electron Scattering

Shaded domain where scattering is restricted solely to correlations. Czyz and Gottfried (1963)

$$\omega_c = \frac{(k+q)^2}{2m} + \frac{q^2}{2m} \quad \omega'_c = \frac{q^2}{2m} - \frac{qk_f}{2m}$$

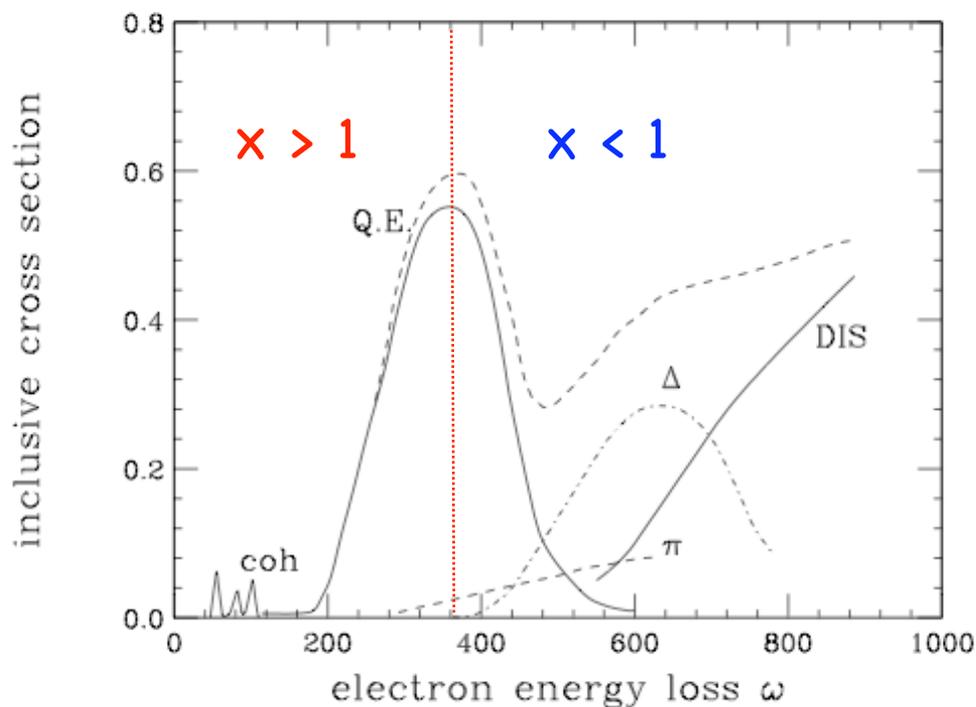
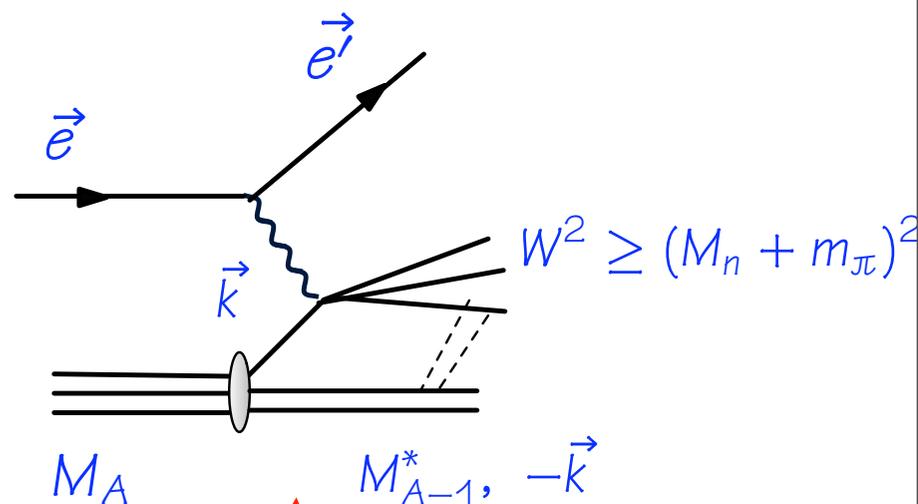
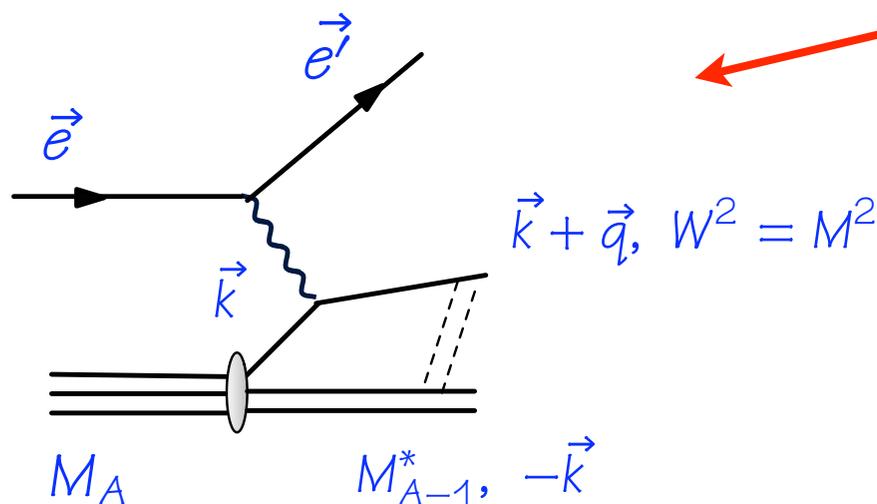
Czyz and Gottfried proposed to replace the Fermi $n(k)$ with that of an actual nucleus. (a) hard core gas; (b) finite system of noninteracting fermions; (c) actual large nucleus.



Inclusive Quasielastic and Deep Inelastic Scattering at High Momentum Transfers

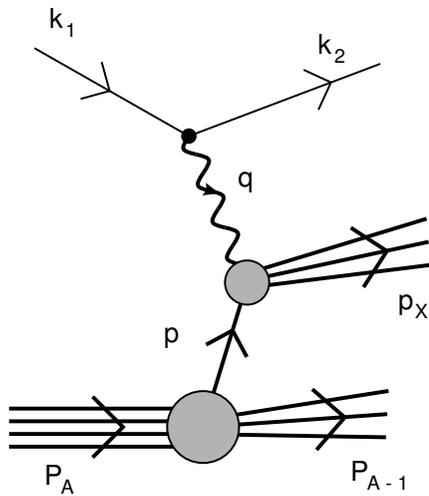
Two distinct processes

Quasielastic from the nucleons in the nucleus



Inelastic, Deep Inelastic from the quark constituents of the nucleon.

Inclusive final state means no separation of two dominant processes



Nonetheless there is a rich, if complicated, blend of nuclear and fundamental QCD interactions available for study from these types of experiments.

The two processes share the same initial state

QES in PWIA $\frac{d^2\sigma}{d\Omega dv} \propto \int d\vec{k} \int dE \sigma_{ei} \underbrace{S_i(k, E)}_{\text{Spectral function}} \delta()$

DIS $\frac{d^2\sigma}{d\Omega dv} \propto \int d\vec{k} \int dE W_{1,2}^{(p,n)} \underbrace{S_i(k, E)}_{\text{Spectral function}}$

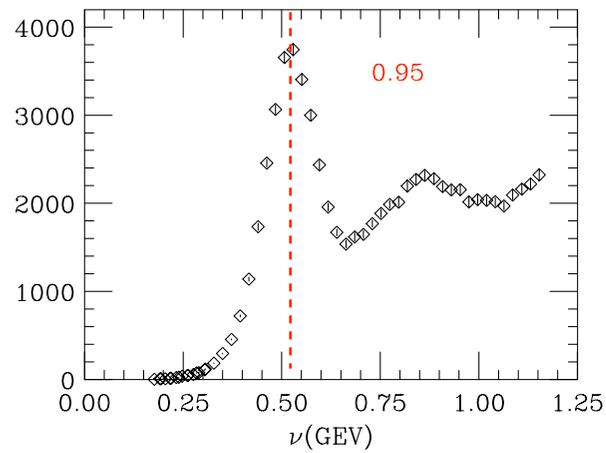
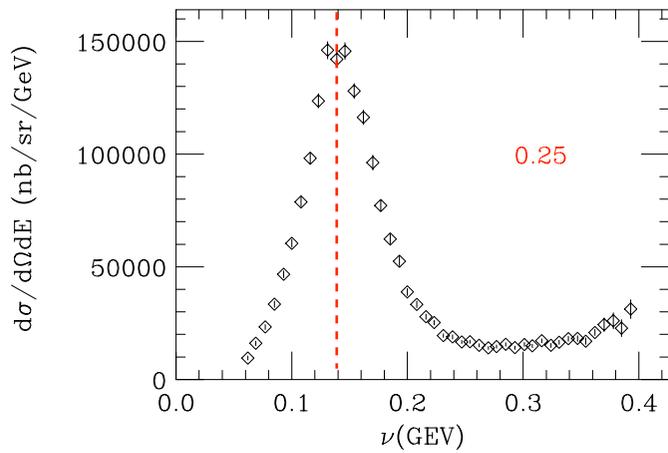
The limits on the integrals are determined by the kinematics. Specific (x, Q^2) select specific pieces of the spectral function.

$$n(k) = \int dE S(k, E)$$

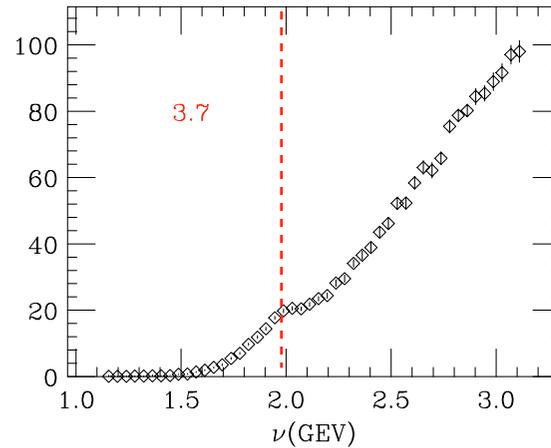
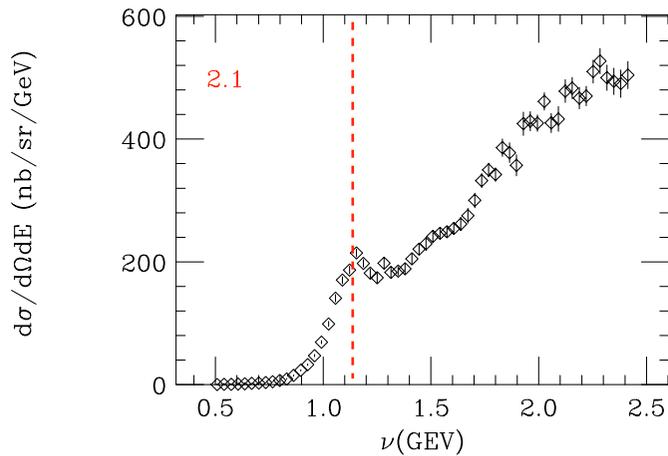
However they have very different Q^2 dependencies

$\sigma_{ei} \propto \text{elastic (form factor)}^2$ $W_{1,2}$ scale with $\ln Q^2$ dependence

Exploit this Q^2 dependence



^3He

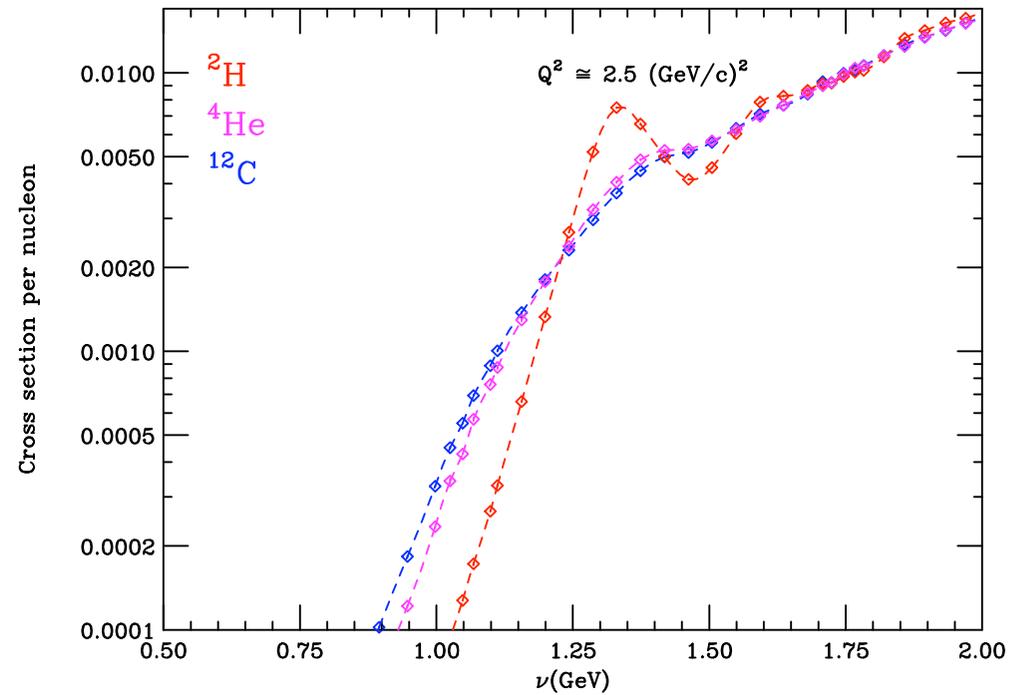
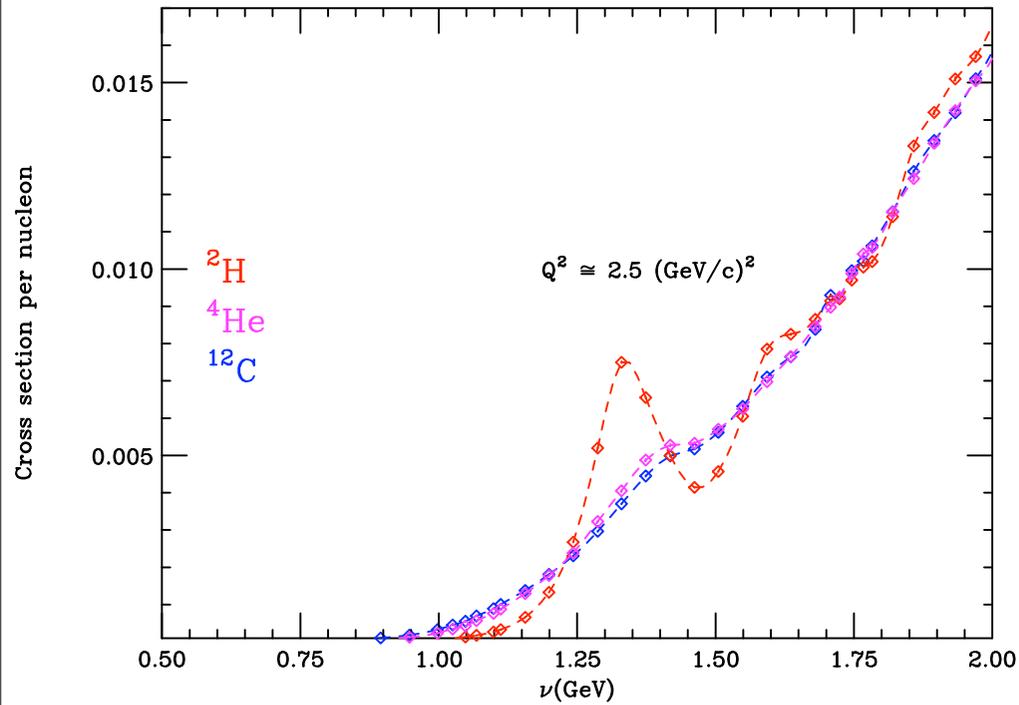


The quasielastic peak (QE) is broadened by the Fermi-motion of the struck nucleon.

The quasielastic contribution dominates the cross section at low energy loss (ν) even at moderate to high Q^2 .

- The shape of the low ν cross section is determined by the momentum distribution of the nucleons.
- As $Q^2 \gg$ inelastic scattering from the nucleons begins to dominate
- We can use x and Q^2 as knobs to dial the relative contribution of QES and DIS.

Exploit A dependence: higher momenta broadens the peak



Short Range Correlations (SRCs)

Mean field contributions: $k < k_F$

Well understood

High momentum tails: $k > k_F$

Calculable for few-body nuclei,
nuclear matter.

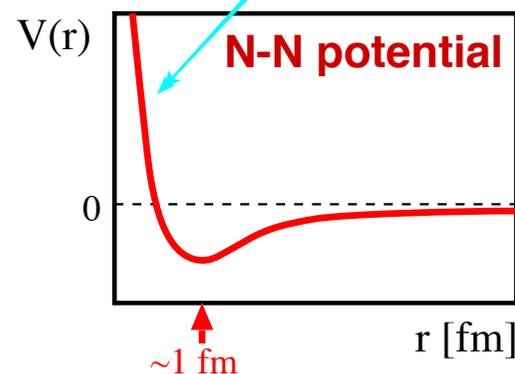
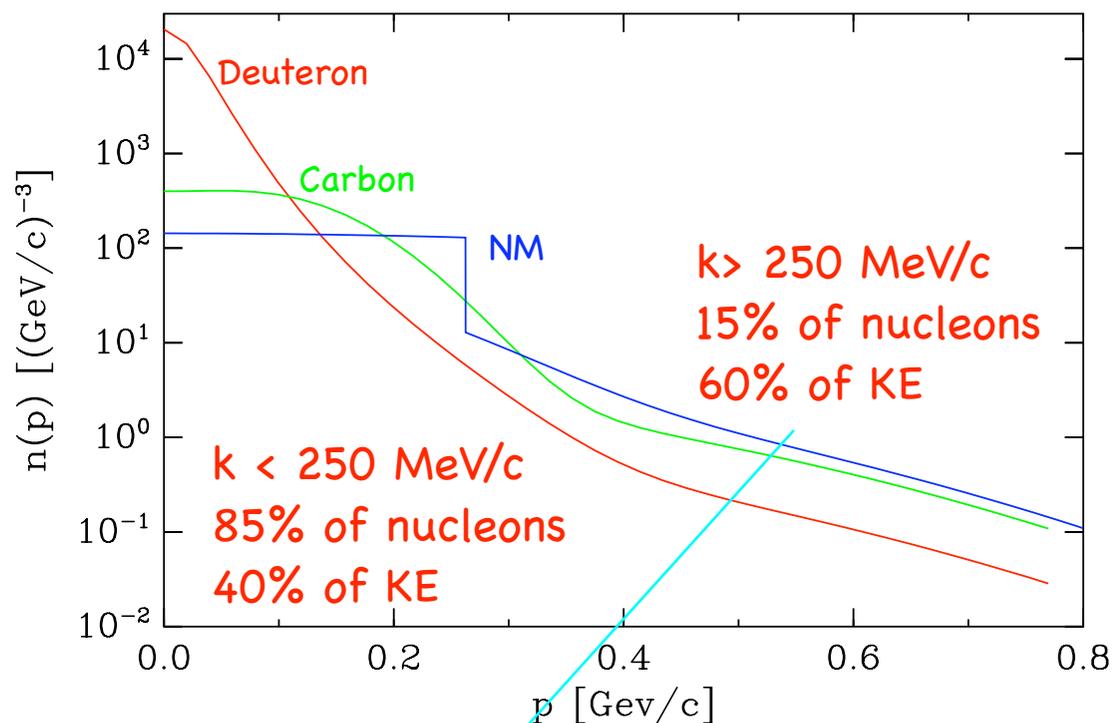
Dominated by two-nucleon
short range correlations

Isolate short range
interactions (and SRC's) by
probing at high p_m

Poorly understood part of
nuclear structure

Sign. fraction have $k > k_F$

Uncertainty in SR interaction leads to
uncertainty at $k \gg$, even for simplest
systems



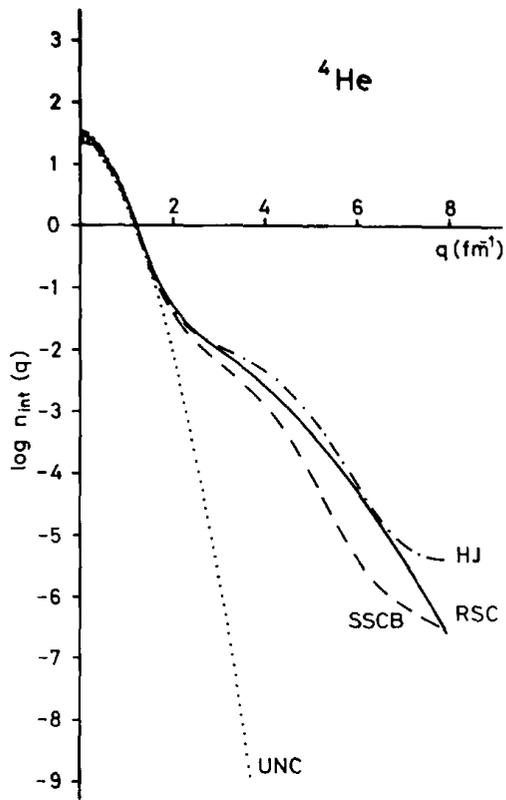


Fig. 2. Momentum distributions for ^4He , HJ: Hamada–Johnston potential, RSC: Reid soft core potential, SSCB: de Tourreil–Sprung super soft core potential B, UNC: uncorrelated, for the RSC potential. The other uncorrelated distributions do not differ appreciably for $q > 2 \text{ fm}^{-1}$.

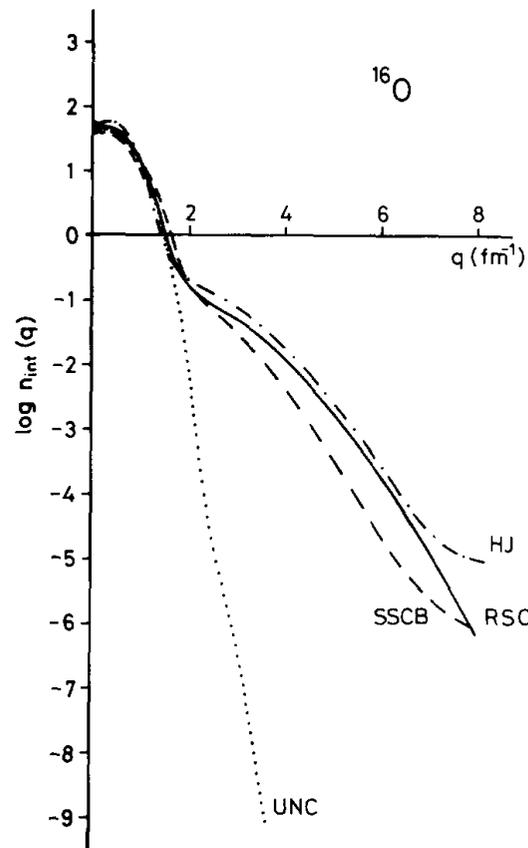
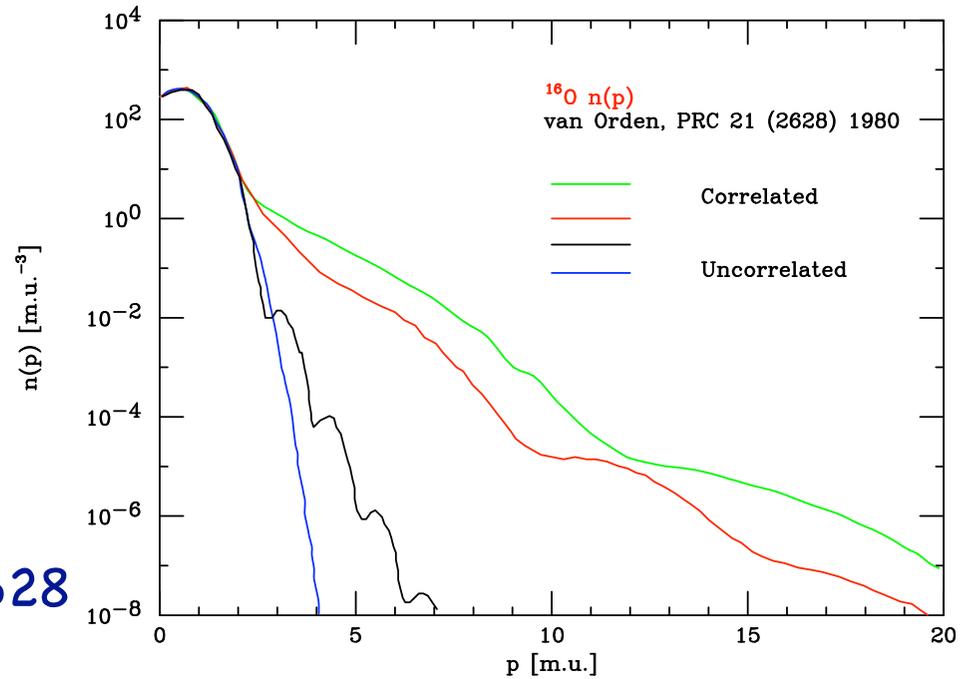


Fig. 3. Same as fig. 2, for ^{16}O .

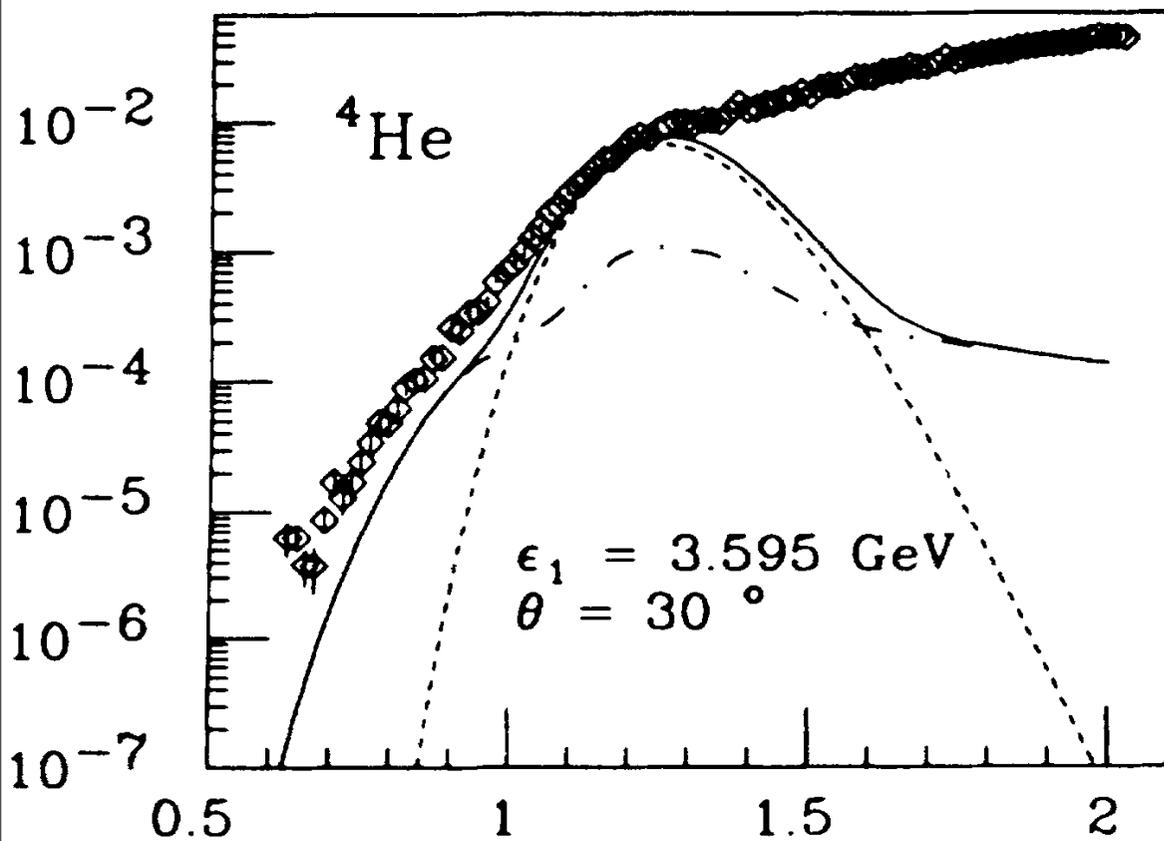
Calculations of SRC

Zabolitzky and Ey, PLB 76, 527

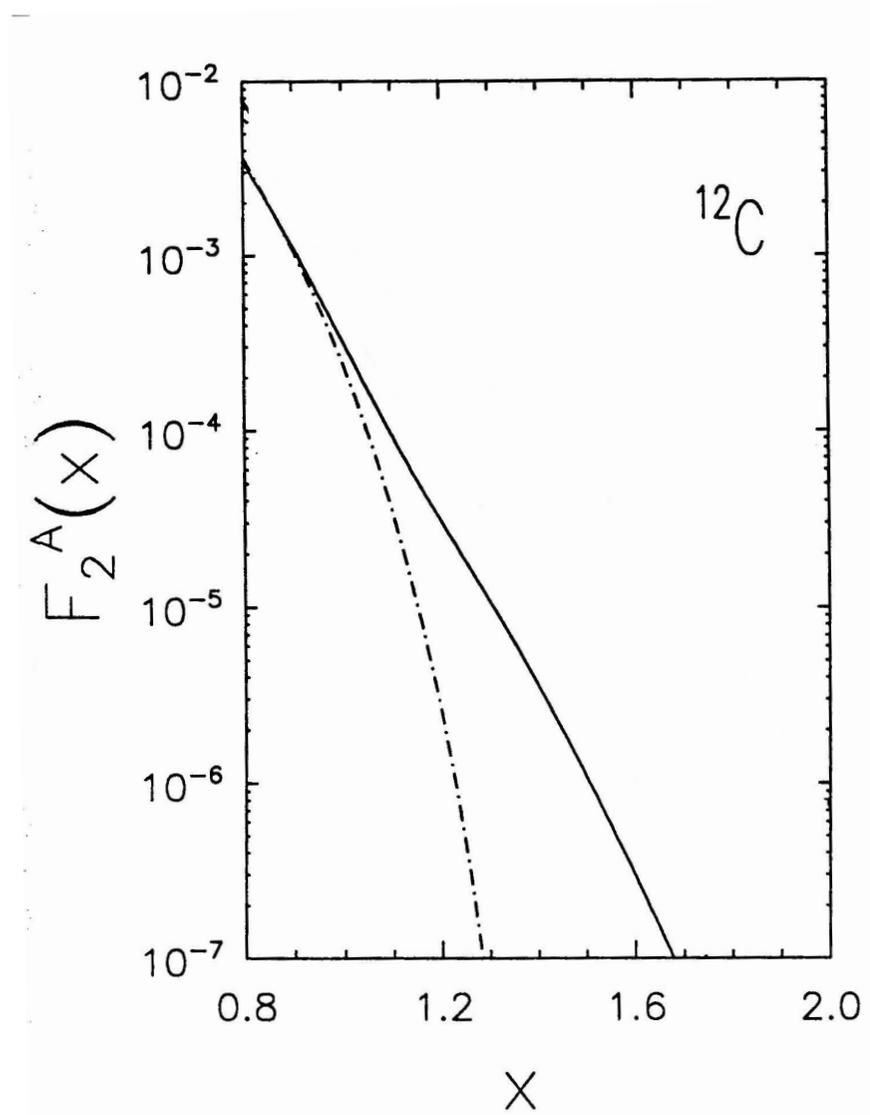


Van Orden et al., PRC21, 2628

Correlations contribute to both in QES and DIS



CdA, Day, Liuti, PRC 46 1992 (1045)

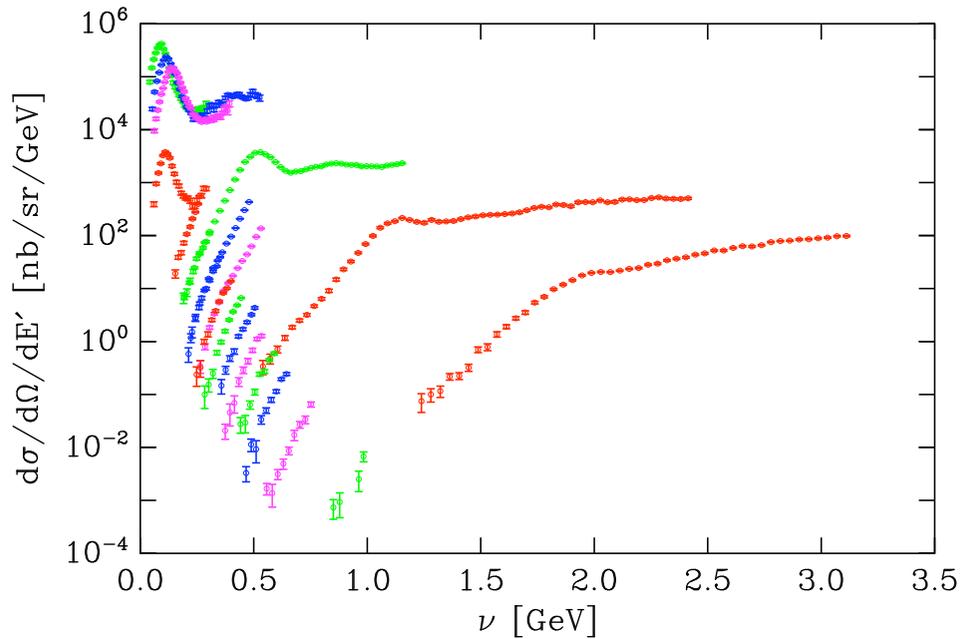


L. Conci and M. Traini, UTF 261/92.

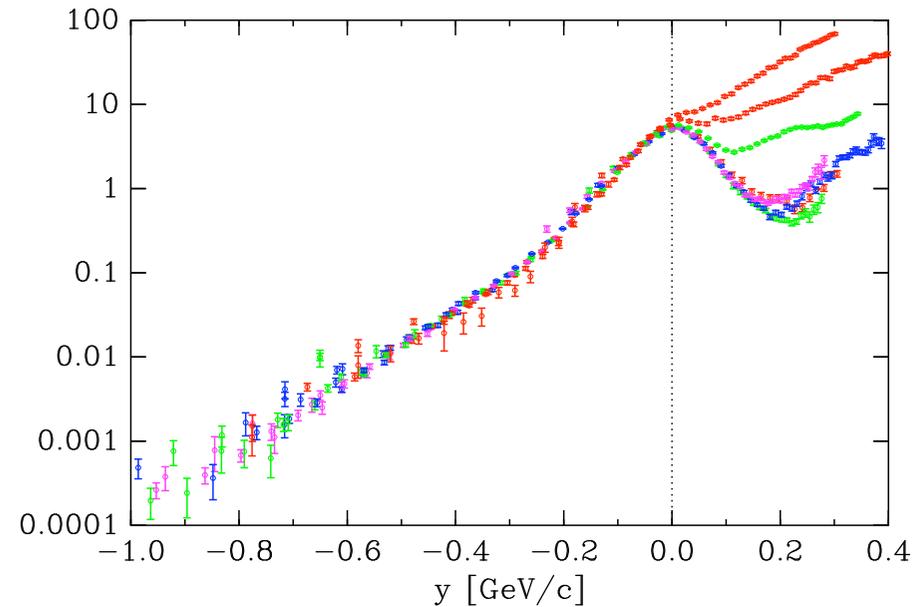
Scaling

- Scaling refers to the dependence of a cross section, in certain kinematic regions, on a single variable. If the data scales in the single variable then it validates the assumptions about the underlying physics and **scale-breaking** provides information about conditions that go beyond the assumptions.
- At moderate Q^2 inclusive data from nuclei has been well described in terms **y -scaling**, one that arises from the assumption that the electron scatters from a quasi-free nucleons.
- **We expect that as Q^2 increases** we should see for evidence (**x -scaling**) that we are scattering from a quark that has obtained its momenta from interactions with partons in other nucleons. **These are super-fast quarks.**

γ -scaling in inclusive electron scattering from ${}^3\text{He}$



$$F(y) = \frac{\sigma^{\text{exp}}}{(Z\tilde{\sigma}_p + N\tilde{\sigma}_n)} \cdot K$$



$$n(k) = -\frac{1}{2\pi y} \frac{dF(y)}{dy}$$

Assumption: scattering takes place from a **quasi-free** proton or neutron in the nucleus.

y is the momentum of the struck nucleon parallel to the momentum transfer:

$$y \approx -q/2 + mv/q$$

Scaling of the response function shows up in a variety of disciplines. Scaling in **inclusive neutron scattering from atoms** provides access to the momentum distributions.

PHYSICAL REVIEW B

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Scaling and final-state interactions in deep-inelastic neutron scattering

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(Received 20 January 1984)

The momentum distributions of atoms in condensed matter can be determined by neutron inelastic scattering experiments if the momentum transfer $\hbar q$ is large enough for the scattering to be described by the impulse approximation. This is strictly true only in the limit $q \rightarrow \infty$ and, in practice, the experimentally determined momentum distributions are distorted by final-state interactions by an amount that is typically 2% to 8%. In this paper we develop a self-consistent method for correcting for the effect of these final-state-interaction effects. We also discuss the Bjorken-scaling and y -scaling properties of the thermal-neutron scattering cross section and demonstrate, in particular, the usefulness of y scaling as an experimental test for the presence of residual final-state interactions.

Momentum distributions are "distorted" by the presence of FSI

y -scaling as a test for presence of FSI

FSI have a $1/q$ dependence

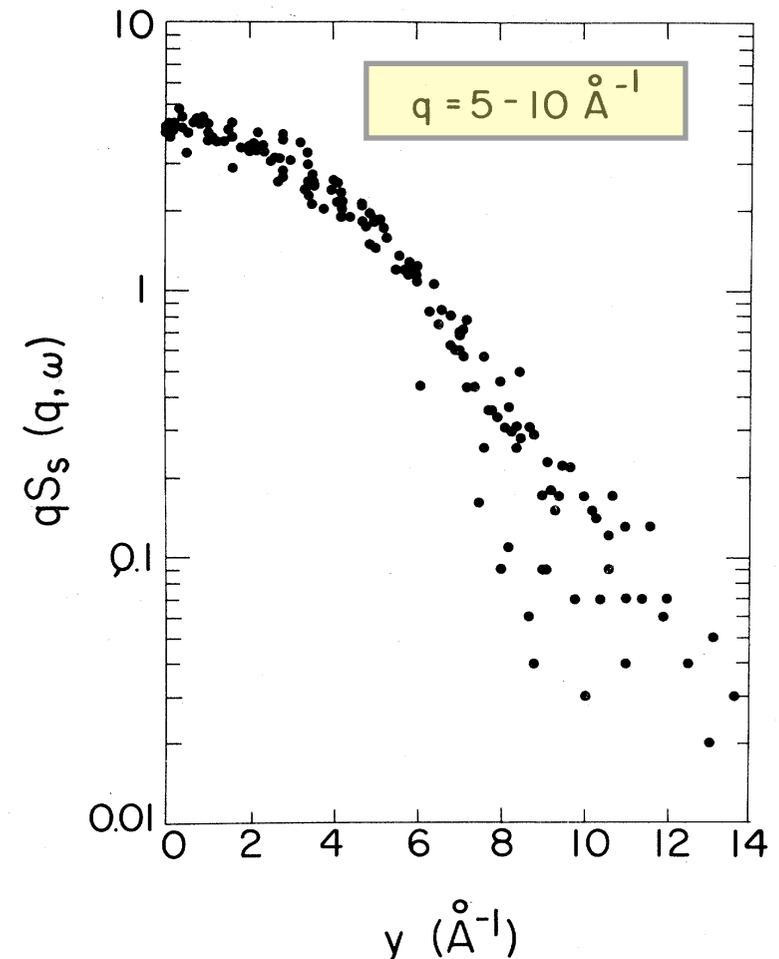
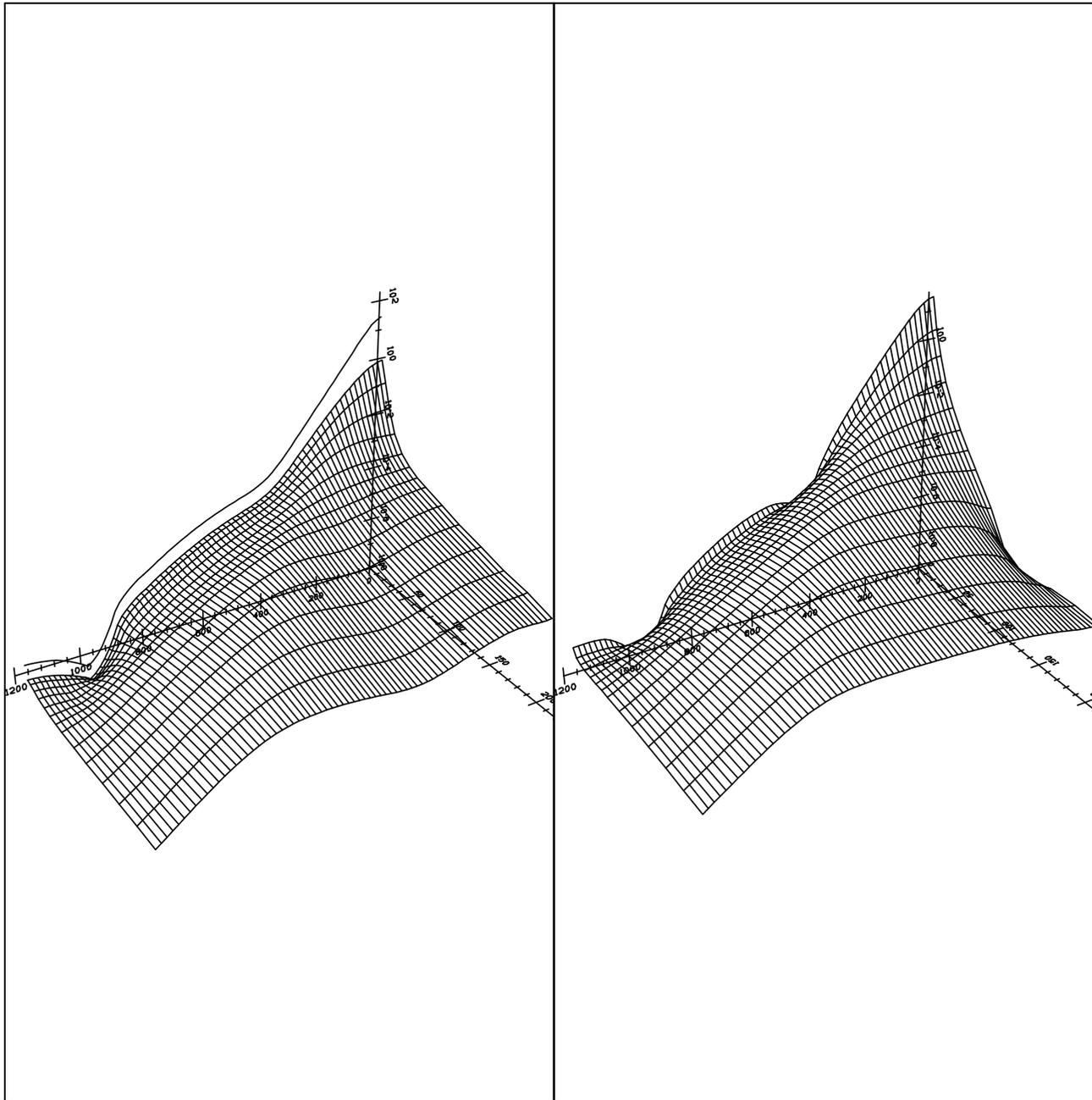


FIG. 1. y scaling in liquid neon. $qS_s(q, \omega)$ is shown in arbitrary units as a function of $y = (m/\hbar q)(\omega - \omega_r)$ for liquid neon at $T = 26.9$ K for the eleven values of q in the range $5.0 - 10.0 \text{ \AA}^{-1}$, which were used in the determination of the momentum distribution in Ref. 7. The data are from Ref. 59.

Helium-3

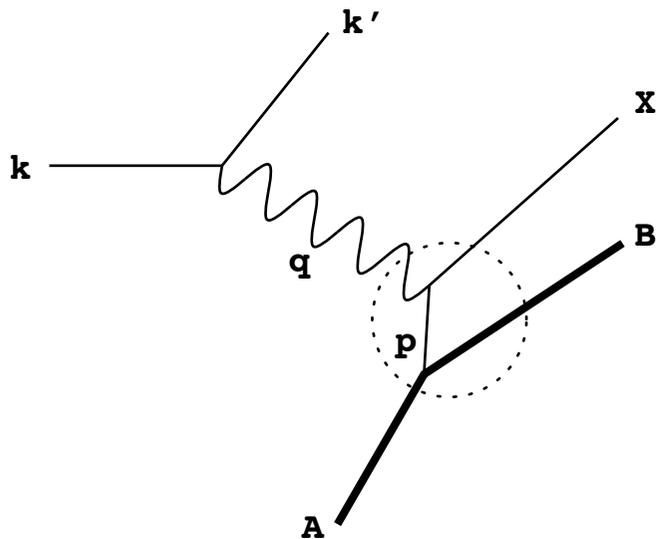


In nuclei the excitation of the residual nucleus complicates the relationship between the scaling function and $n(k)$. The spectral function $S(k,E)$

Hanover group, $T = 0$ and $T = 1$ pieces (right)

Scaling and Correlations

$$\frac{d\sigma^{QE}}{d\Omega d\omega} = \int dE_m d\mathbf{p} (Z\tilde{\sigma}_p + N\tilde{\sigma}_n) \times \delta(M_A - E_B + \nu - E_X) P(p, E_m).$$



E_m is the missing energy and determines the invariant mass of M_B such that $E_m = M_B + M - M_A$ and $E_X = \sqrt{M^2 + (p^2 + q^2)}$.

When B is the (A-1) ground state then the missing energy is minimal: $E_m \equiv M_{A-1} + M - M_A$.

This specific case defines the y-scaling variable.

$$y = -\frac{W^2 + M_{A-1}^2 - M^2}{2W^2} \left(|\mathbf{q}| - (M_A + \nu) \left[1 - \frac{4W^2 M_{A-1}^2}{(W^2 + M_{A-1}^2 - M^2)^2} \right]^{1/2} \right)$$

Integrating and pulling out of the integral terms that change slowly over range of integration.

$$\frac{d\sigma^{QE}}{dQd\omega} \simeq (Z\tilde{\sigma}_p + N\tilde{\sigma}_n) \cdot \frac{E_x}{|\mathbf{q}|} \cdot \underbrace{2\pi \int_{E_{\min}}^{E_{\max}} dE_m \int_{p_{\min}(E_m)}^{p_{\max}(E_m)} pdp \mathcal{P}(p, E_m)}_{F(y, |\mathbf{q}|)}$$

We should study the properties of $F(y, q)$ as $|\mathbf{q}| \Rightarrow \infty$ limit.

The upper bounds both tend toward infinity so the kinematic dependence comes from the lower bound, which as $|\mathbf{q}| \Rightarrow \infty$

becomes

$$p^{\min}(E_m) \rightarrow \frac{(M_A + \nu - |\mathbf{q}|)^2 - M_B^2}{2(M_A + \nu - |\mathbf{q}|)}$$

Because $|y| \neq |p_{\min}(E_m)|$, ($|p_{\min}(E_m)| > |y|$) it is instructive to rewrite $F(y, |q|)$ by breaking integral into two parts:

$$\begin{aligned}
 F(y, |q|) &= 2\pi \int_{E_{\min}}^{\infty} dE_m \left[\int_{|y|}^{\infty} - \int_{|y|}^{p_{\min}(E_m)} \right] p dp P(p, E_m) \\
 &= 2\pi \int_{|y|}^{\infty} n(p) p dp - B(y, |q|)
 \end{aligned}$$

The subtracted term

$$B(y, |q|) = 2\pi \int_{E_{\min}}^{\infty} dE_m \int_{|y|}^{p_{\min}(E_m)} p dp P(p, E_m)$$

is the "binding" correction. The continuum missing energy distribution in the spectral function **spoils a direct connection between the y -scaling function and the momentum distribution** by an amount $B(y, |q|)$.

Note that $B(y) = 0$ from 2H and that $B(y)$ scales!

Original motivation has been the possibility of using those measurements where the quasi-elastic function has reached the scaling limit

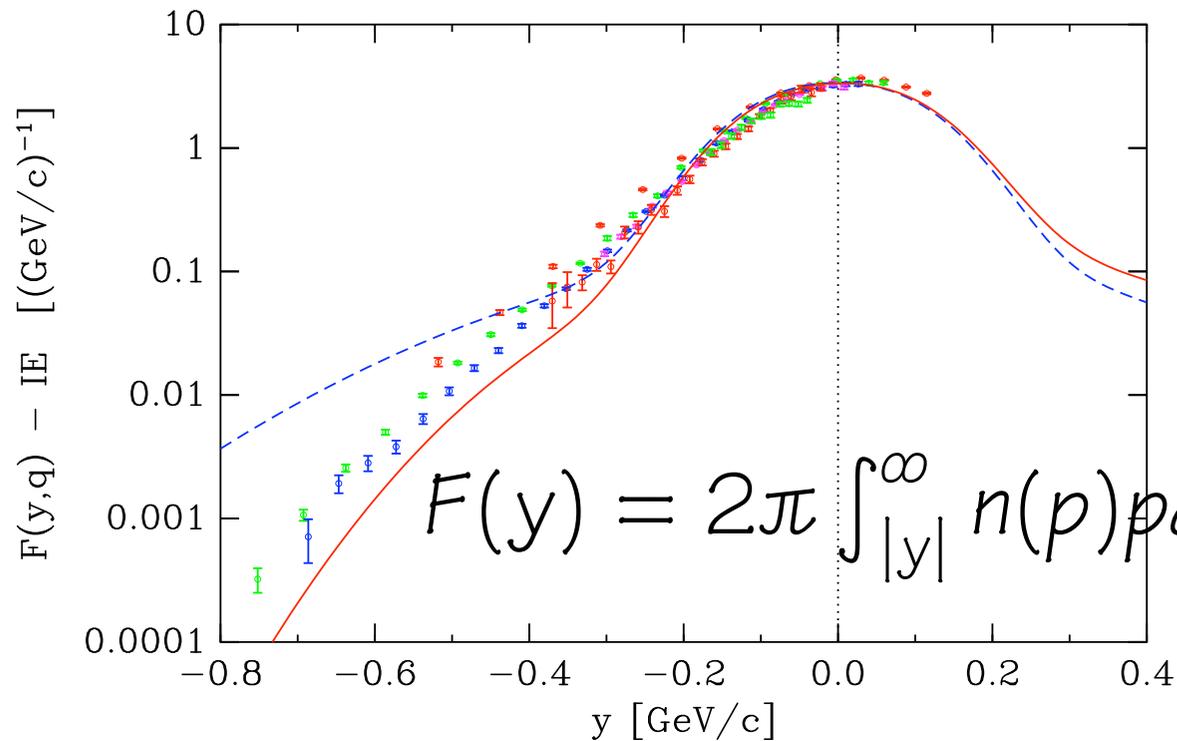
$$F(y, |\mathbf{q}|) = \frac{\sigma^{\text{exp}}}{Z\tilde{\sigma}_p + N\tilde{\sigma}_n} \cdot \frac{|\mathbf{q}|}{\sqrt{M^2 + (y + |\mathbf{q}|)^2}}$$

$$F(y) \equiv F(y, |\mathbf{q}| \rightarrow \infty)$$

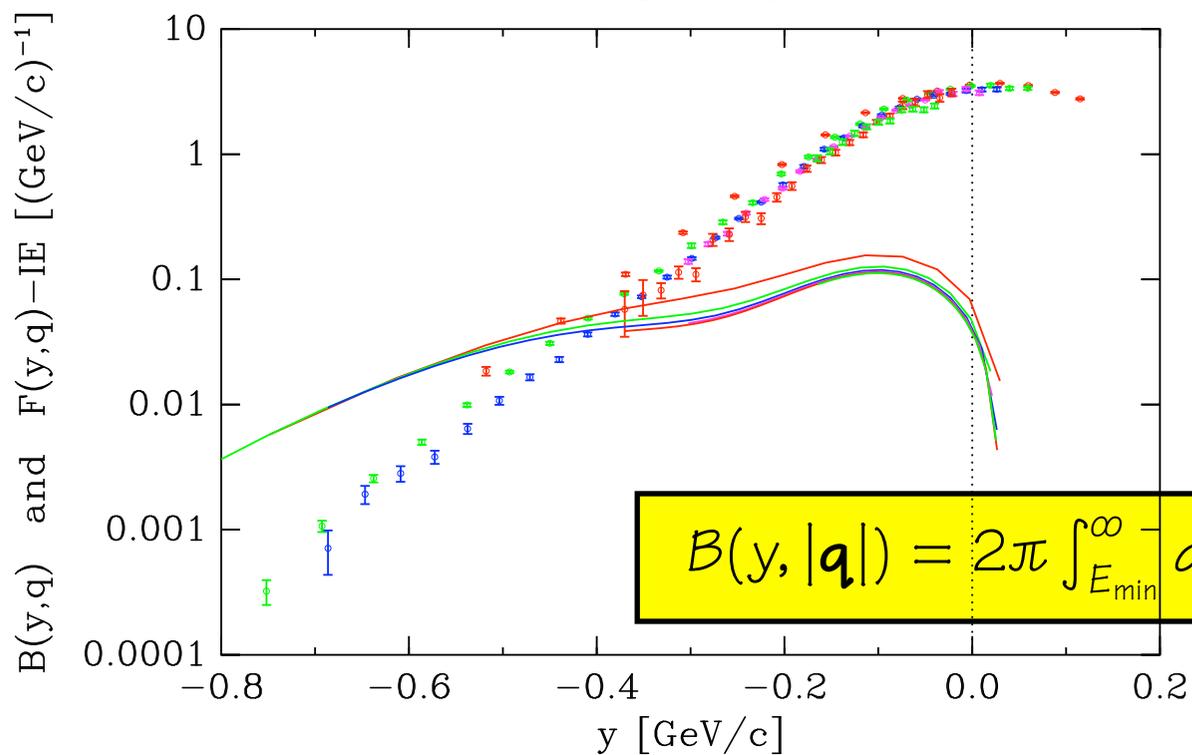
$$F(y) = 2\pi \int_{|y|}^{\infty} n(p) p dp - B(y).$$

$B(y)$ must be calculated from a model spectral function and is small for moderate values of $|y|$.

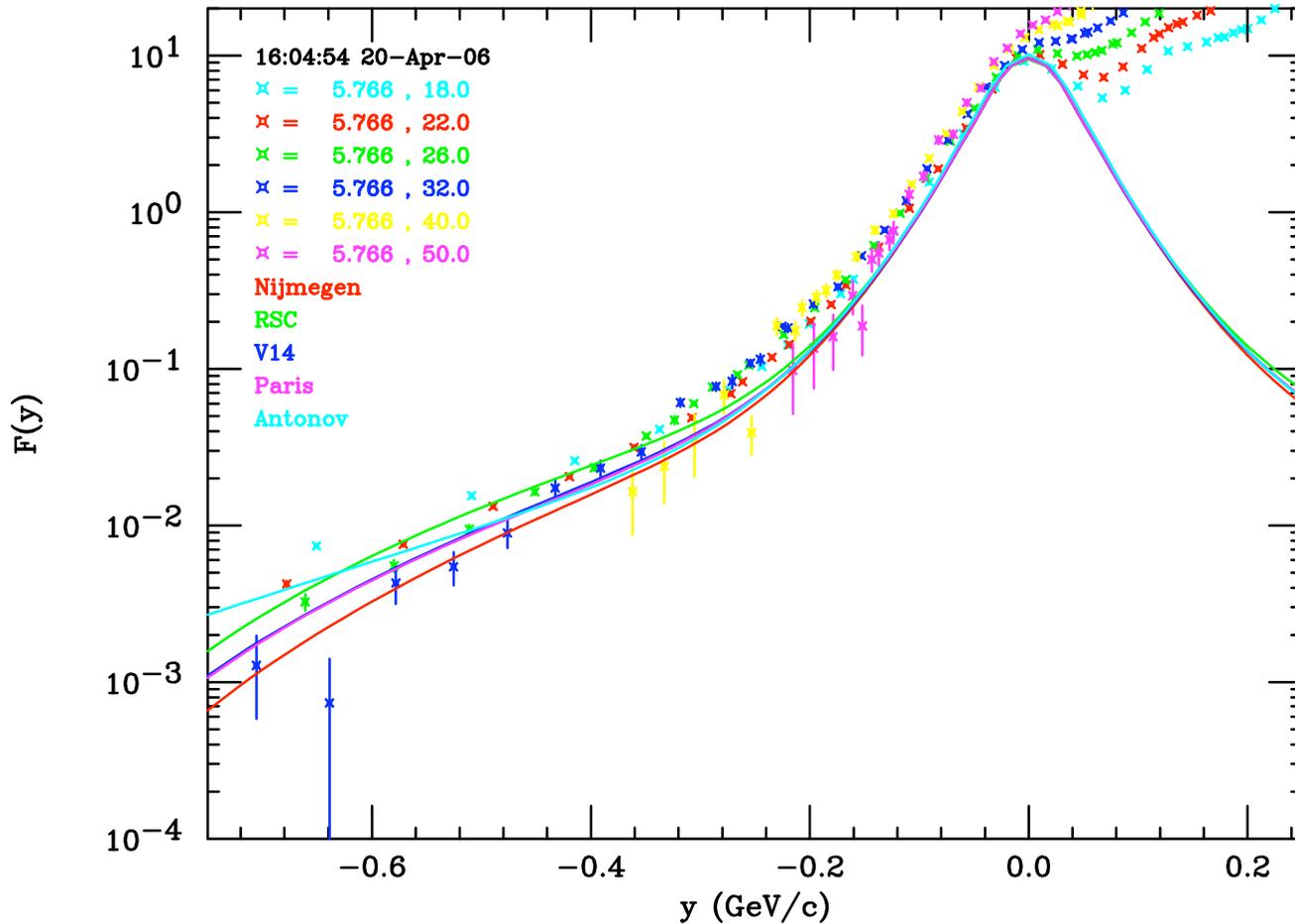
However, for large $|y|$ this correction is large and sabotages the connection between $F(y)$ and $n(p)$ just where we wish to learn about the correlation part of the momentum distribution!



$$f(y) \equiv 2\pi \int_{|y|}^{\infty} n(p) p dp$$



y-scaling Deuteron (E-02-019)



Deuteron

$$F(y) = \frac{\sigma^{\text{exp}}}{(Z\sigma_p + N\sigma_n)} \cdot K$$

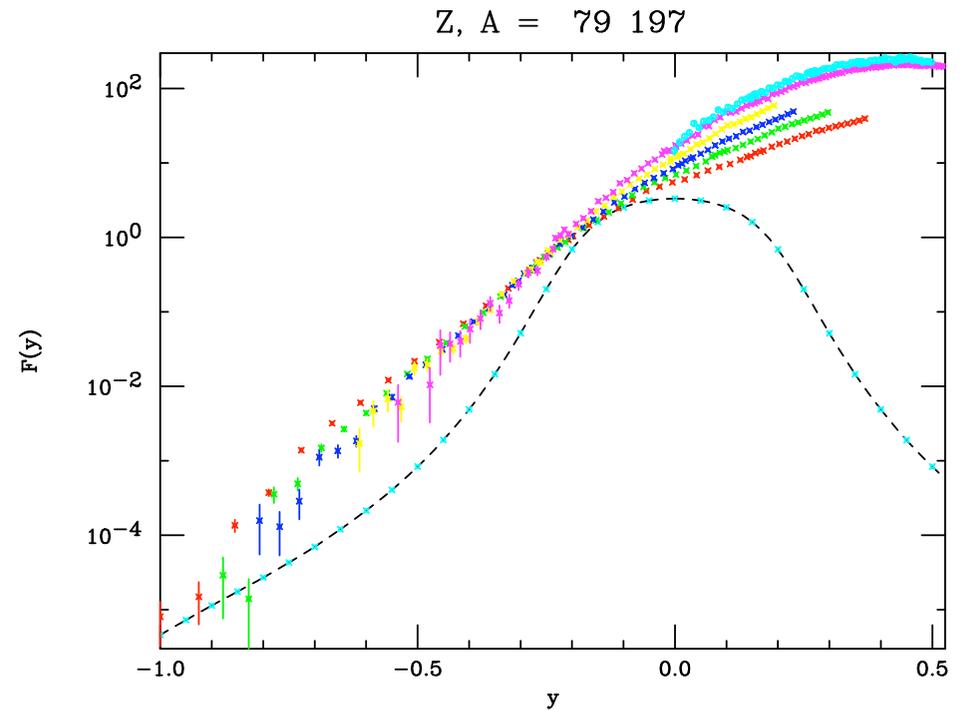
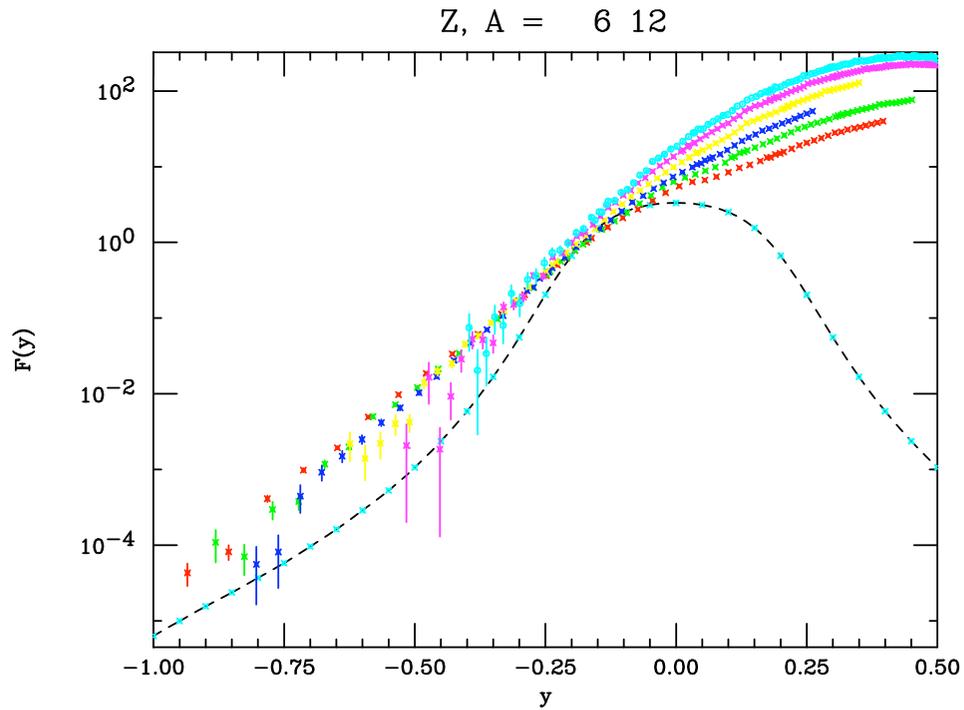
$$n(k) = -\frac{1}{2\pi y} \frac{dF(y)}{dy}$$

Assumption: scattering takes place from a quasi-free proton or neutron in the nucleus.

y is the momentum of the struck nucleon parallel to the momentum transfer:

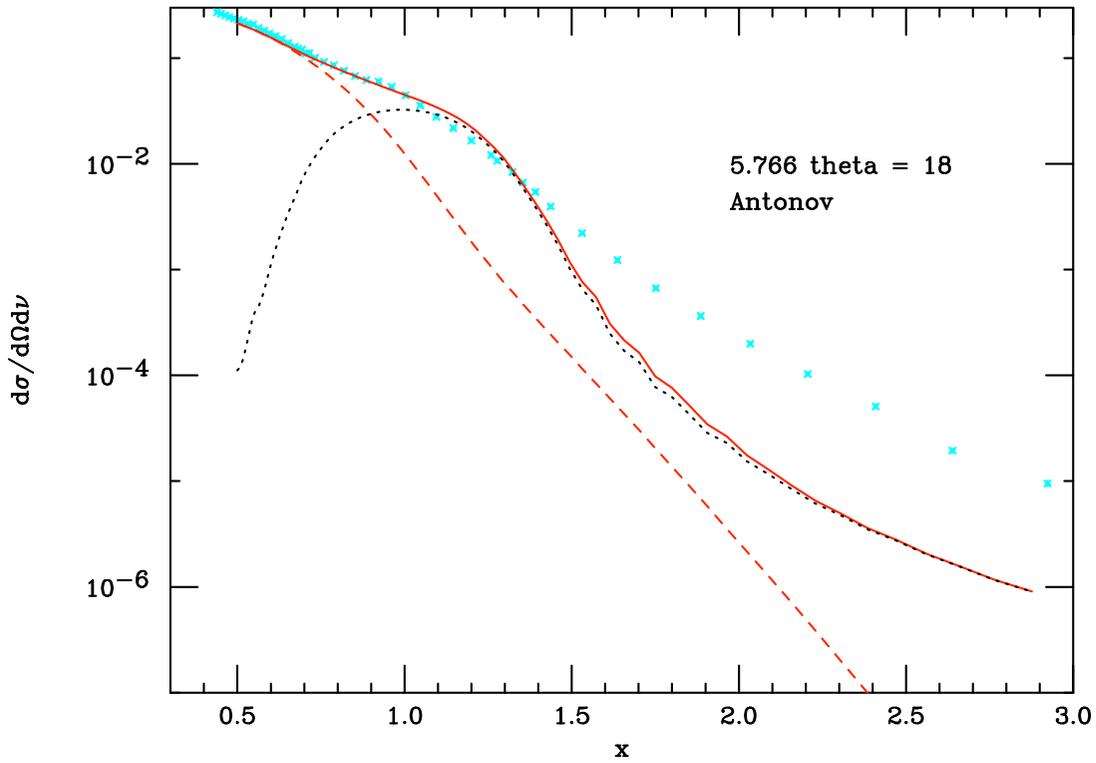
$$y \approx -q/2 + mv/q$$

$F(y)$ for heavier nuclei and theory



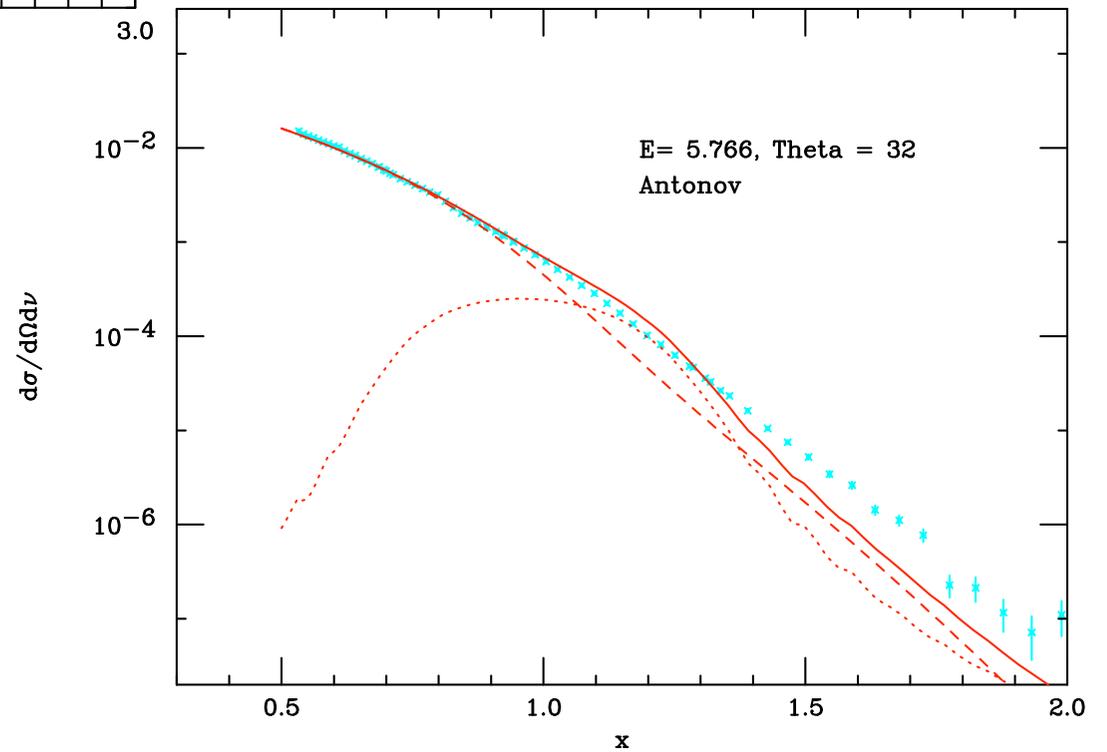
Gaidarov, Antonov (CDFM)

Z, A = 6 12



Cross section and
convolution model

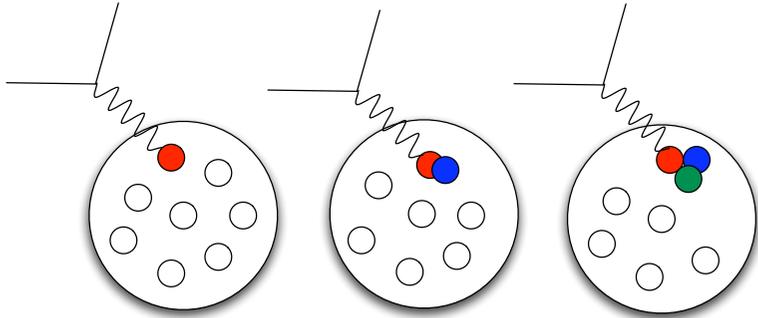
Z, A = 6 12



Gaidarov, Antonov (CDFM)

Short Range Correlations

In the region where correlations should dominate, **large x**,



$$\begin{aligned} \sigma(x, Q^2) &= \sum_{j=1}^A A \frac{1}{j} a_j(A) \sigma_j(x, Q^2) \\ &= \frac{A}{2} a_2(A) \sigma_2(x, Q^2) + \\ &\quad \frac{A}{3} a_3(A) \sigma_3(x, Q^2) + \\ &\quad \vdots \end{aligned}$$

$a_j(A)$ are proportional to finding a nucleon in a **j-nucleon** correlation. It should fall rapidly with **j** as nuclei are dilute.

$$\sigma_2(x, Q^2) = \sigma_{eD}(x, Q^2) \quad \text{and} \quad \sigma_j(x, Q^2) = 0 \quad \text{for} \quad x > j.$$

$$\Rightarrow \frac{2 \sigma_A(x, Q^2)}{A \sigma_D(x, Q^2)} = a_2(A) \Big|_{1 < x \leq 2}$$

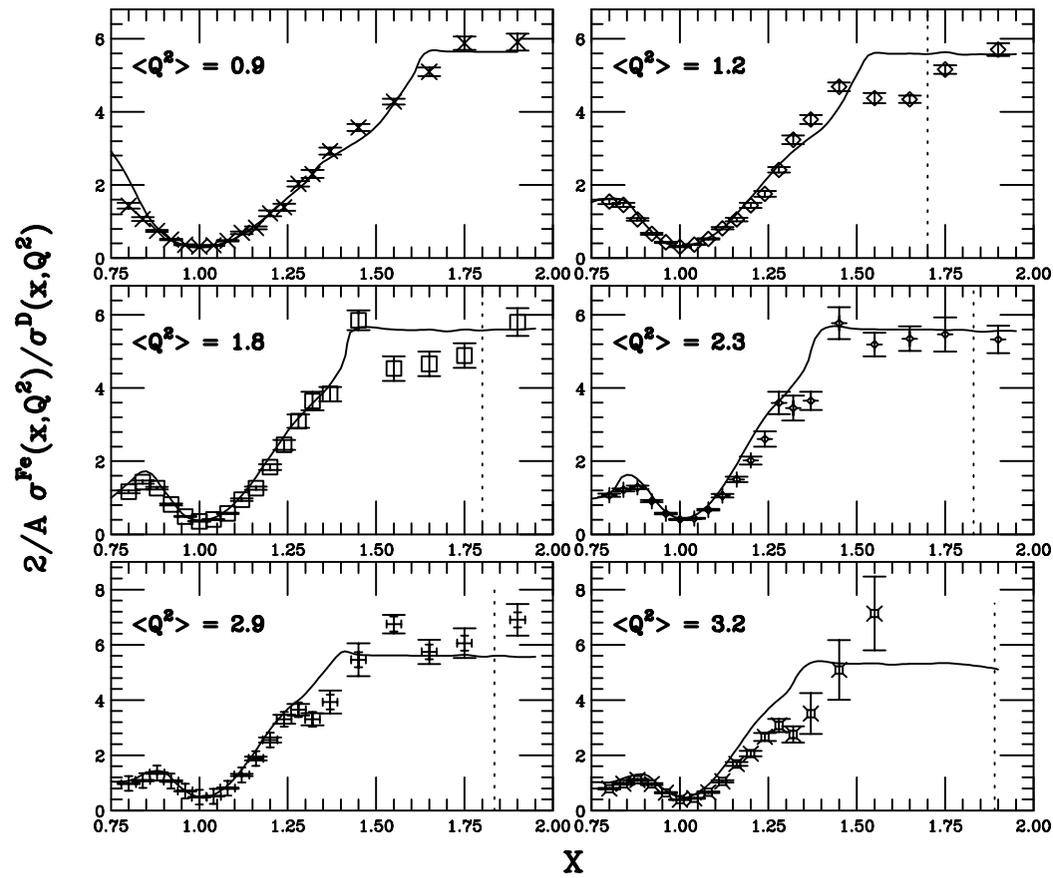
$$\frac{3 \sigma_A(x, Q^2)}{A \sigma_{A=3}(x, Q^2)} = a_3(A) \Big|_{2 < x \leq 3}$$

In the ratios, off-shell effects and FSI largely cancel.

$a_j(A)$ is proportional to probability of finding a **j-nucleon** correlation

Short Range Correlations

$$\frac{2 \sigma_A}{A \sigma_D} = a_2(A); \quad (1.4 < x < 2.0)$$

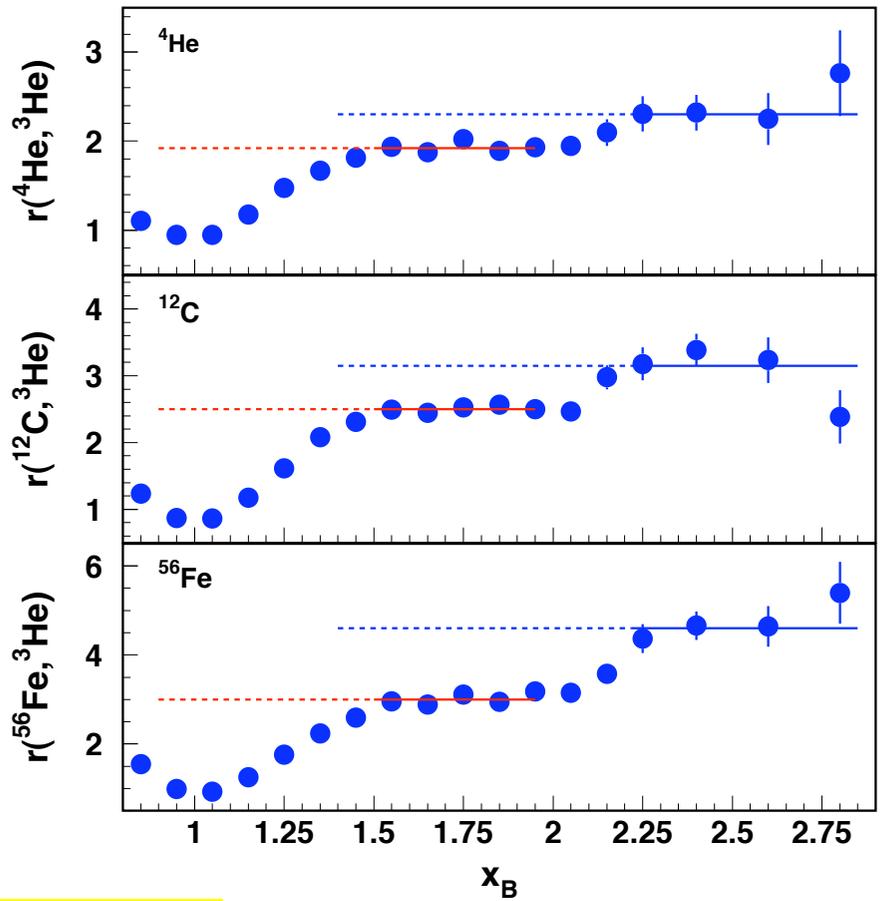


FSDS, Phys.Rev.C48:2451-2461,1993

$a_j(A)$ is proportional to probability of finding a j -nucleon correlation

$\alpha_{2N} \approx 20\%$
 $\alpha_{3N} \approx 1\%$

$A(e, e'), 1.4 < Q^2 < 2.6$

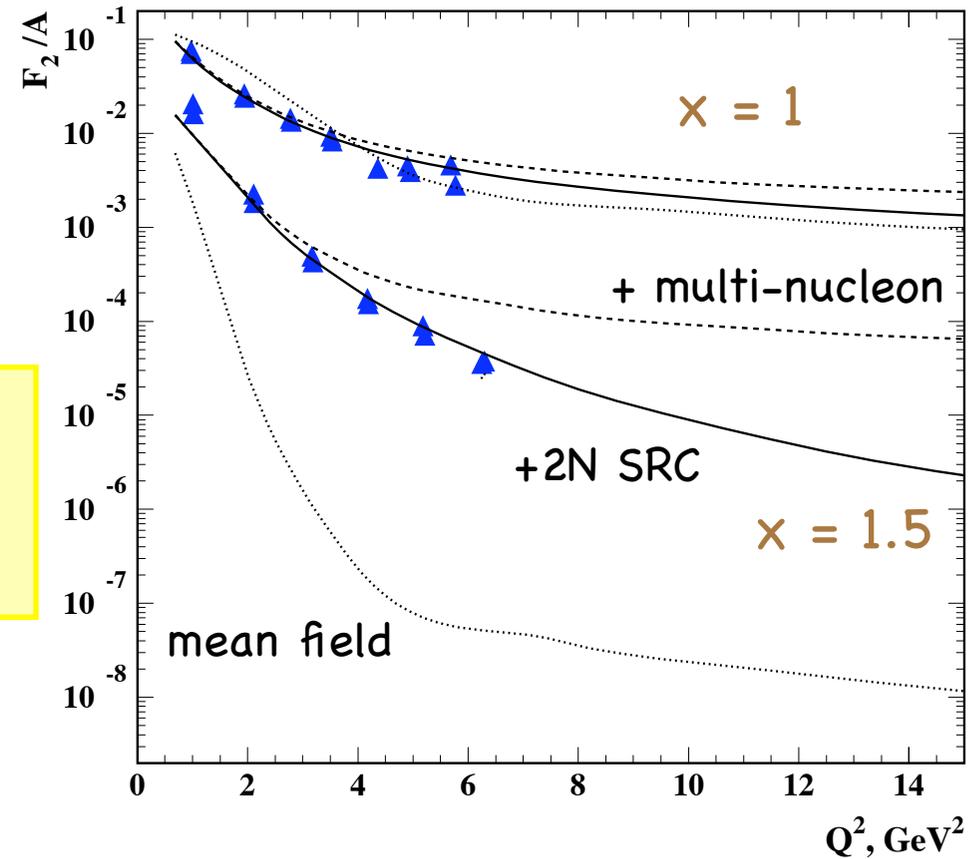


CLAS data
 Egiyan et al., PRL 96, 082501, 2006

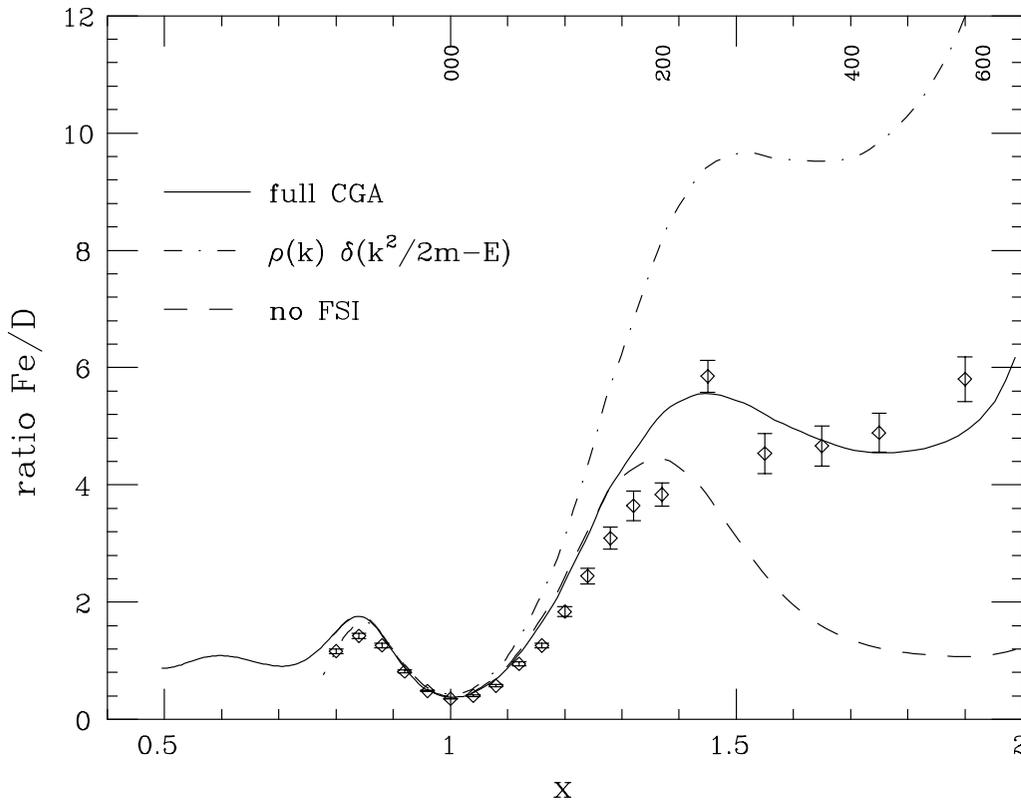
Sensitivity to SRC increase with Q^2 and x

We want to be able to isolate and probe two-nucleon and multi-nucleon SRCs

Dotted = mean field approx.
Solid = +2N SRCs.
Dashed = +multi-nucleon.



11 GeV can reach $Q^2 = 20$ (13) GeV^2 at $x = 1.3$ (1.5)
- very sensitive, especially at higher x values



Arguments about role of FSI

Benhar et al.: FSI includes a piece that has a weak Q^2 dependence

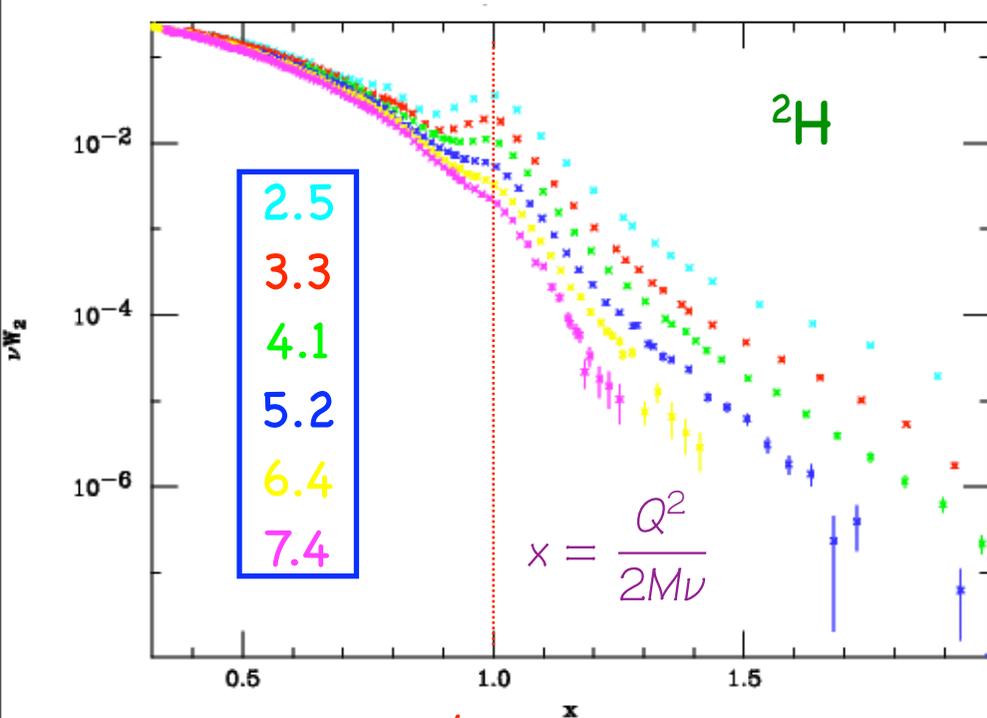
There is the cancellation of two large factors (≈ 3) that bring the theory to describe the data. These factors are Q^2 and A dependent

The solution

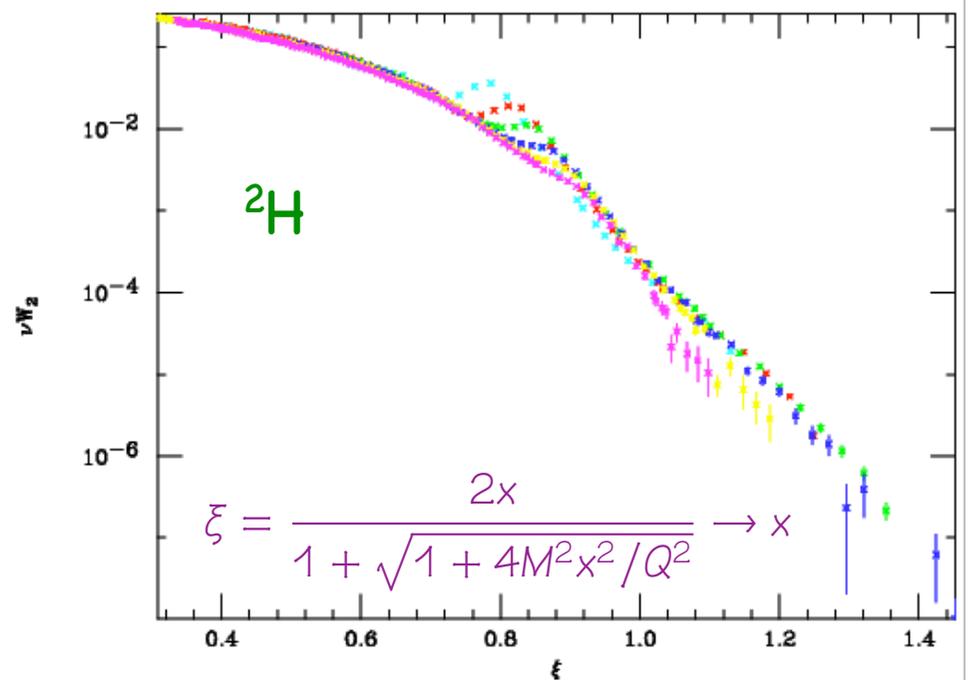
- Direct ratios to ^2H , ^3He , ^4He out to large x and over wide range of Q^2
- Study Q^2 , A dependence (FSI)
- Absolute Cross section to test exact calculations and FSI
- Extrapolation to NM

x and ξ scaling

An alternative view is suggested when the data (deuteron) is presented in terms of scattering from individual quarks

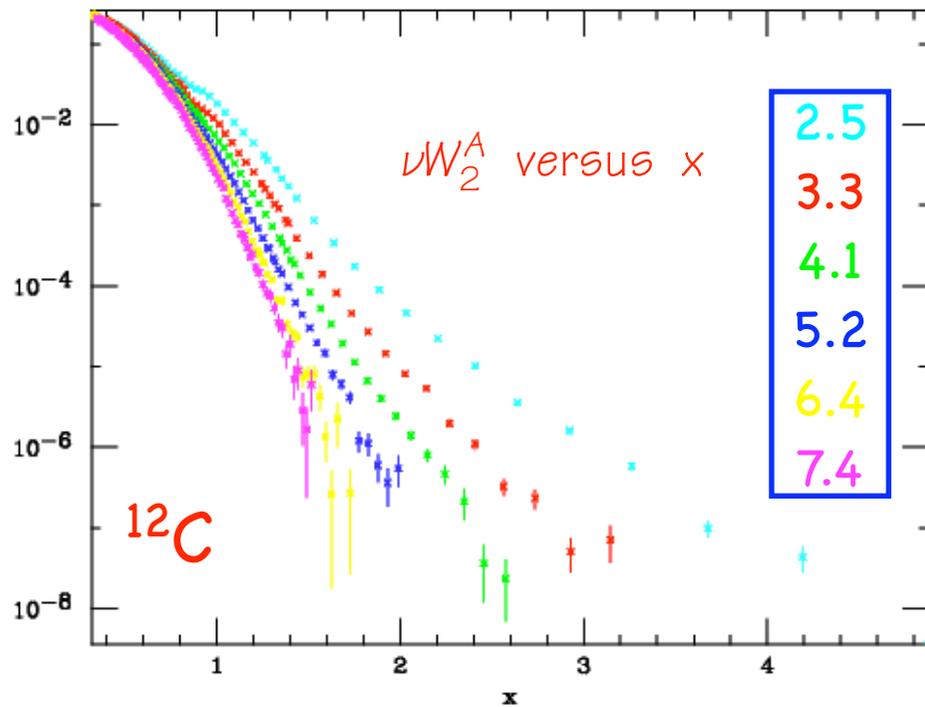


νW_2^A versus x



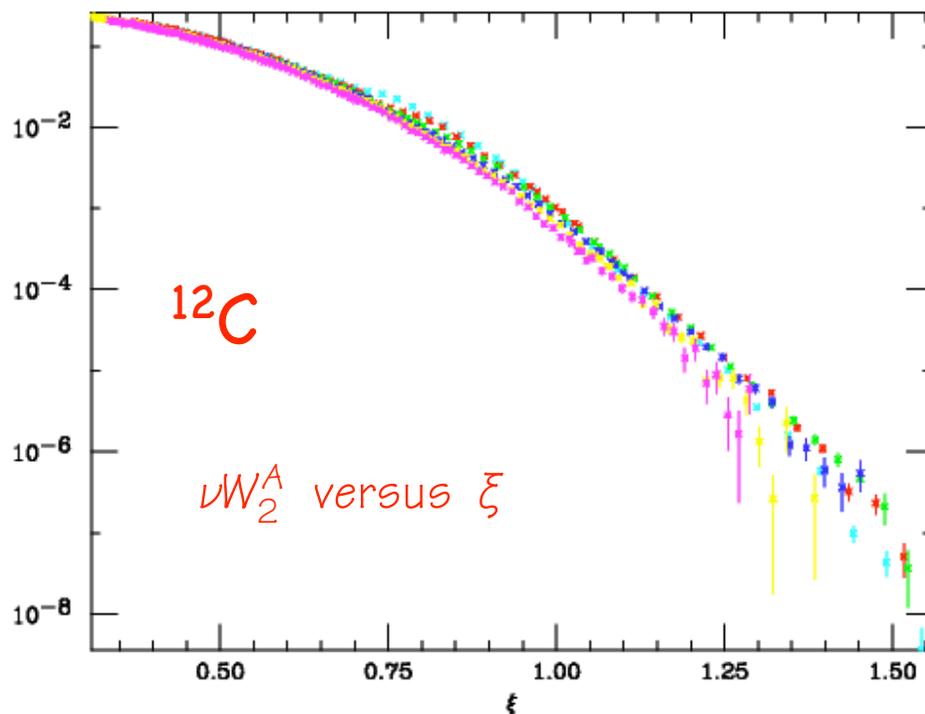
νW_2^A versus ξ

$$\nu W_2^A = \nu \cdot \frac{\sigma^{\text{exp}}}{\sigma_M} \left[1 + 2 \tan^2(\theta/2) \cdot \left(\frac{1 + \nu^2/Q^2}{1 + R} \right) \right]^{-1}$$



The Nachtmann variable (fraction ξ of nucleon **light cone** momentum p^+) has been shown to be the variable in which logarithmic violations of scaling in DIS should be studied.

Local duality (averaging over finite range in x) should also be valid for elastic peak at $x = 1$ if analyzed in ξ

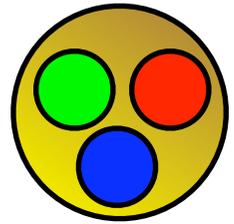
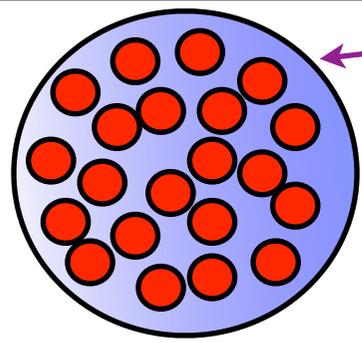


$$F_2^A(\xi) = \underbrace{\int_{\xi}^A dz F(z) F_2^n(\xi/z)}_{\text{averaging}}$$

Evidently the inelastic and quasielastic contributions cooperate to produce ξ scaling. **Is this duality?**

Medium Modifications generated by high density configurations

Gold nucleus

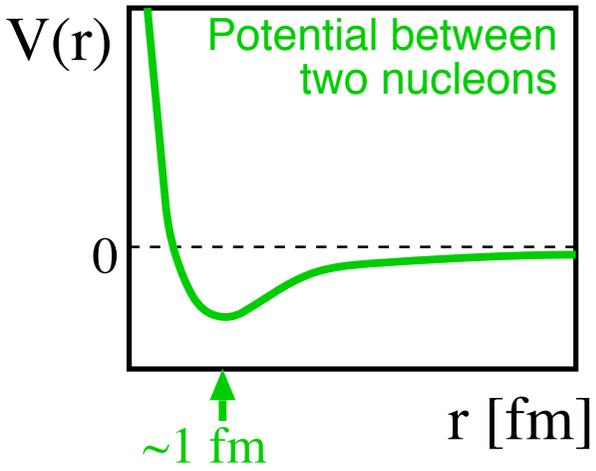


$$R = 1.2A^{1/3}$$

$$\text{Volume} = \frac{4}{3}\pi R^3 \approx 1400\text{fm}^3$$

A single nucleon, $r = 1 \text{ fm}$, has a volume of 4.2 fm^3
 197 times $4.2 \text{ fm}^3 \approx 830 \text{ fm}^3$

60% of the volume is occupied - very closely packed!



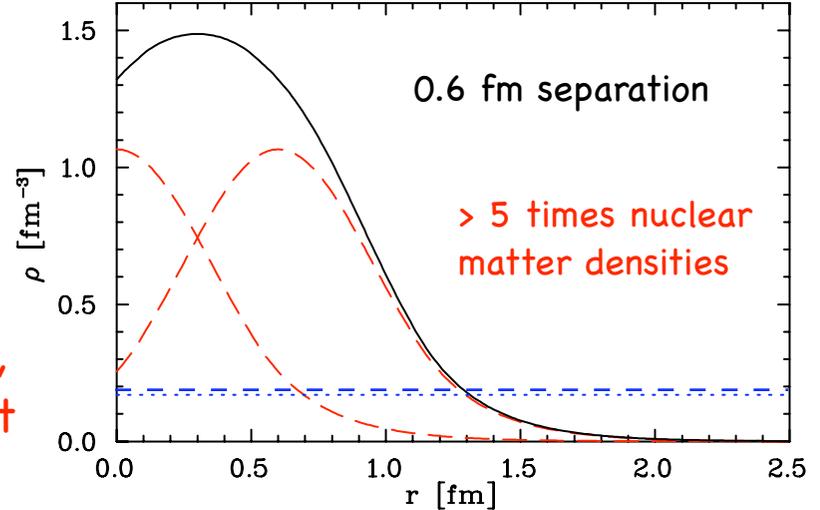
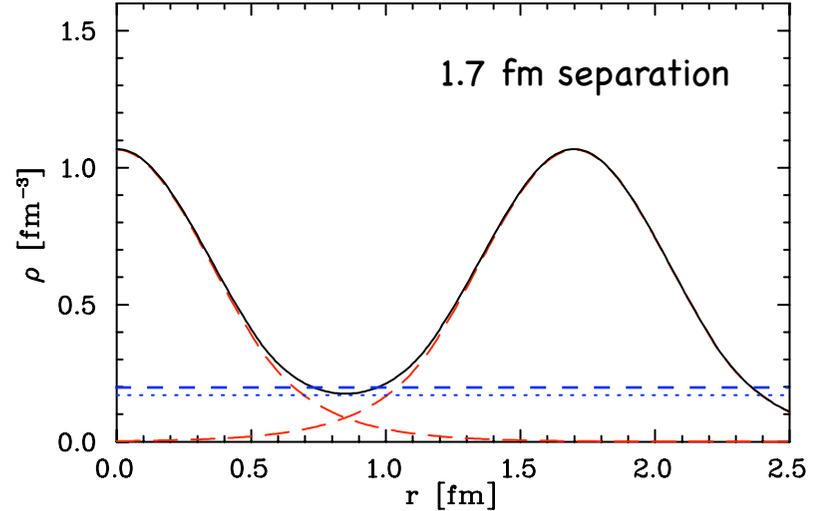
Nucleon separation is limited by the short range repulsive core

Even for a 1 fm separation, the central density is about 4x nuclear matter

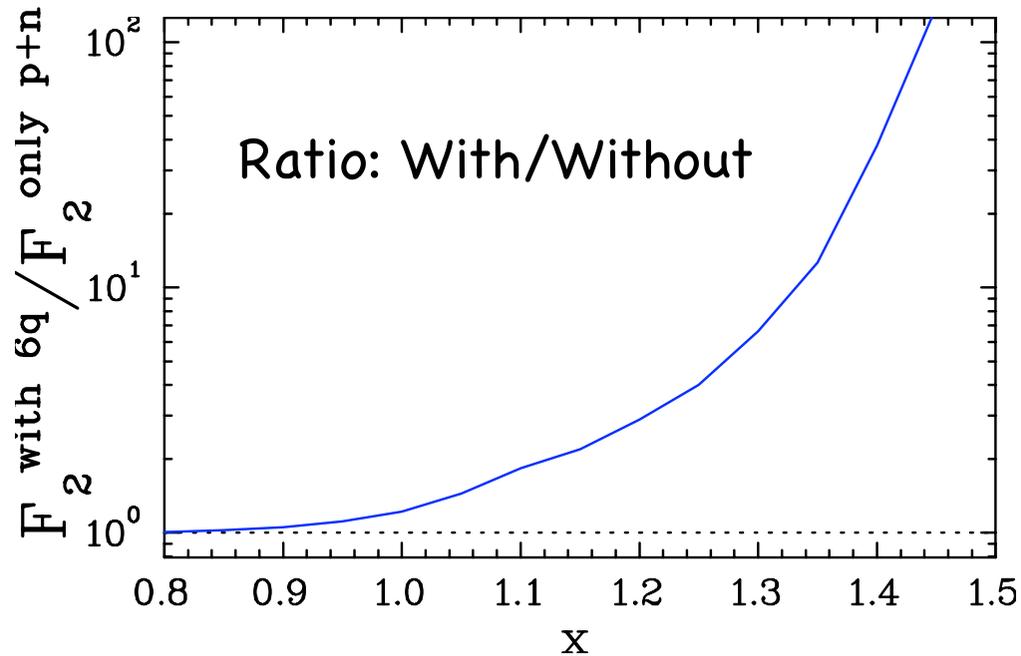
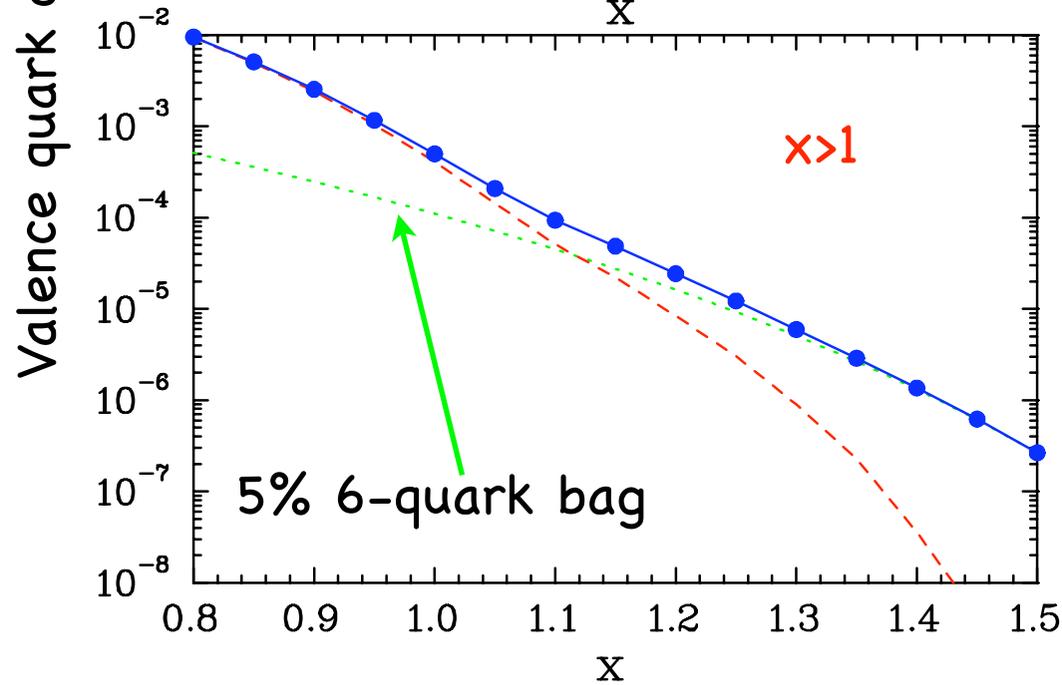
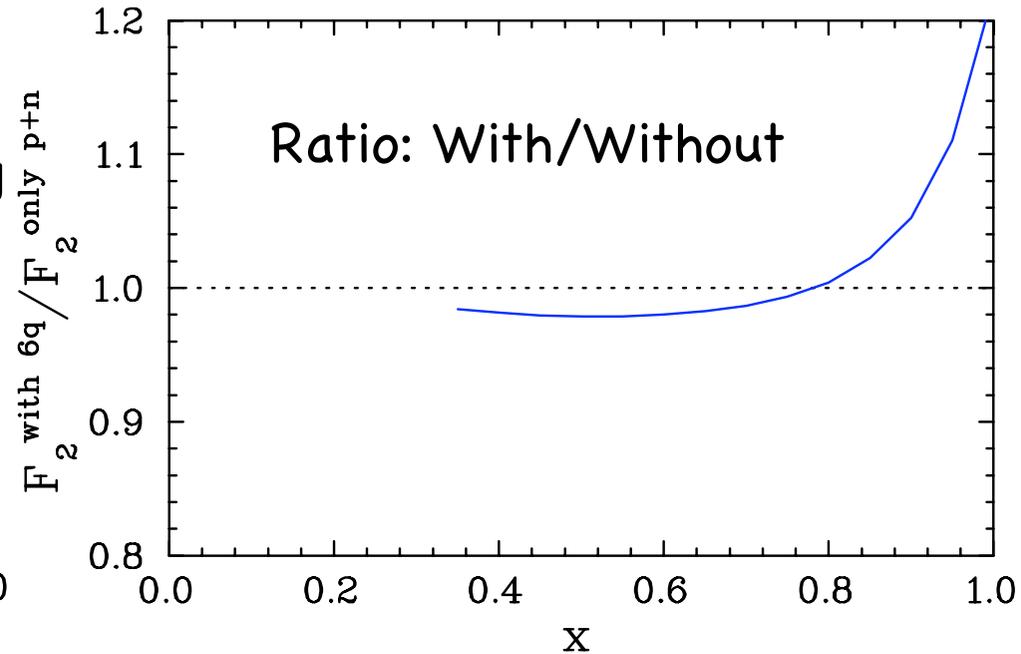
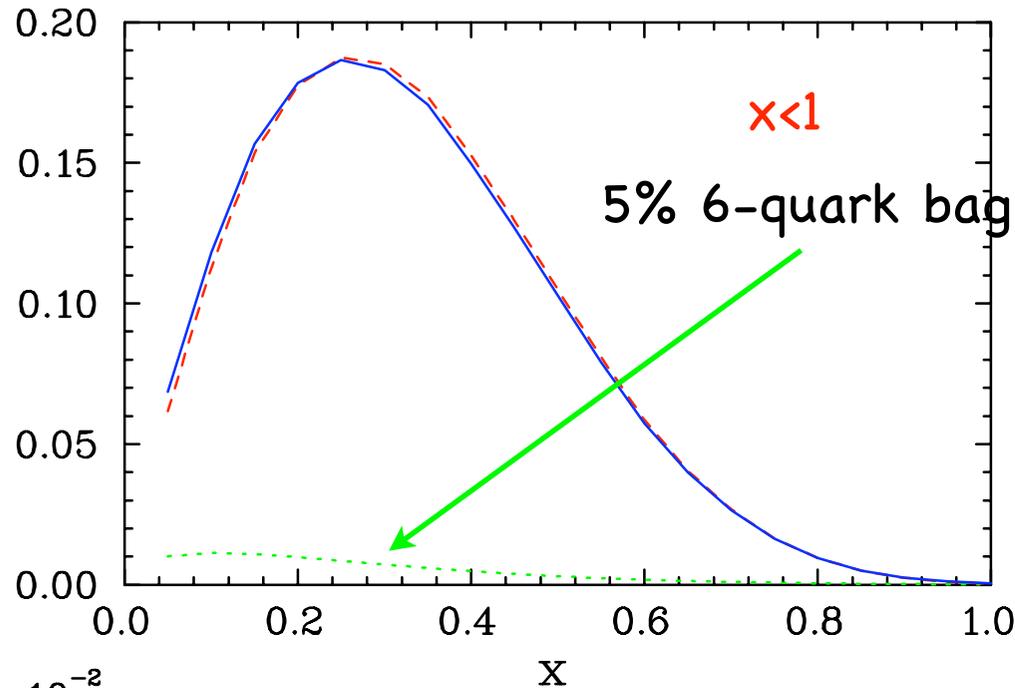
Comparable to neutron star densities!

High enough to modify nucleon structure?

To which nucleon does the quark belong?



Sensitivity to non-hadronic components



Quark distributions at $x > 1$

Two measurements (very high Q^2) exist so far:

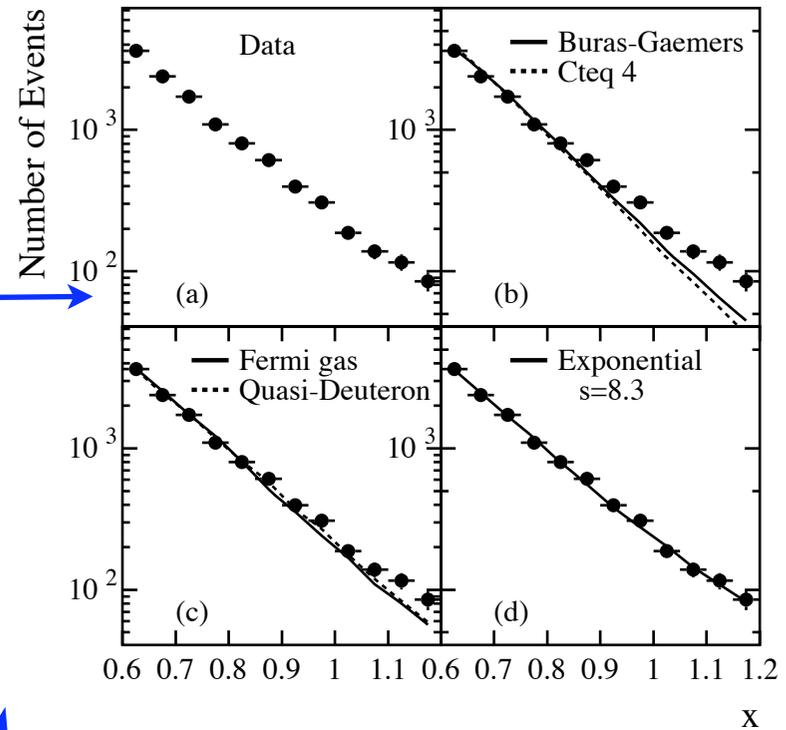
CCFR (ν -C): $F_2(x) \propto e^{-sX}$

$s = 8$

BCDMS (μ -Fe): $F_2(x) \propto e^{-sX}$

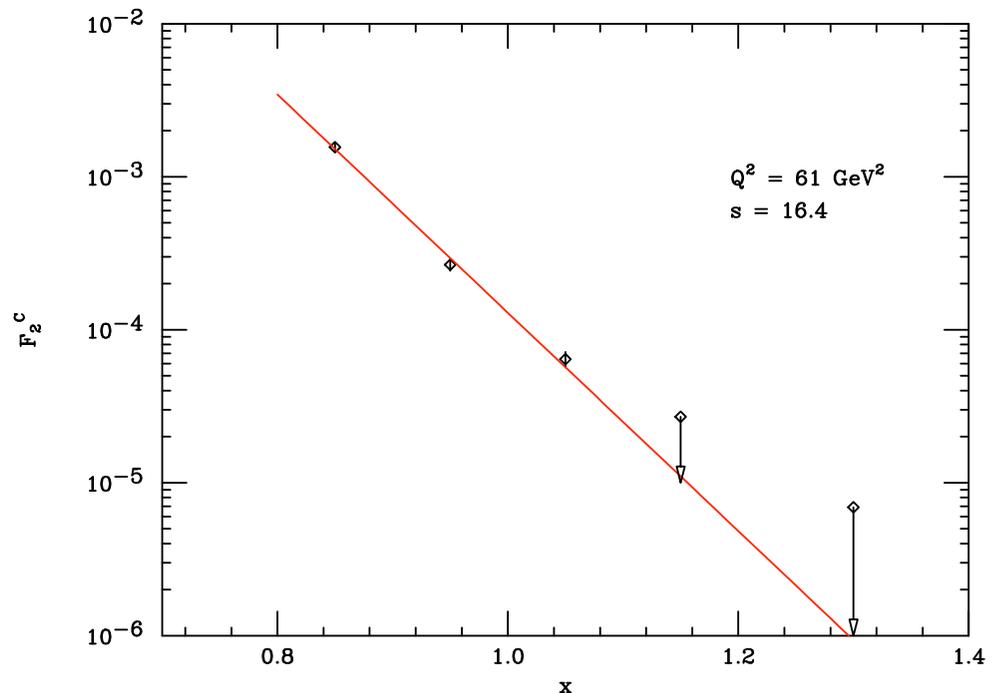
$s = 16$

Limited x range, poor resolution
Limited x range, low statistics



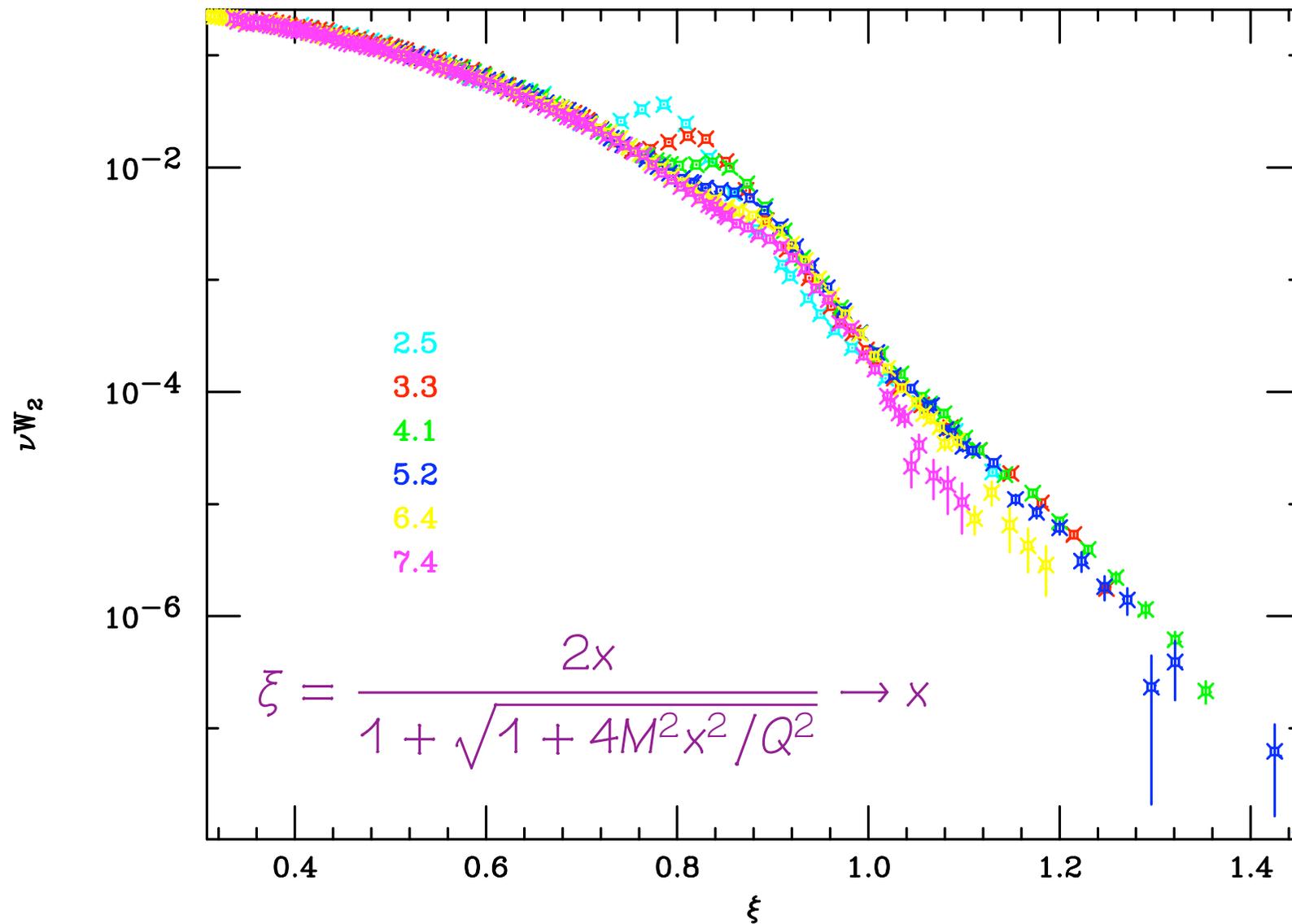
BCDMS 200 GeV muon

With 11 GeV beam, we should be in the scaling region up to $x \approx 1.4$



Quark Distribution Functions

Deuterium

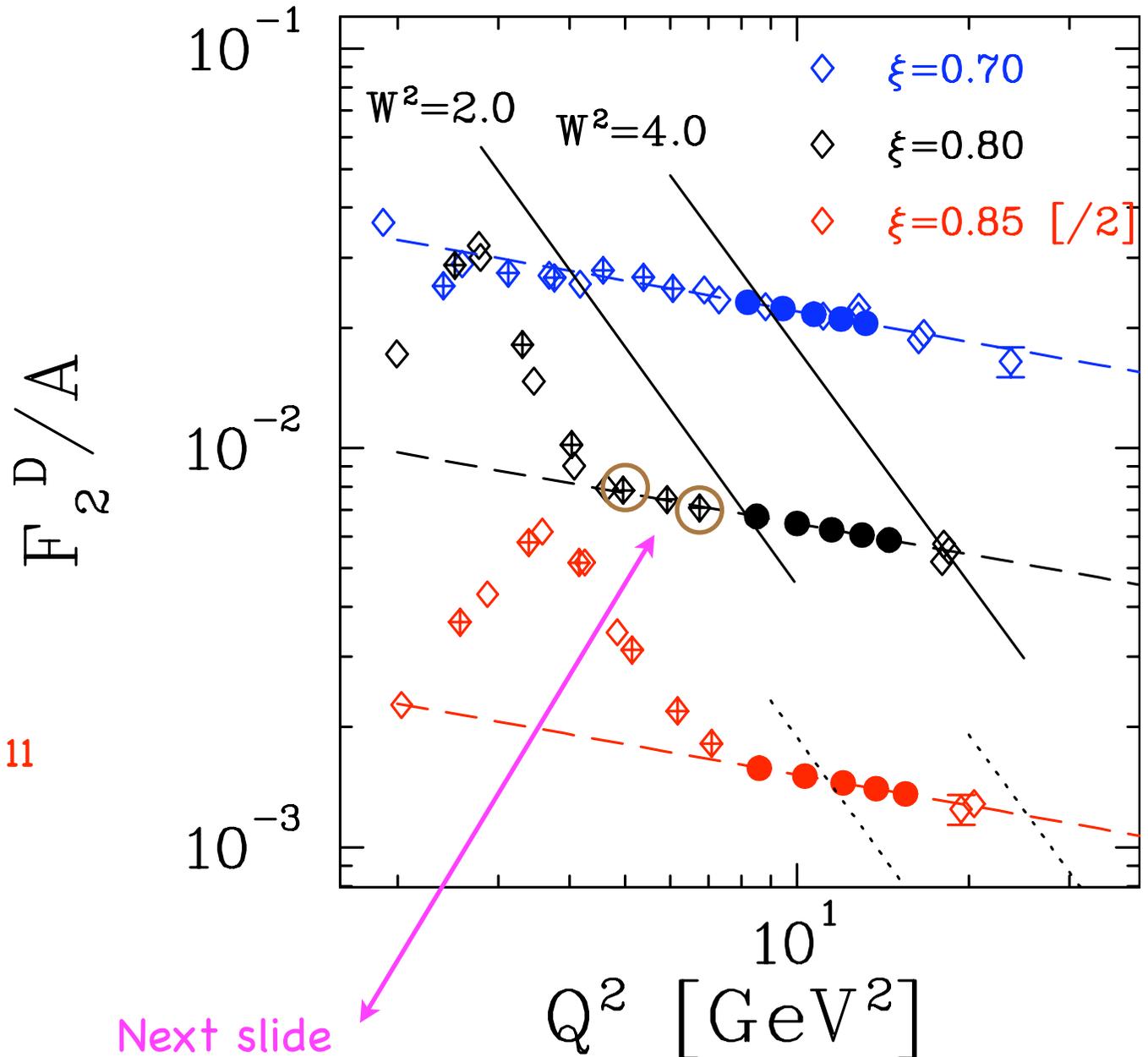


νW_2^A versus ξ

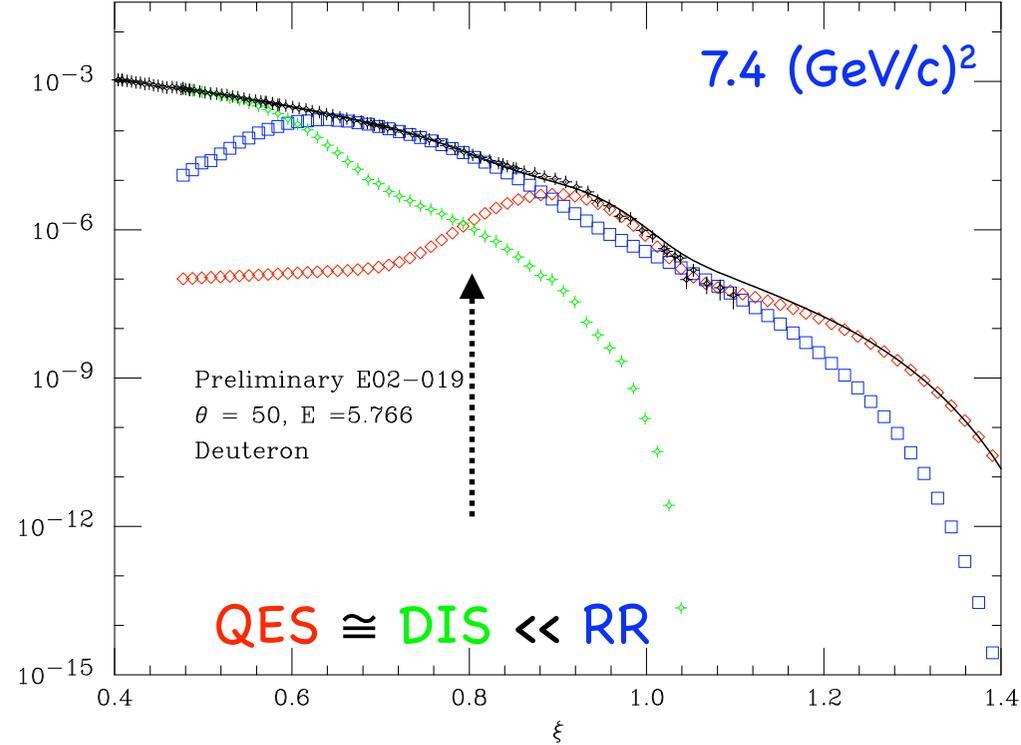
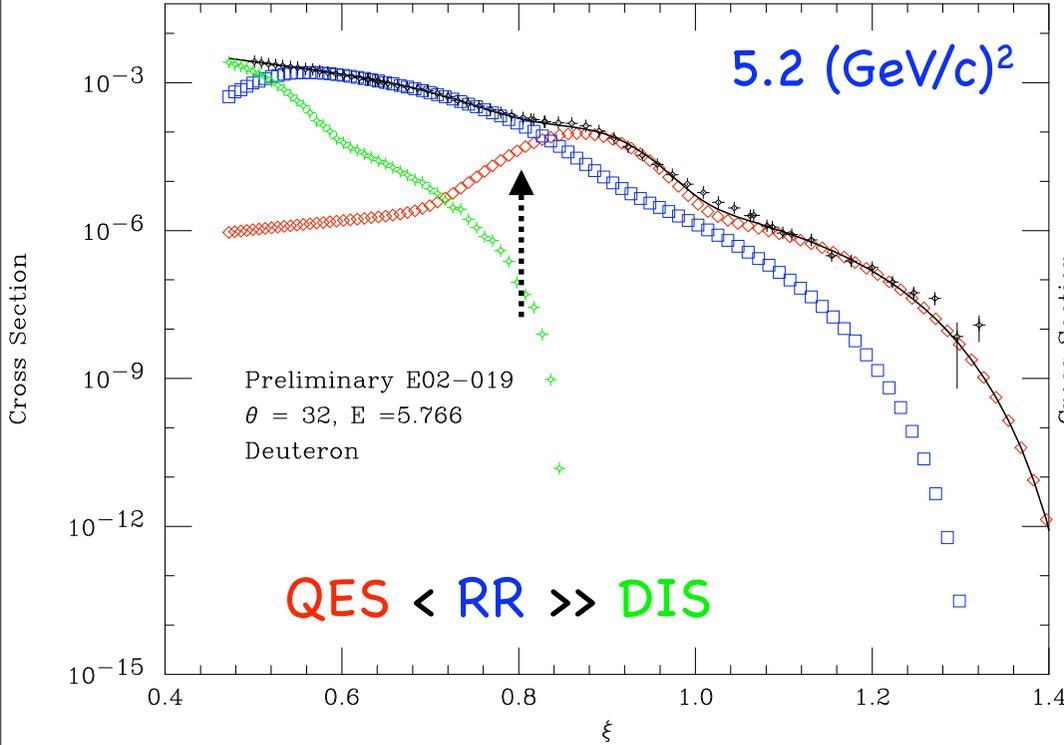
Approach to Scaling - Deuteron

Dashed lines are arbitrary normalization (adjusted to go through the high Q^2 data) with a constant value of $d\ln(F_2)/d\ln(Q^2)$

filled dots - experiment with 11 GeV



Approach to Scaling (Deuteron)



Convolution model

QES

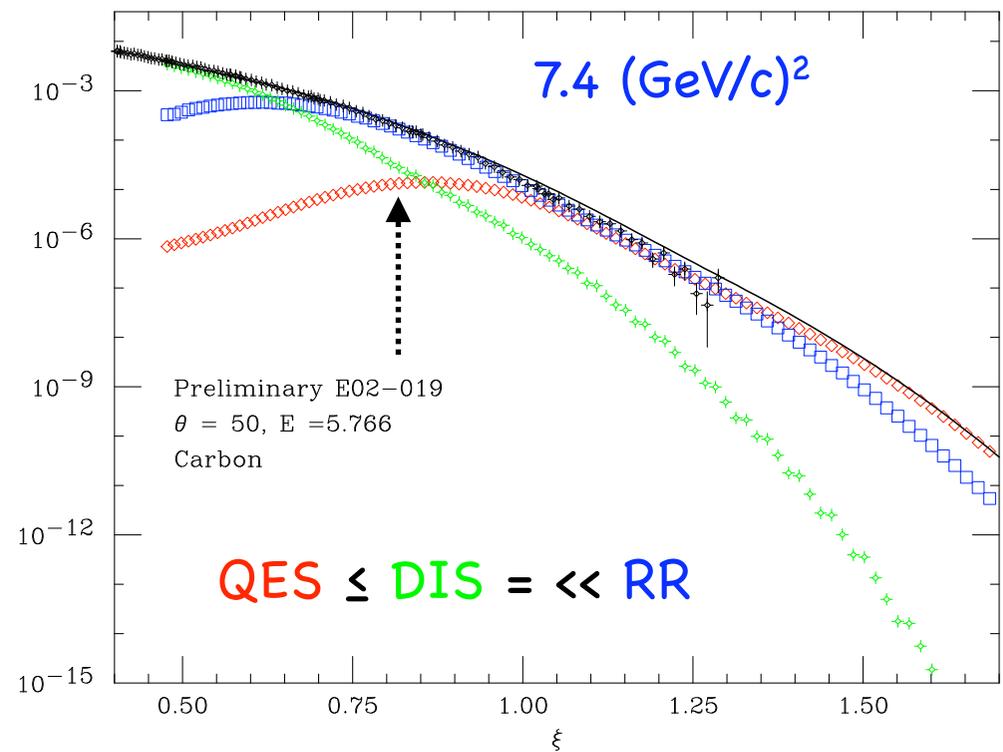
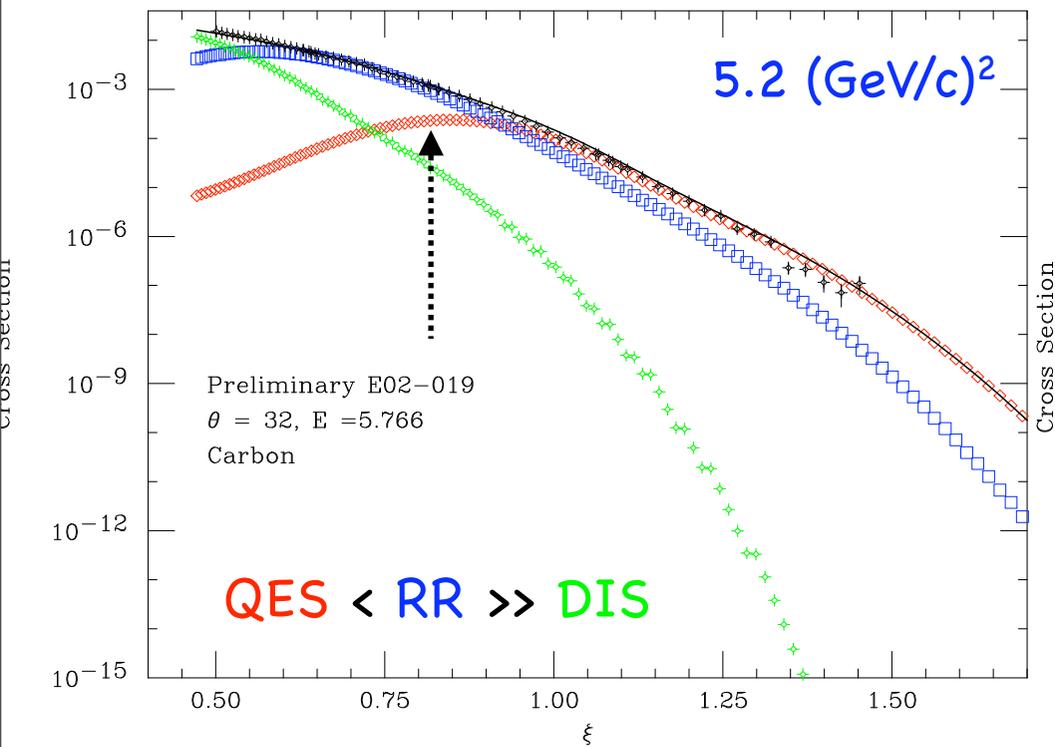
RR ($W^2 < 4$)

DIS ($W^2 > 4$)

Scaling appears to work well even in regions where the DIS is not the dominate process

We can expect that any scaling violations will melt away as we go to higher Q^2

Approach to Scaling (Carbon)



Convolution model

QES

RR ($W^2 < 4$)

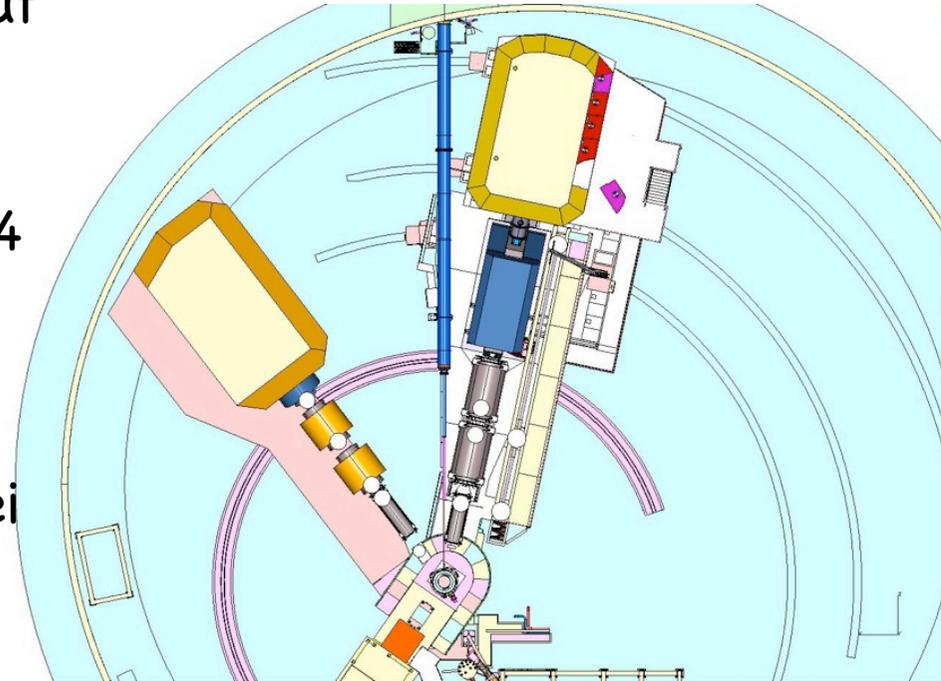
DIS ($W^2 > 4$)

Scaling appears to work well even in regions where the DIS is not the dominate process

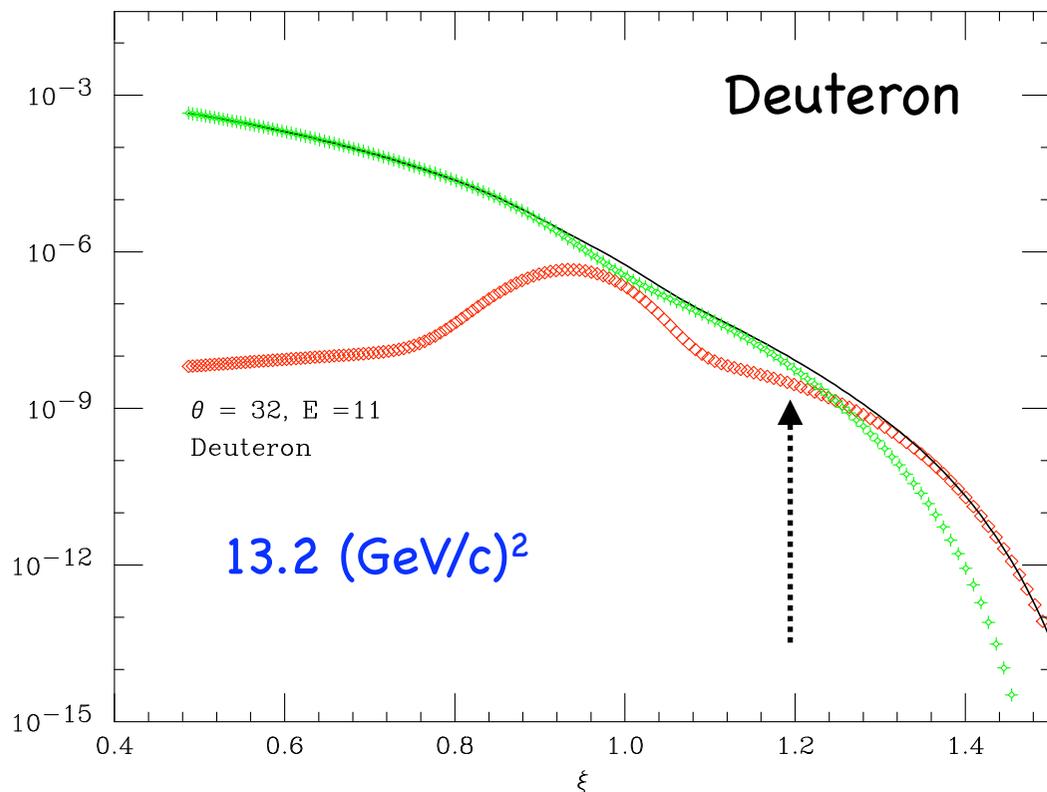
We can expect that any scaling violations will melt away as we go to higher Q^2

Inclusive DIS at $x > 1$ at 12 GeV

- New proposal approved at last JLAB PAC
- Target ratios (and absolute cross sections) in quasielastic regime: map out 2N, 3N, 4N correlations
- Measure nuclear structure functions (parton distributions) up to $x = 1.3 - 1.4$
 - Extremely sensitive to non-hadronic configurations
- Targets include several few-body nuclei allowing precise test of theory.
- Extend measurements to large enough Q^2 to fully suppress the quasielastic contribution
- Extract structure functions at $x > 1$
- $Q^2 \approx 20$ at $x=1$, $Q^2 \approx 12$ at $x = 1.5$



Cross Section



Quark distributions at $x > 1$

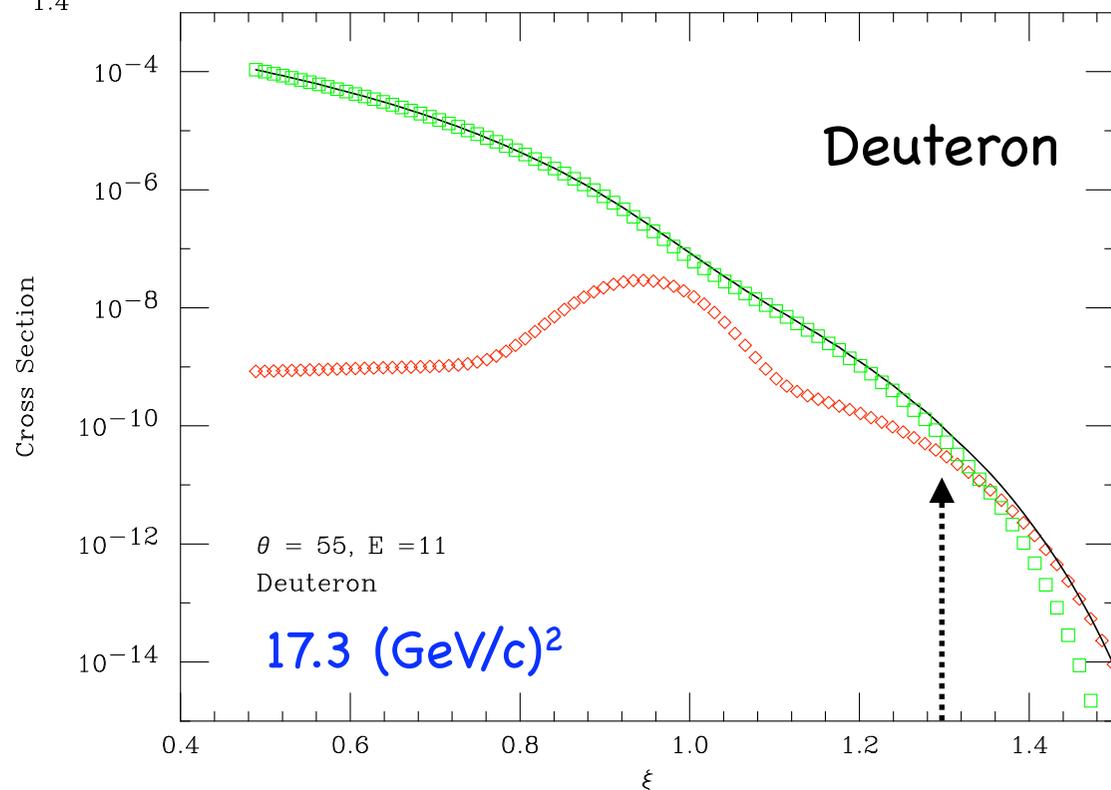
Predictions for 11 GeV

Convolution model

QES

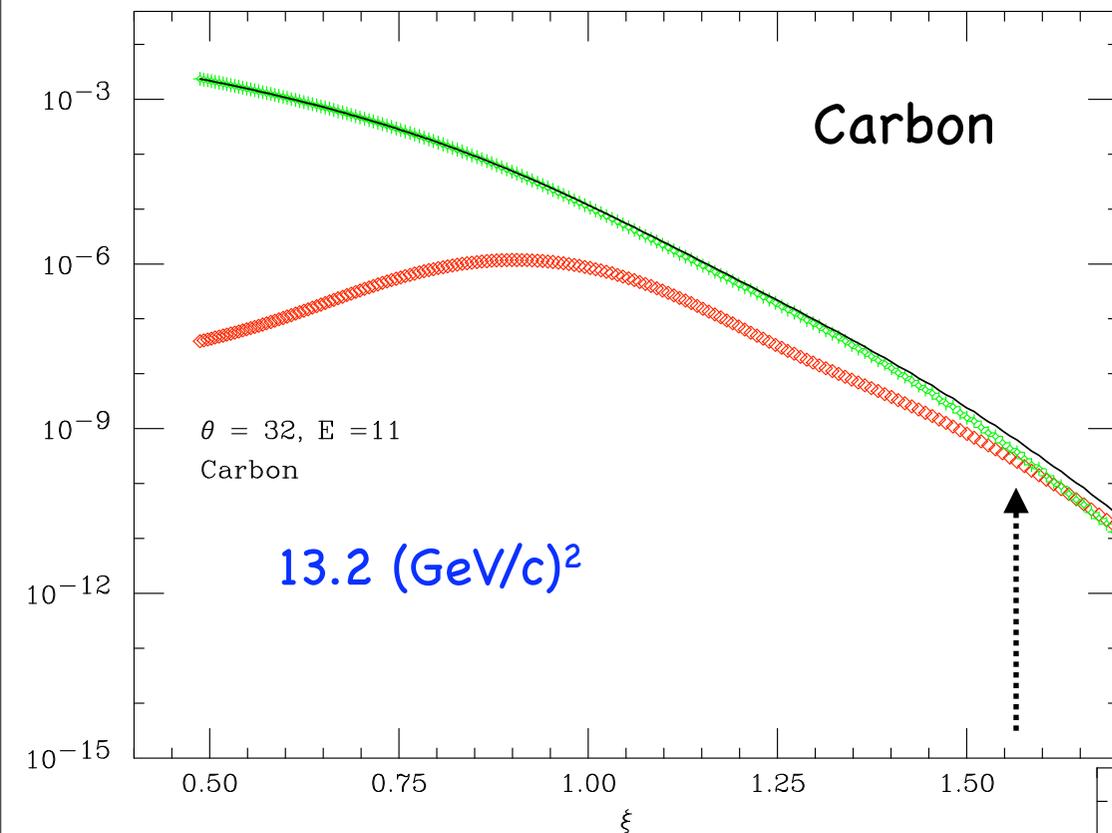
DIS + RR

Deuteron is worst case
as narrow QE peak
makes for larger scaling
violations



Quark distributions at $x > 1$

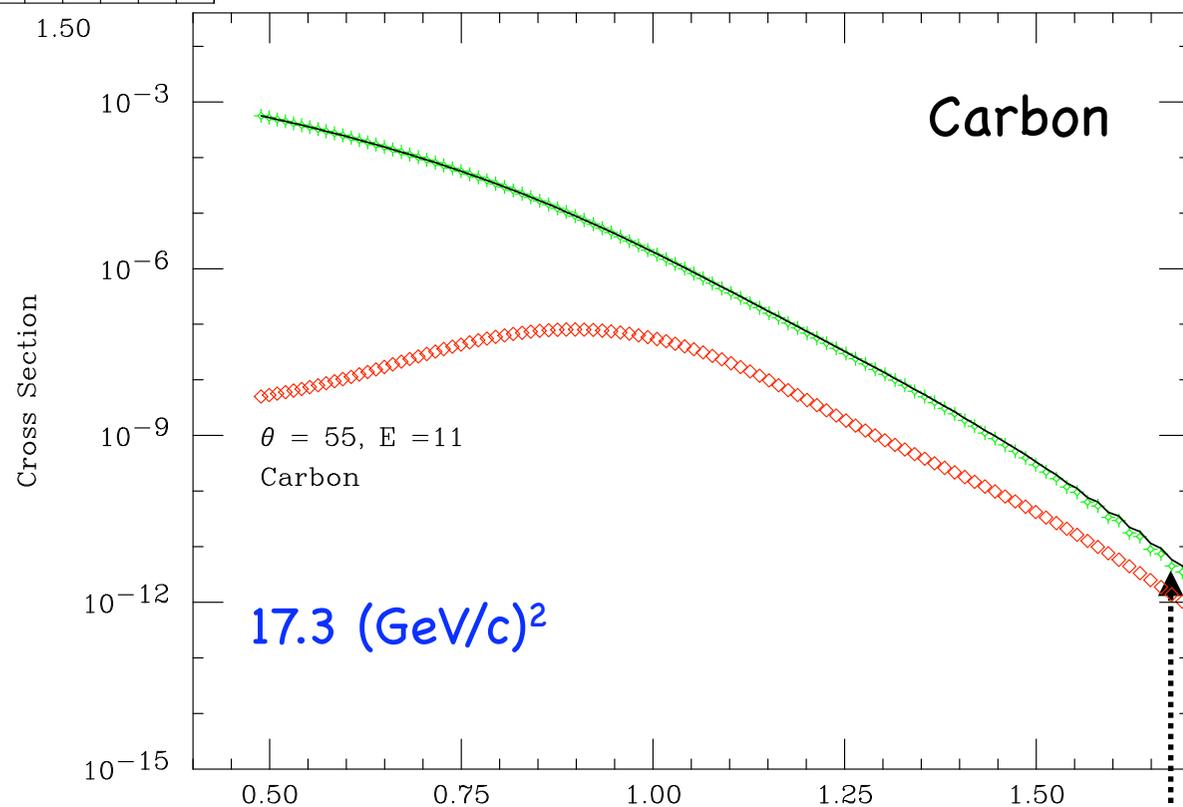
Predictions for 11 GeV



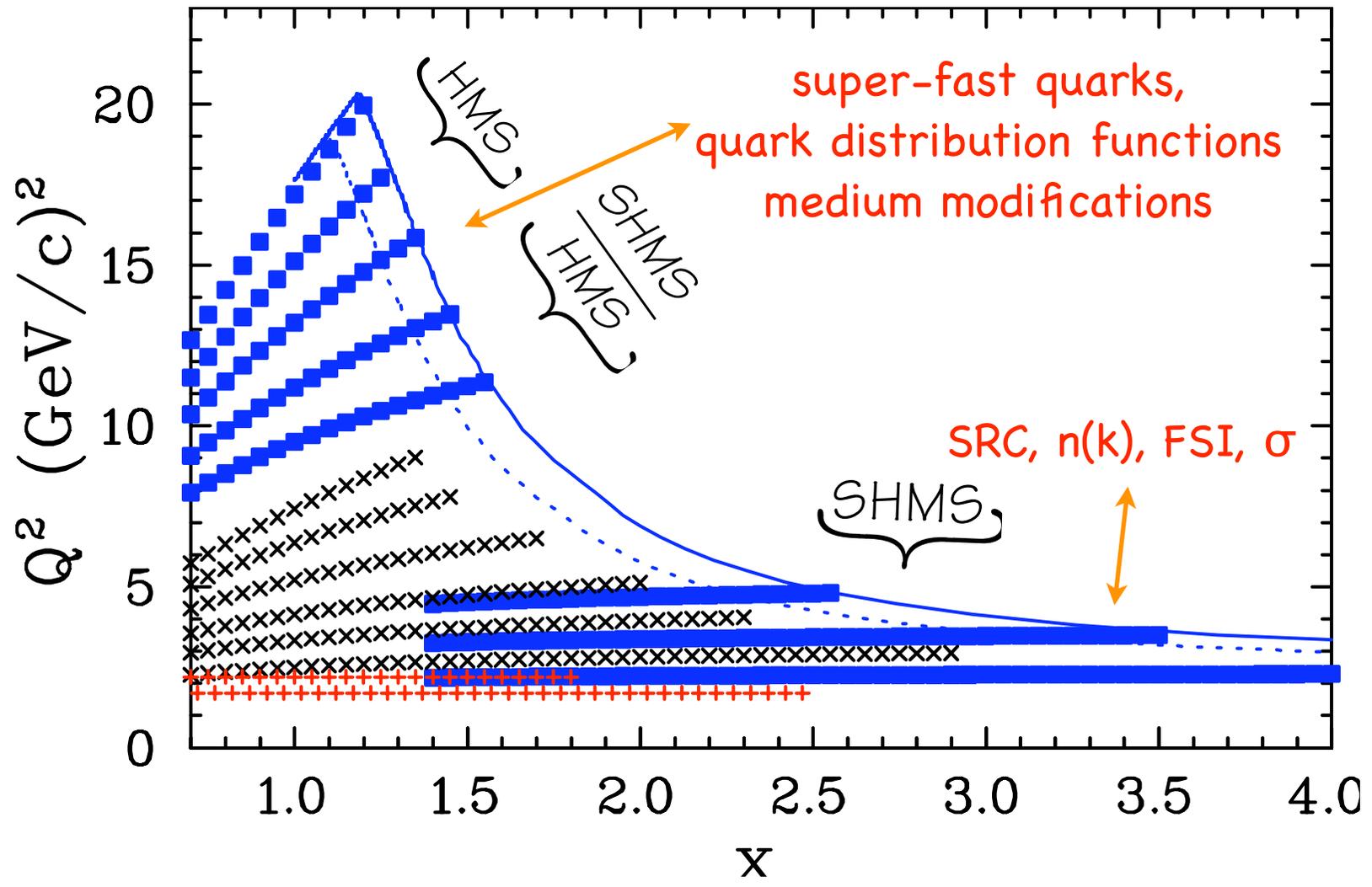
Convolution model

QES

DIS + RR



Kinematic range to be explored



Black - 6 GeV, red - CLAS, blue - 11 GeV

Summary

- High Q^2 scattering at $x > 1$ holds great promise and is not nearly fully exploited.
 - Window on wide variety of interesting physics.
 - Provides access to SRC and high momentum components through y -scaling, ratios of heavy to light nuclei, φ' -scaling
 - Testing ground for EMC models of medium modification, [quark clusters, and other non-hadronic components](#)
- DIS does not dominate over QES at 6 GeV but should be at 11 GeV and at $Q^2 > 10 - 15 \text{ (GeV/c)}^2$.
- Experiments are relatively straightforward. JLAB at 12 GeV will significantly expand the coverage in x - Q^2

Quasielastic Electron Nucleus Scattering Archive

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[Utilities](#)

[Bibliography](#)

[Acknowledgements](#)

Welcome to Quasielastic Electron Nucleus Scattering Archive

In connection with a review article (Quasielastic Electron-Nucleus Scattering, by O. Benhar, D. Day and I. Sick) to be submitted to Reviews of Modern Physics, we have collected here an extensive set of quasielastic electron scattering data in order to preserve and make available these data to the nuclear physics community.

We have chosen to provide the cross section only and not the separated response functions. Unless explicitly indicated the data do not include Coulomb corrections.

Our criteria for inclusion into the data base is the following:

1. Data published in tabular form in journal, thesis or preprint.
2. Radiative corrections applied to data.
3. No known or acknowledged pathologies

At present there are about 600 different combinations of targets, energies and angles consisting of some 19,000 data points.

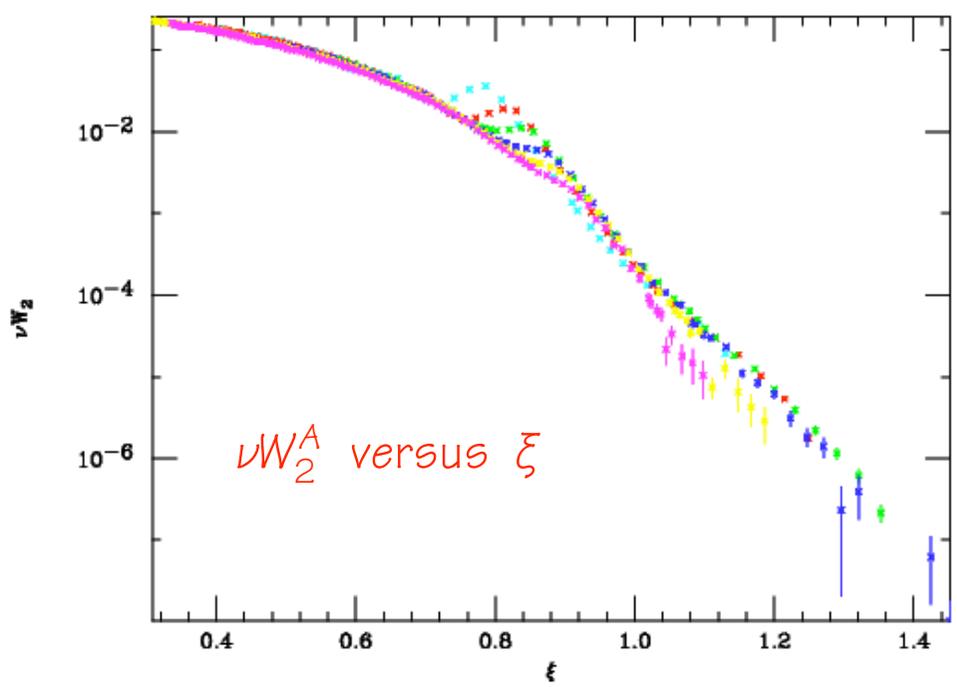
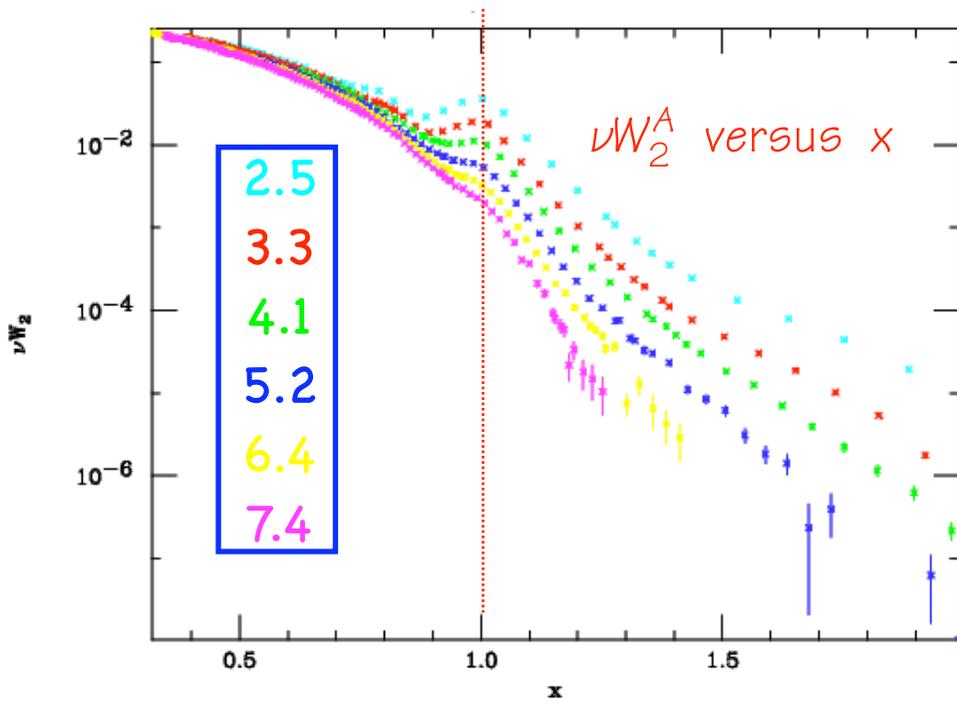
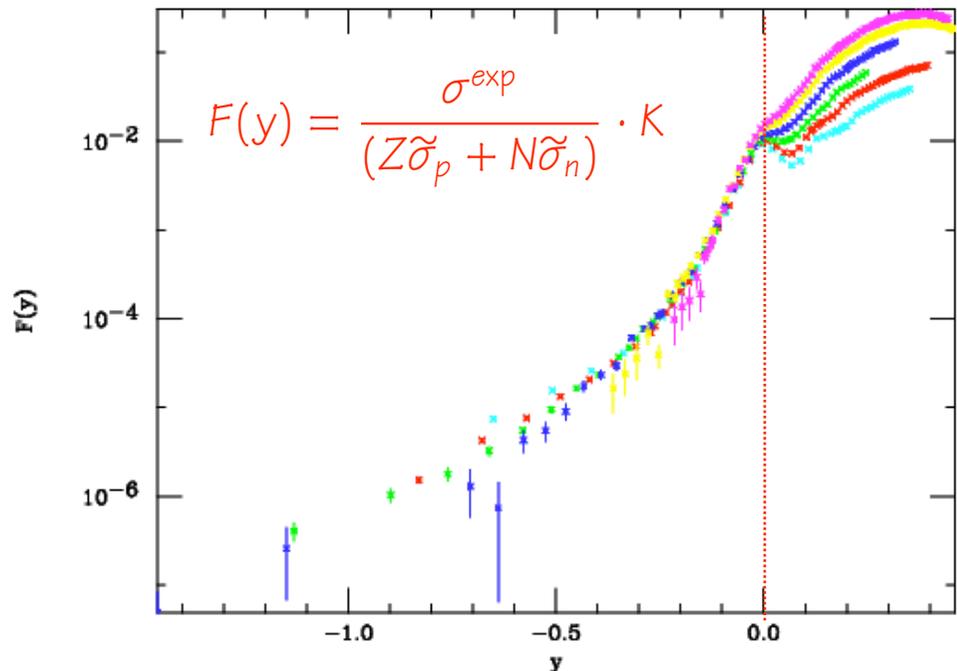
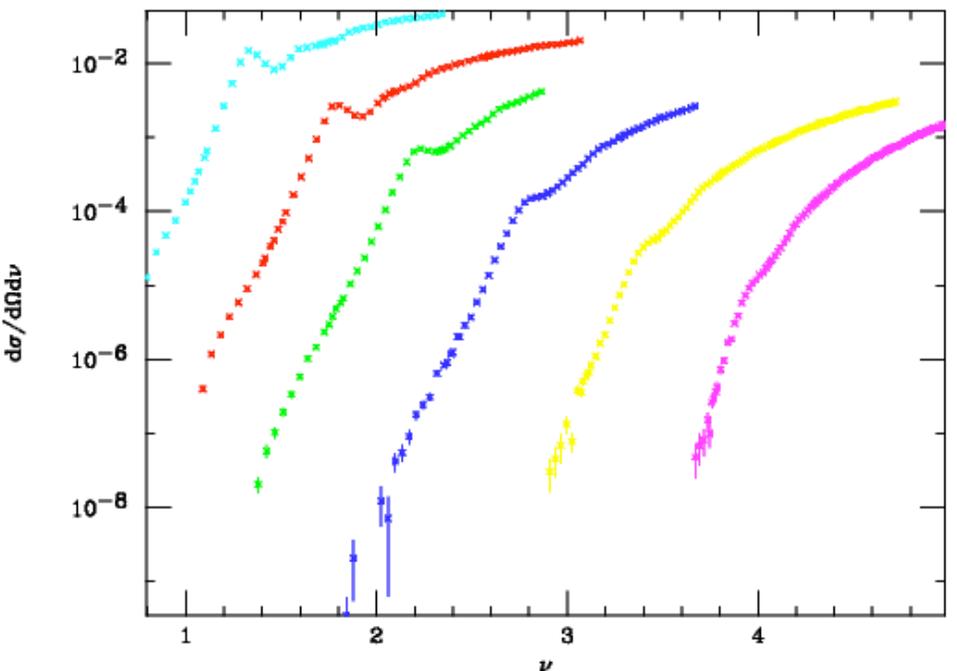
In the infrequent event that corrections were made to the data after the original publications, we included the latest data set, adding an additional reference, usually a private communication.

As additional data become known to us, we will add to the data sets.

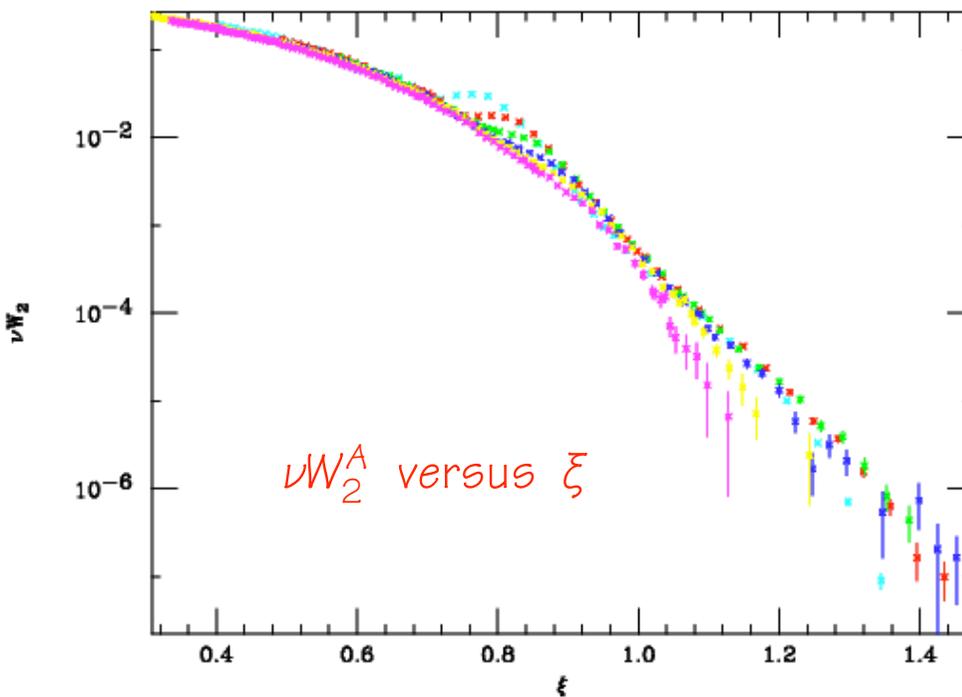
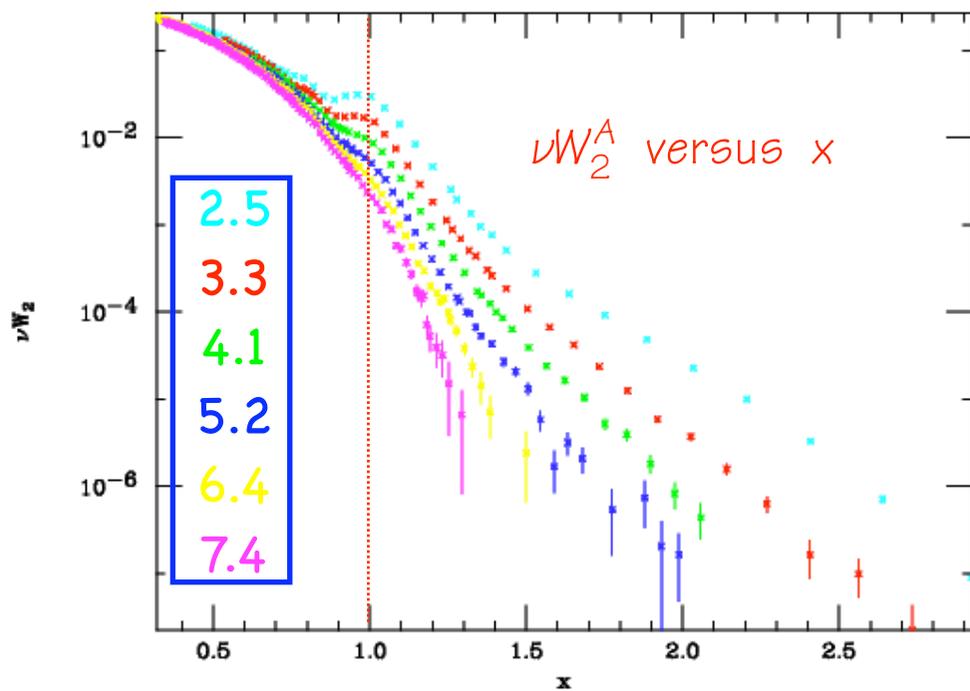
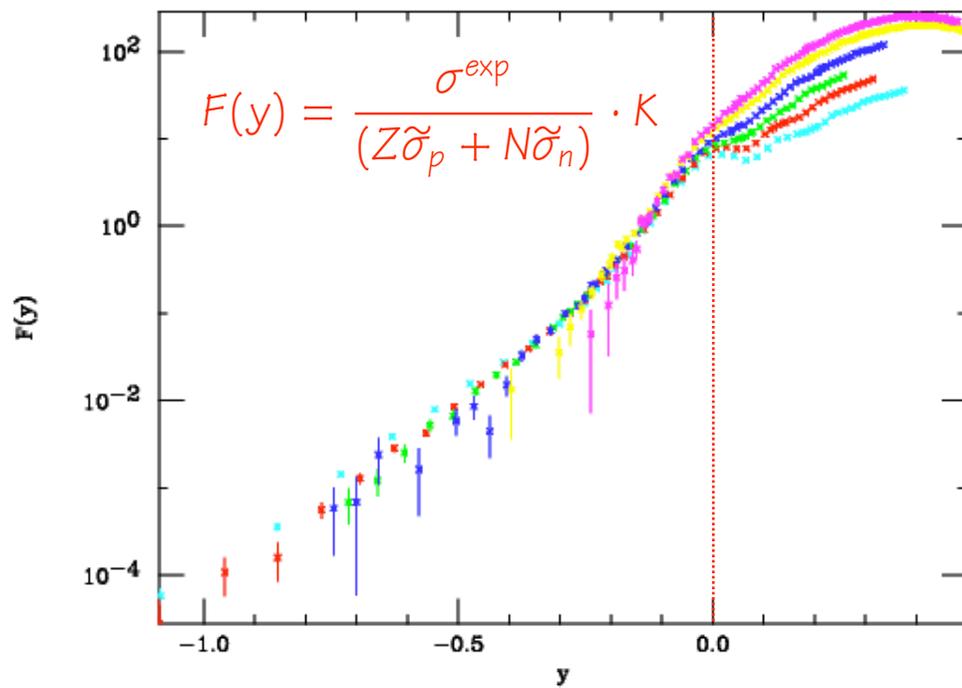
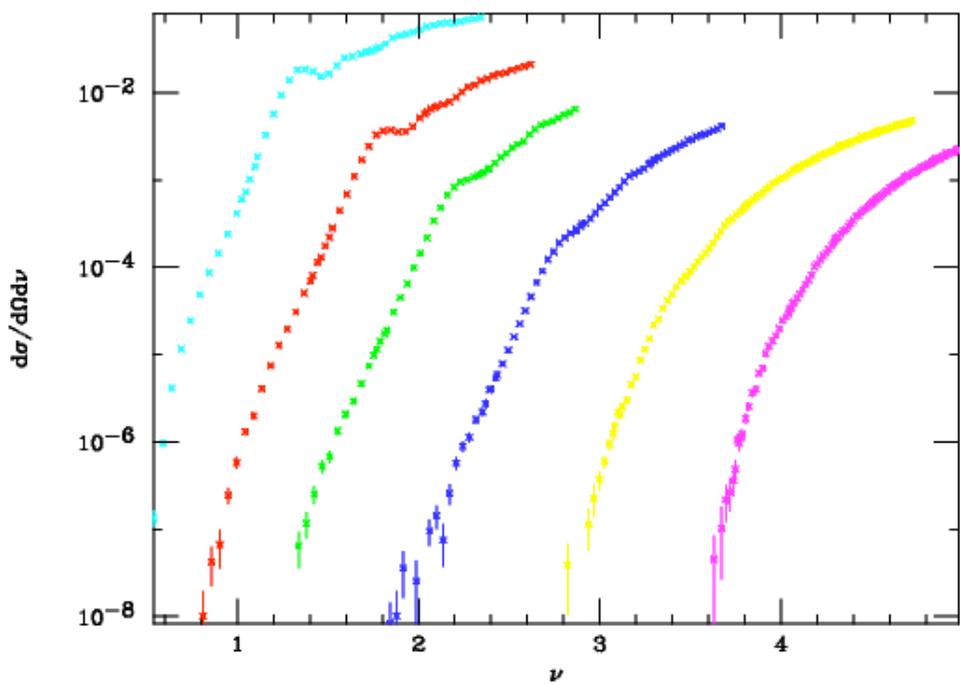
If you wish to be alerted to changes in the archive or to the inclusion of new data, send an email to me. Send any comments or corrections you might have as well.

Preliminary Results - Deuteron

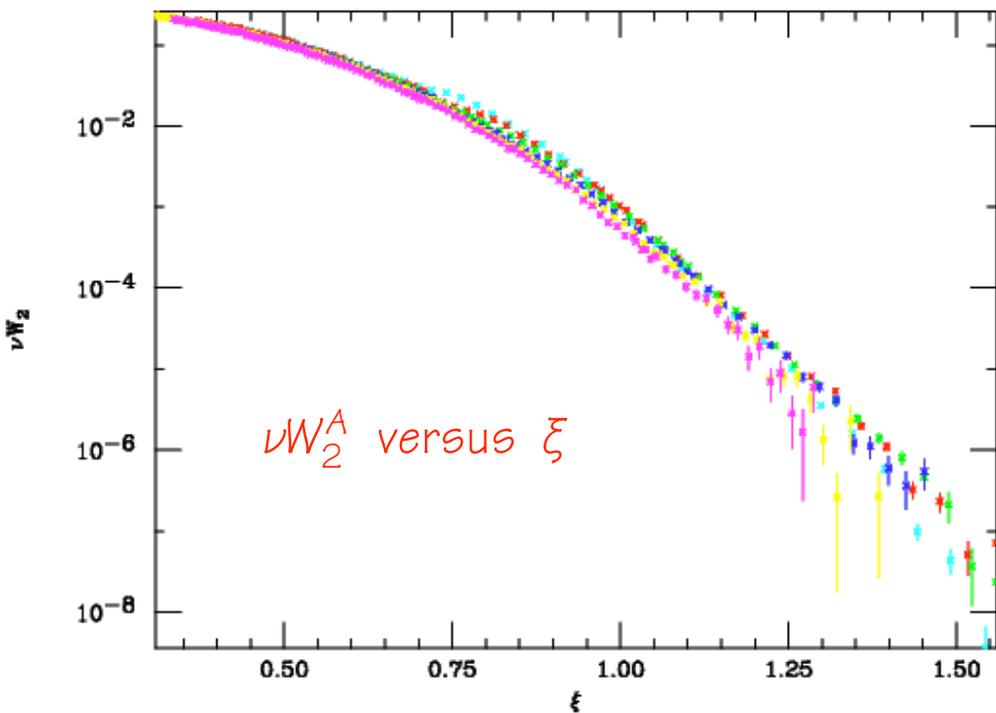
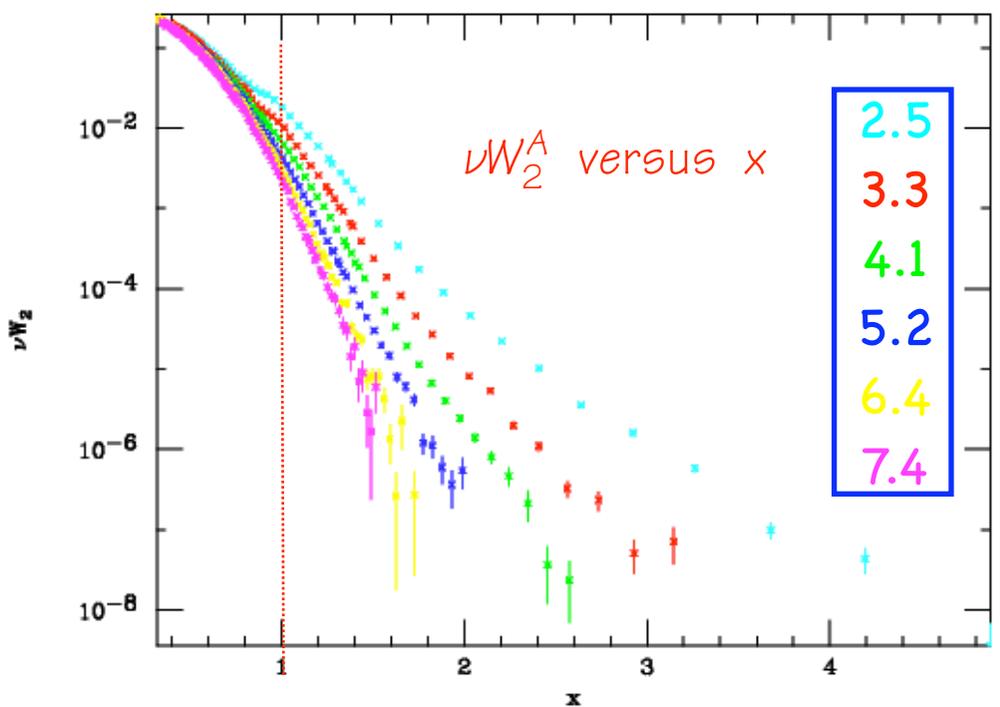
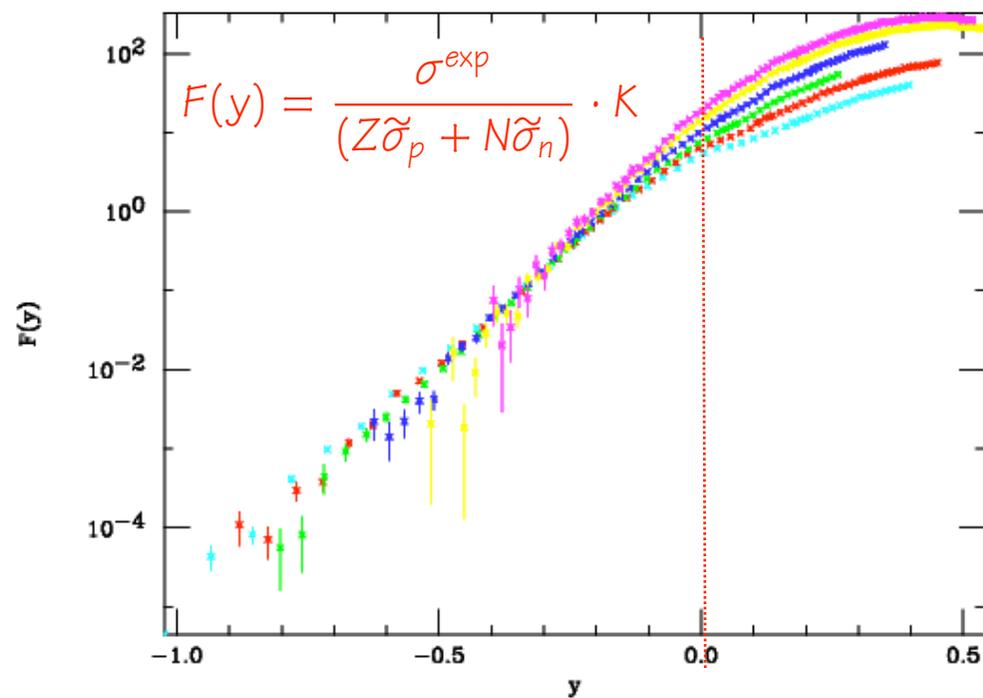
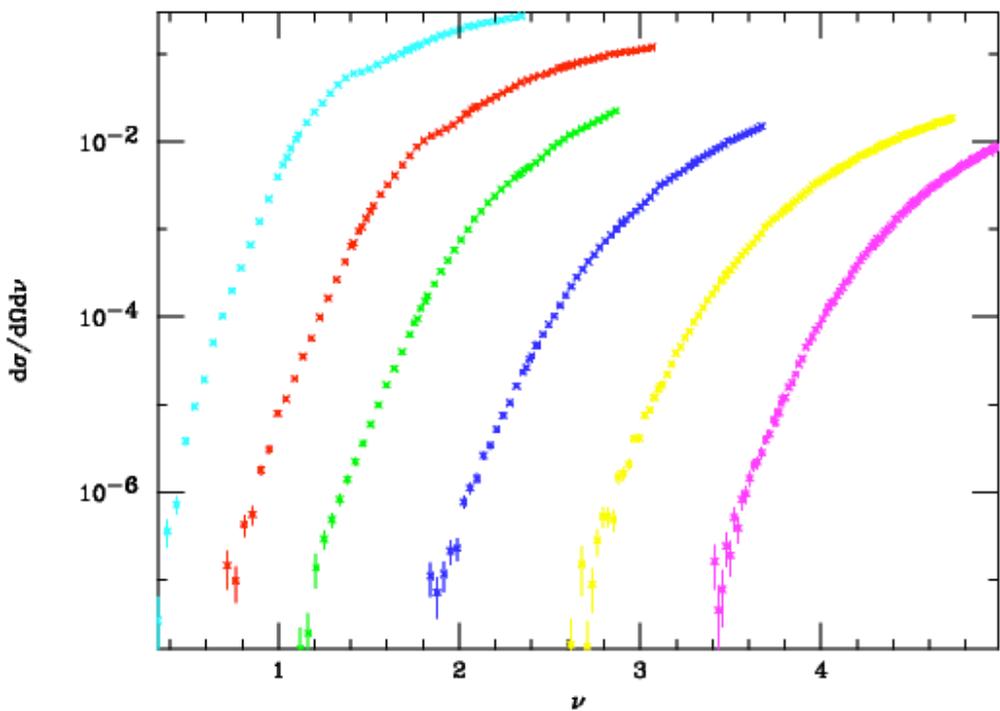
Z, A = 1 2



Preliminary Results - ^3He



Preliminary Results - ^{12}C



Preliminary Results - ^{197}Au

