Electron scattering from nuclei in the quasielastic region and beyond

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Part 1

HUGS 2007

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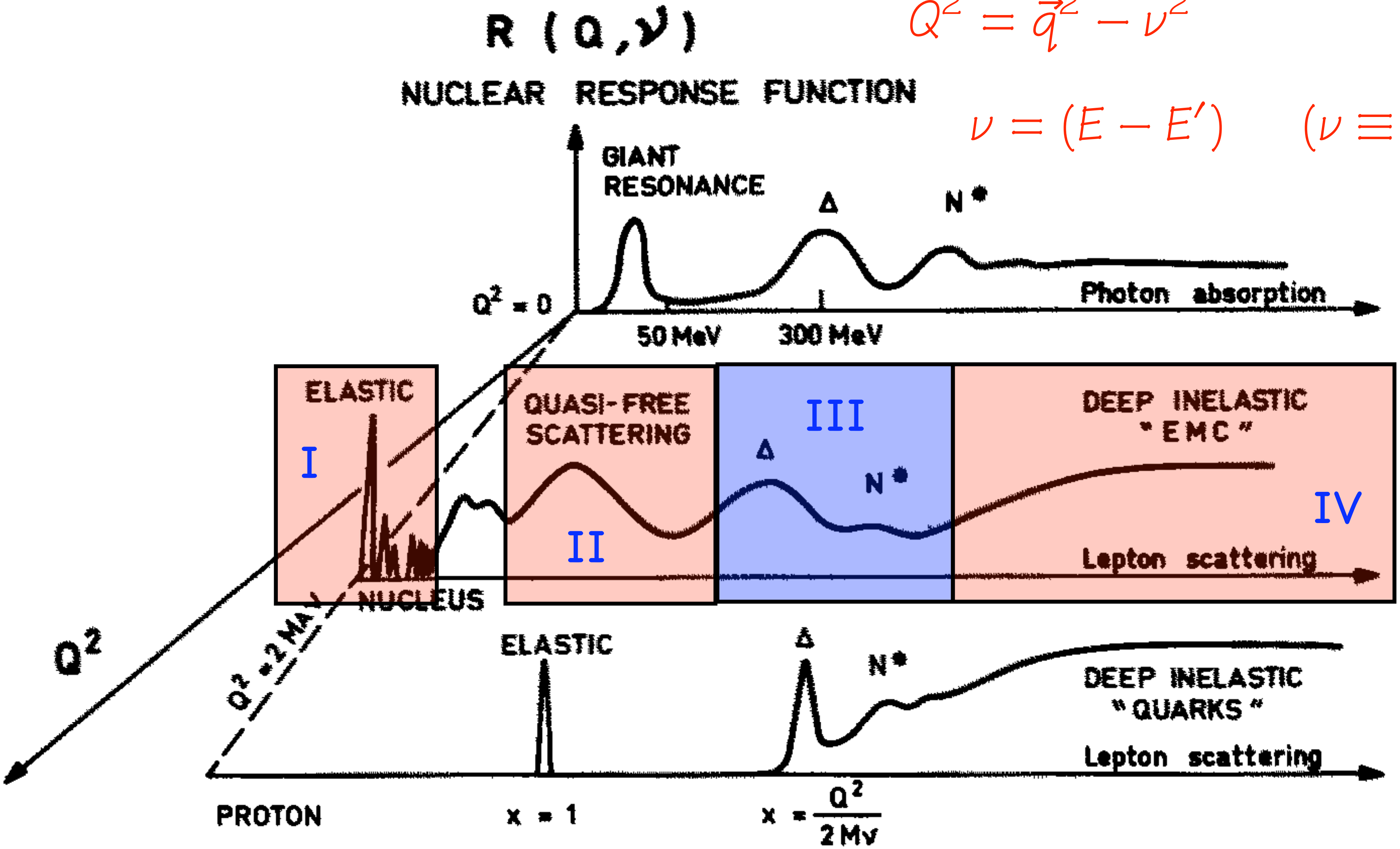
Newport News, VA

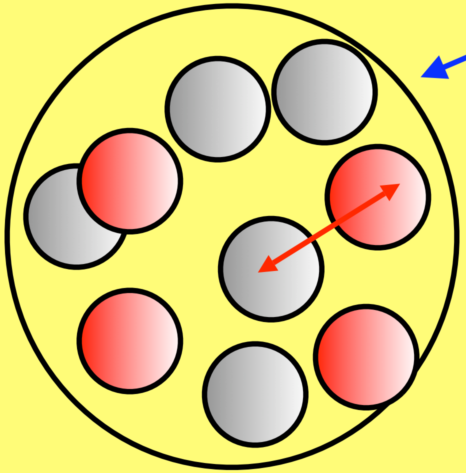
Nuclear Response Function

$$Q^2 = \vec{q}^2 - \nu^2$$

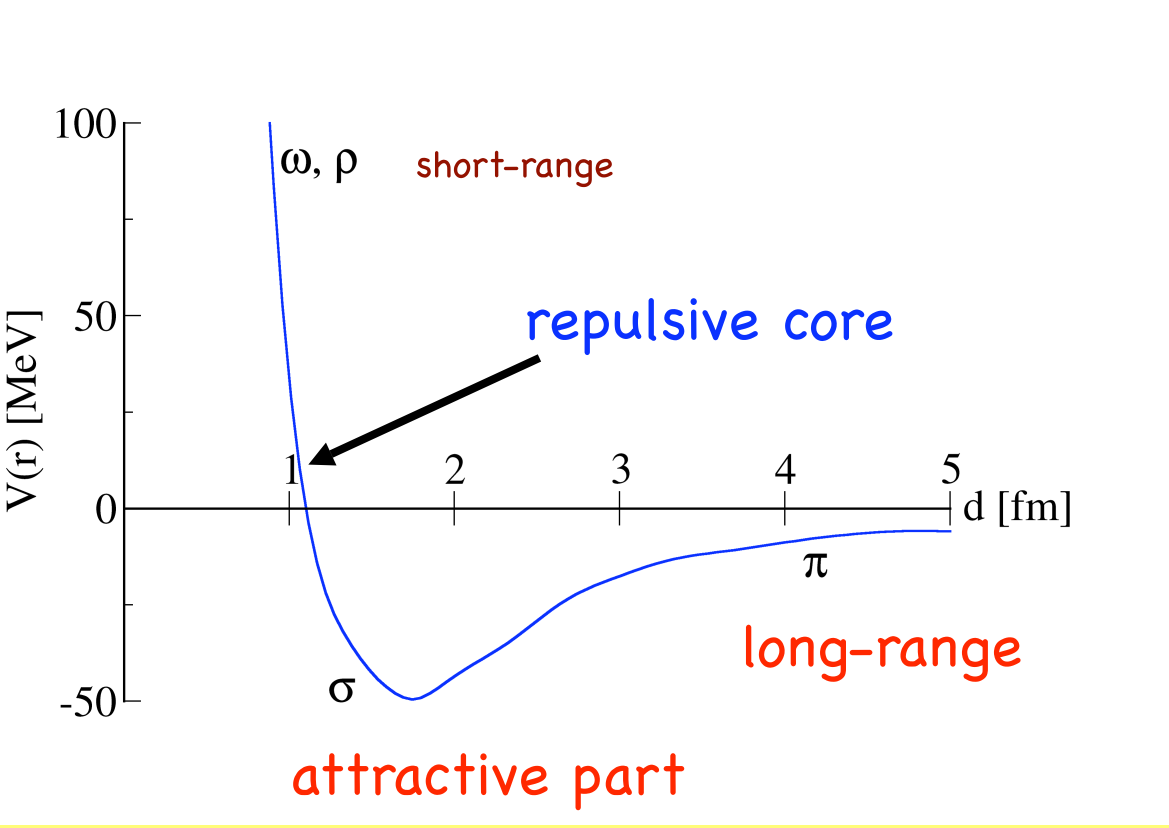
$$\nu = (E - E') \qquad (\nu \equiv \omega)$$



Structure of the nucleus



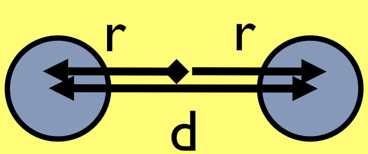
- nucleons are bound
  - energy (E) distribution
  - shell structure
- nucleons are not static
  - momentum (k) distribution



determined by
N-N potential

on average:
binding energy: ~ 8 MeV

distance: ~ 2 fm



How well do we understand nuclear structure ?

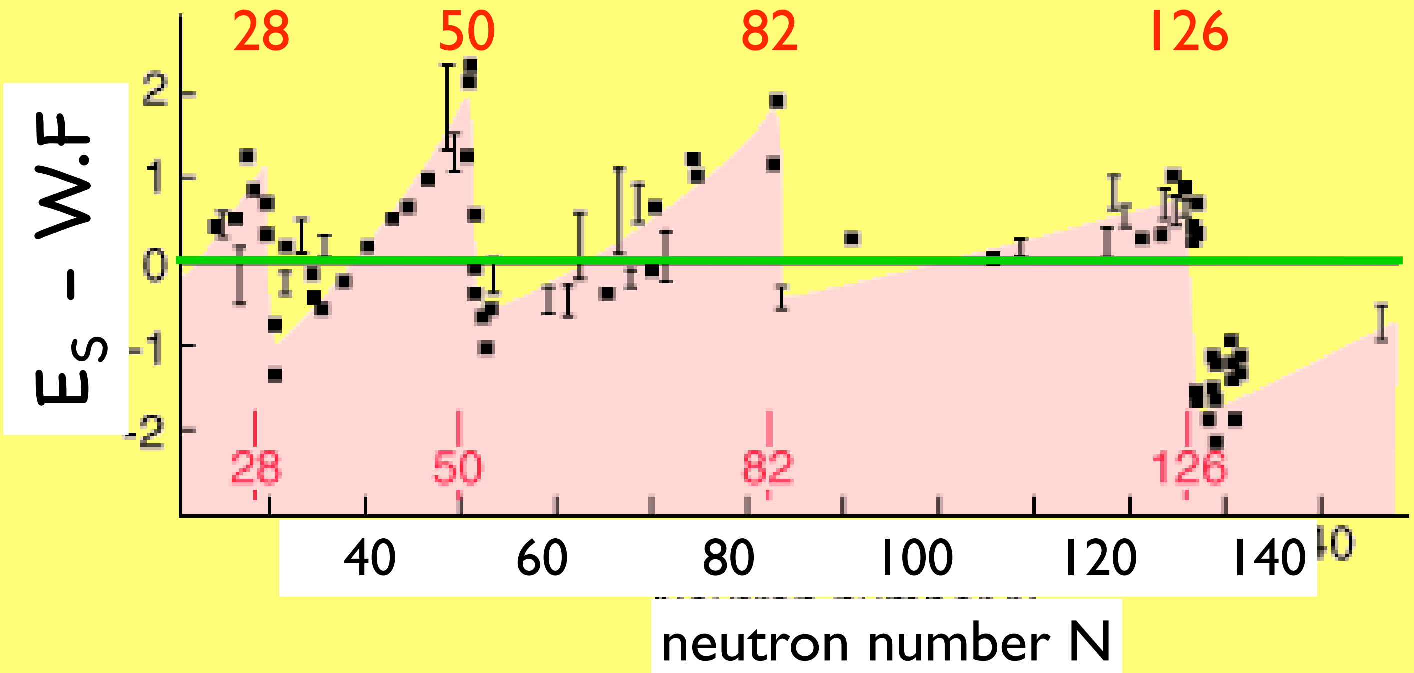
- The shell model
  - Basis upon which most model calculations of nuclear structure rely.
- The underlying physical picture
  - Dense system of fermions whose motions to first order can be treated as independent particles moving in a mean field.
- Electromagnetic interactions
  - Best probe for investigating the validity of the independent particle picture because they are sensitive to a much larger fraction of the nuclear volume

Early hint on shell structure in the nucleus

particular stable nuclei with Z,N = 2, 8, 20, 28, 50, 82, 126
(magic numbers)

↓

large separation energy E_S



"average"
(Weizsäcker Formula,
no shell structure,
bulk properties)

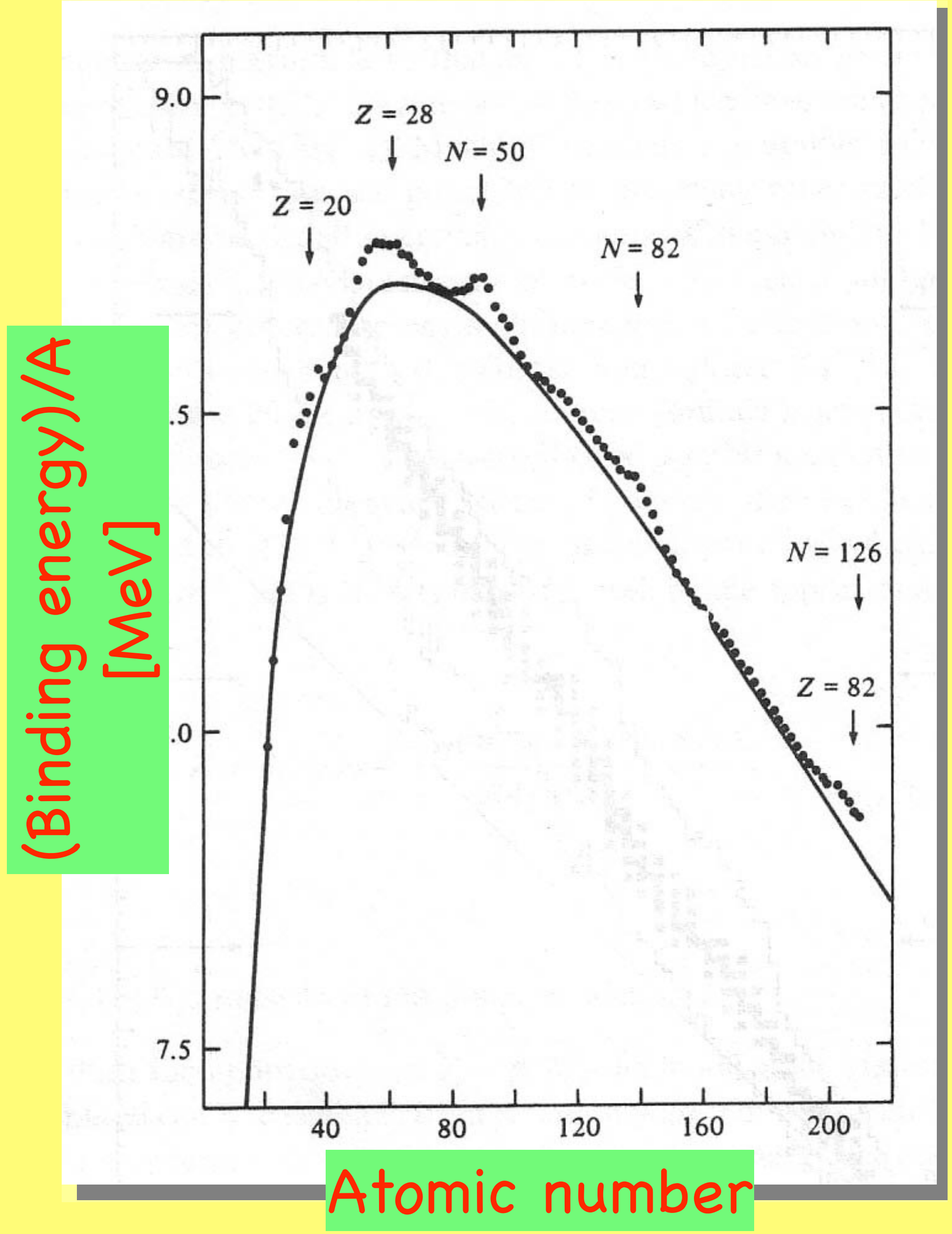
⟶ shell closure

Weizsäcker Formula == Semi Empirical Mass Formula

Large deviations from the SEMF curve at small mass number, e.g. A = 4.

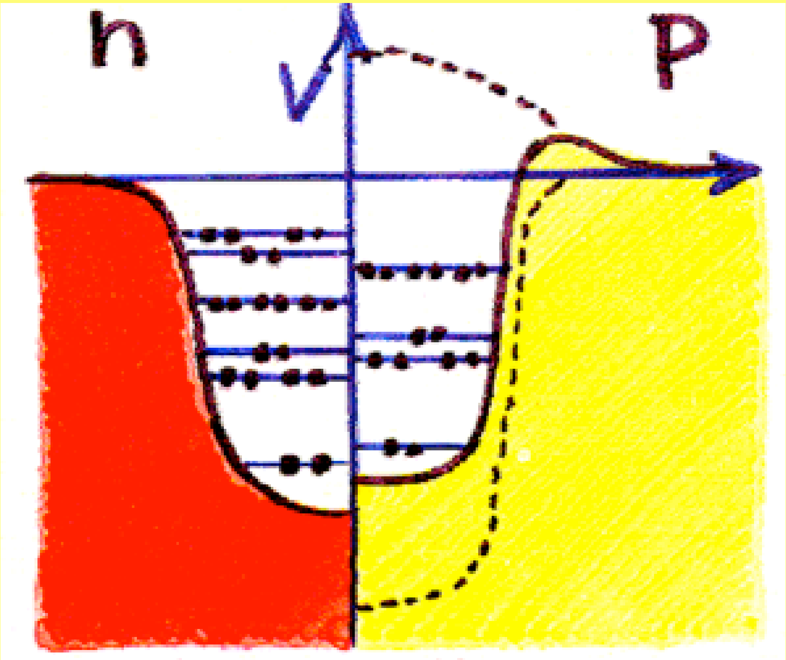
Systematic pattern of deviations occurs, with maxima in B occurring for certain "magic" values of N and Z, given by:
N/Z = 2, 8, 20, 28, 50, 82, 126.

These values of neutron and proton number are anomalously stable with respect to the average –the pattern must therefore reflect something important about the average nuclear potential V(r) that the neutrons and protons are bound in....



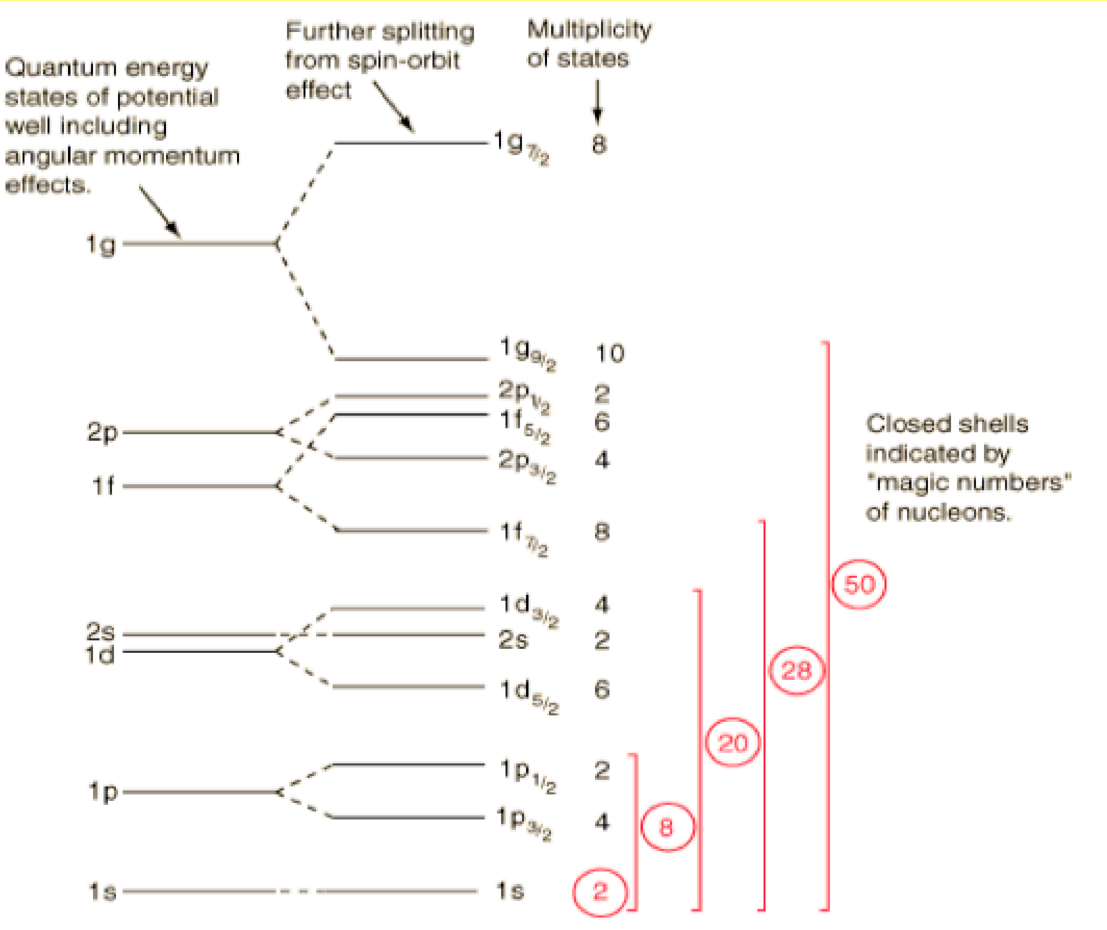
Shell structure (Maria Goeppert-Mayer, Jensen, 1949)

Nobel Prize, 1963



nuclear density $10^{18} kg/m^3$

With the enormous strong force acting between them and with so many nucleons to collide with, how can nucleons possibly complete whole orbits without interacting?



But: there is experimental evidence for shell structure

Pauli Exclusion Principle: → nucleons can not scatter into occupied levels: Suppression of collisions between nucleons

Independent Particle Shell model (IPSM)

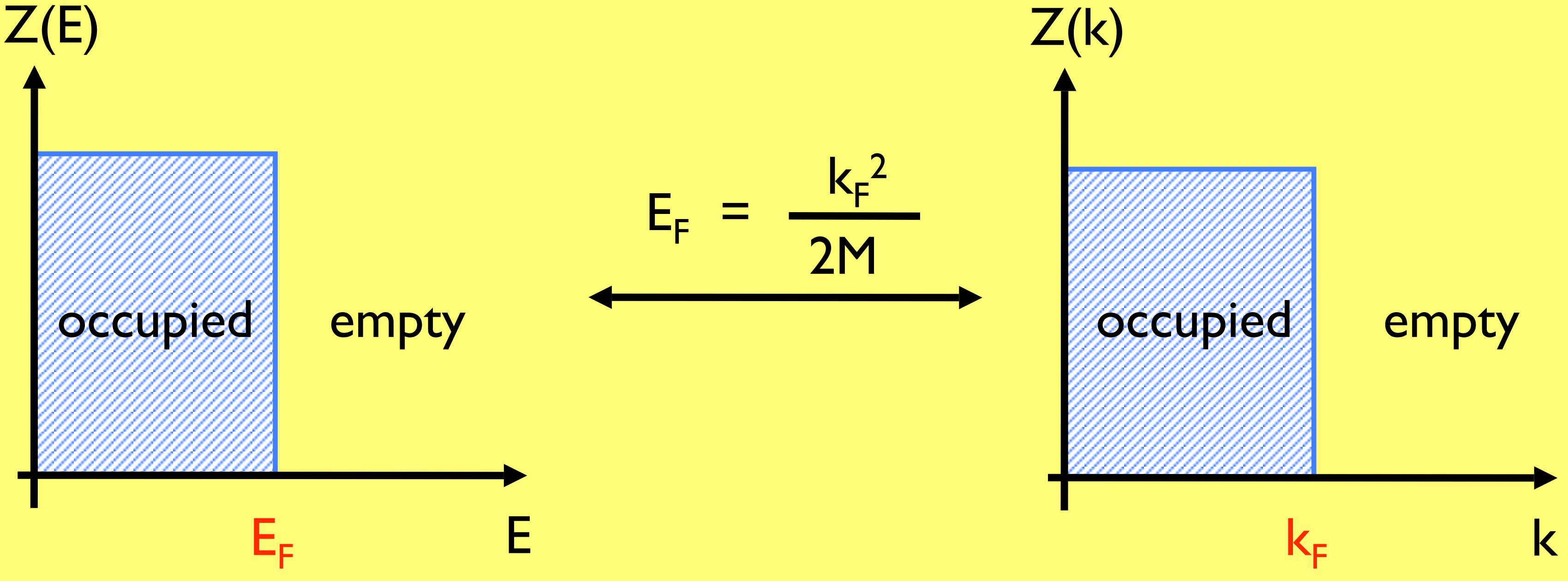
- single particle approximation:

  nucleons move independently from each other
  in an average potential created by the surrounded nucleons (mean field)

  spectral function S(E, k):
  probability of finding a proton with initial momentum k and energy E in the nucleus

- factorizes into energy & momentum part

nuclear matter:



$$E_F = \frac{k_F^2}{2M}$$

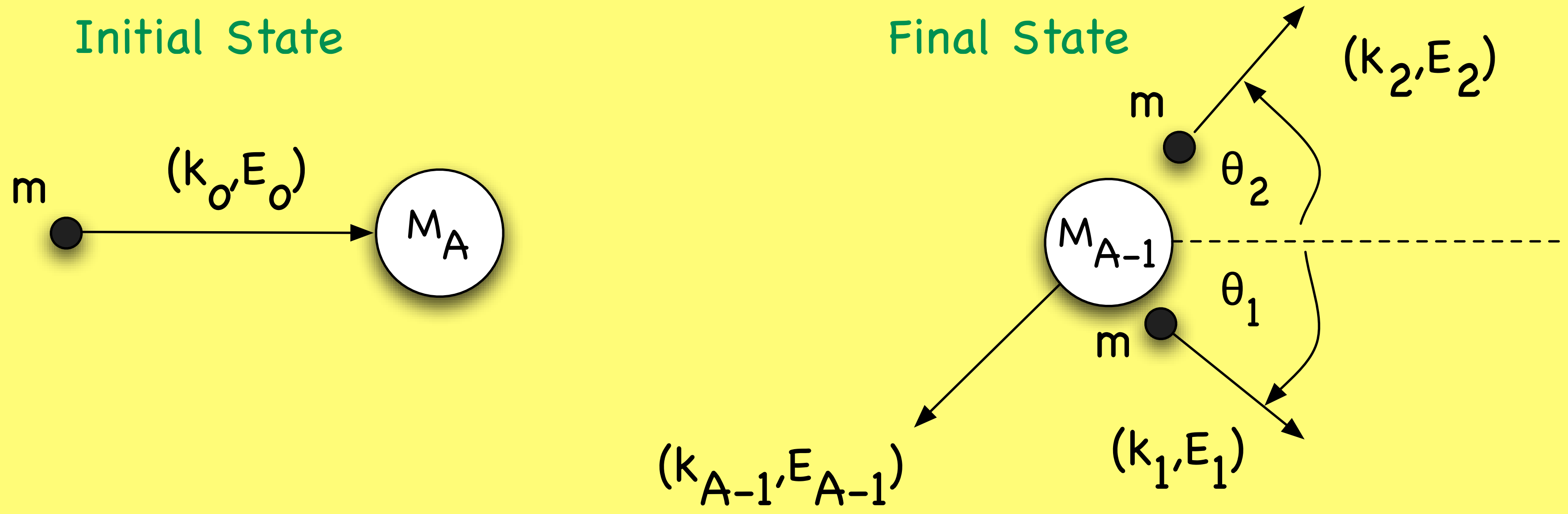
IPSM

- Simple model yet excellent first approximation to structure of the nucleus
- The single-particle energies ξ_α and wave function Φ_α are the basic quantities in IPSM
- In high energy knockout reaction we can directly measure ξ_α and Φ_α
- Observed first in Uppsala in 1957 in (p,2p) reactions on $^{12}C(p,2p)^{11}B$

$$S(\vec{p},E) = \sum_{i} \mid \Phi_a(p) \mid^2 \delta(E+\epsilon_a)$$

The spectral function should exhibit a structure at fixed energies with momentum distributions characteristic of the shell (orbit).

Quasi free Knockout Reactions



$$\vec{k}_{A-1} = \vec{k}_o - \vec{k}_1 - \vec{k}_2 = -\vec{k}$$

Momentum of particle in target nucleus that is knocked out

$$E_s = T_1 + T_2 + T_{A-1} - T_o$$

Separation energy (missing energy)

Energy required for separation of the nucleon from the target nucleus [Includes possible excitation of residual nucleus]

Tyren, Hillman & Maris ($p+C^{12} \rightarrow p + p + B^{11}$) Uppsala in 1957

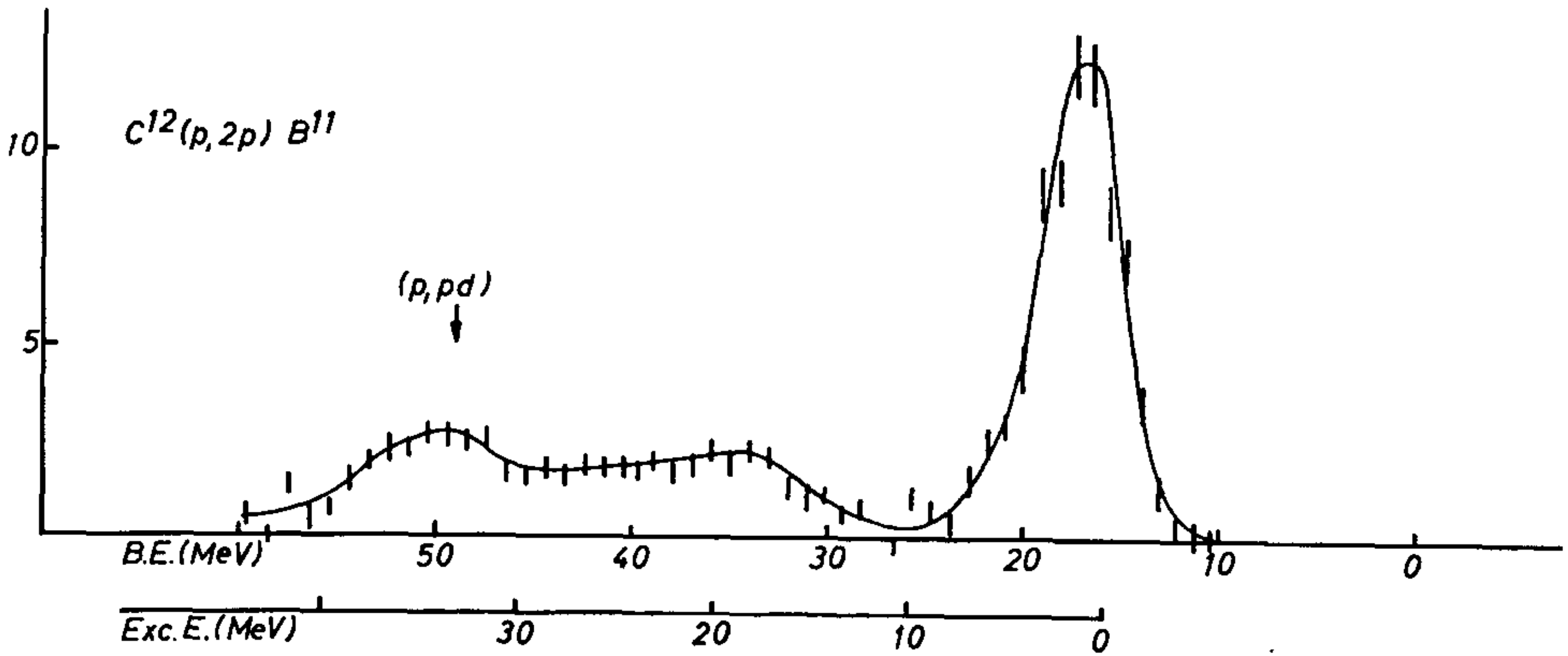


Fig. 4 and Fig. 5. Absolute cross sections for the (p, 2p) reaction at 185 MeV versus binding energy of the removed proton. Separate energy scales for each target also show the corresponding excitation energy of the residual nuclei. Relative errors should not be much larger than the statistical errors shown on each point; absolute errors should be less than 40 %, and the error in comparing two spectra somewhat less.

(p,2p) experiments provided information on the binding energies of the inner shells of nuclei and their momentum. These experiments suffered from distortion of the proton (strongly interacting): Jacob and Maris (1966) suggested using high energy electrons - nucleus is almost transparent to them

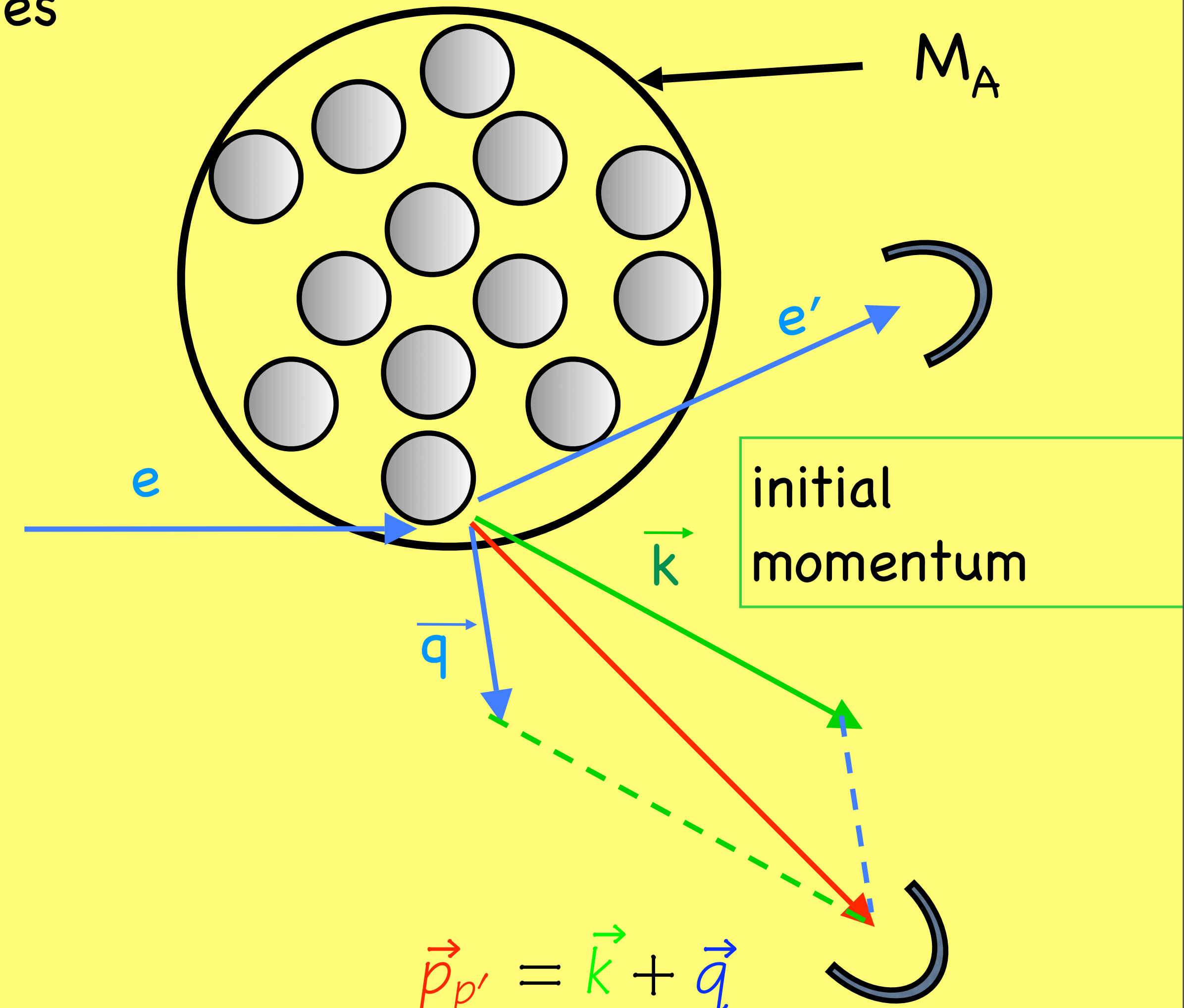
(e,e′p)-reaction: coincidence experiment

measured values: momentum, angles

electron energy: E_e

proton: $\vec{p}_{p'}$

electron: $\vec{k}_{e'}$, $E_{e'} = |\vec{k}_{e'}|$



reconstructed quantites:

missing energy:

$$E_m = E_e - E_{e'} - T_{p'} - T_{A-1}$$

missing momentum:

$$\vec{p}_m = \vec{q} - \vec{p}_{p'}$$

$$\vec{p}_{p'} = \vec{k} + \vec{q}$$

in PWIA:

direct relation between measured quantities and theory:

$$|\,E\,| \equiv E_m \quad \vec{k} \equiv -\vec{p}_m$$

IA and IPSM

The QFS reaction cross section

$$\frac{d\sigma^{fi}}{dE_1 d\Omega_1 dE_2 d\Omega_2} = KS(\vec{k}, E)\frac{d\sigma^{free}}{d\Omega}$$ factorized

Other reaction proportional S(p,E) are single nucleon pickup [(p,d), (d, ^{3}He), (γ,p)]

Provides complimentary information but... strong absorption in nucleus hinders mapping out the spectral function.

IA and IPSM is a considerable simplification

- Assumption that asymptotic p_m and E_m are equal to values just before knockout
- Elementary reaction = free
- No FSI

Factorized form is preserved when strong interaction effects are considered - DWIA

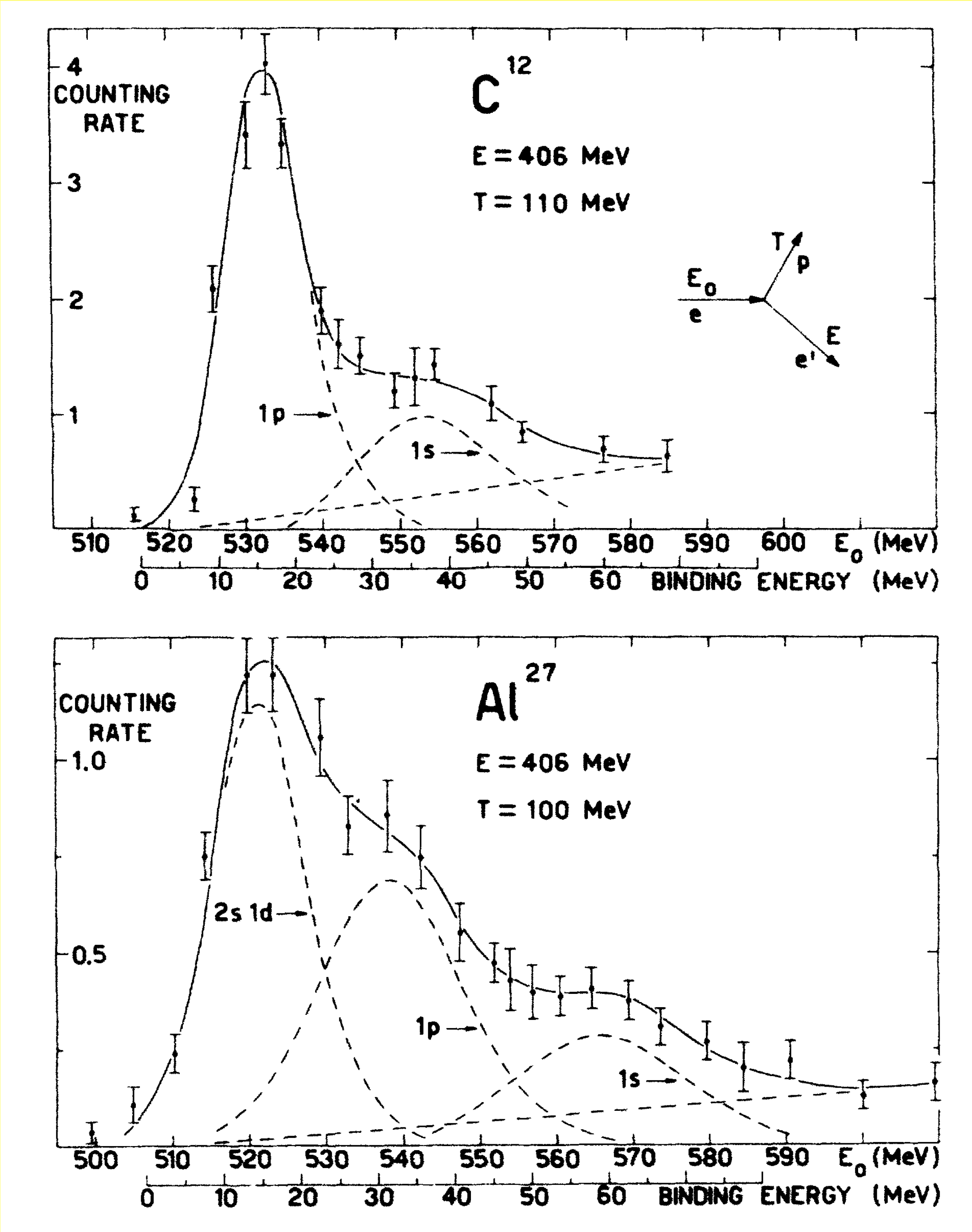
The first (e,e'p) measurement: identification of different orbits

Frascati Synchrotron, Italy

U. Amaldi, Jr. et al., Phys. Rev. Lett. 13, 341 (1964).

$^{12}C(e,e'p)$

$^{27}Al(e,e'p)$



moderate resolution:

FWHM: 20 MeV

Saclay

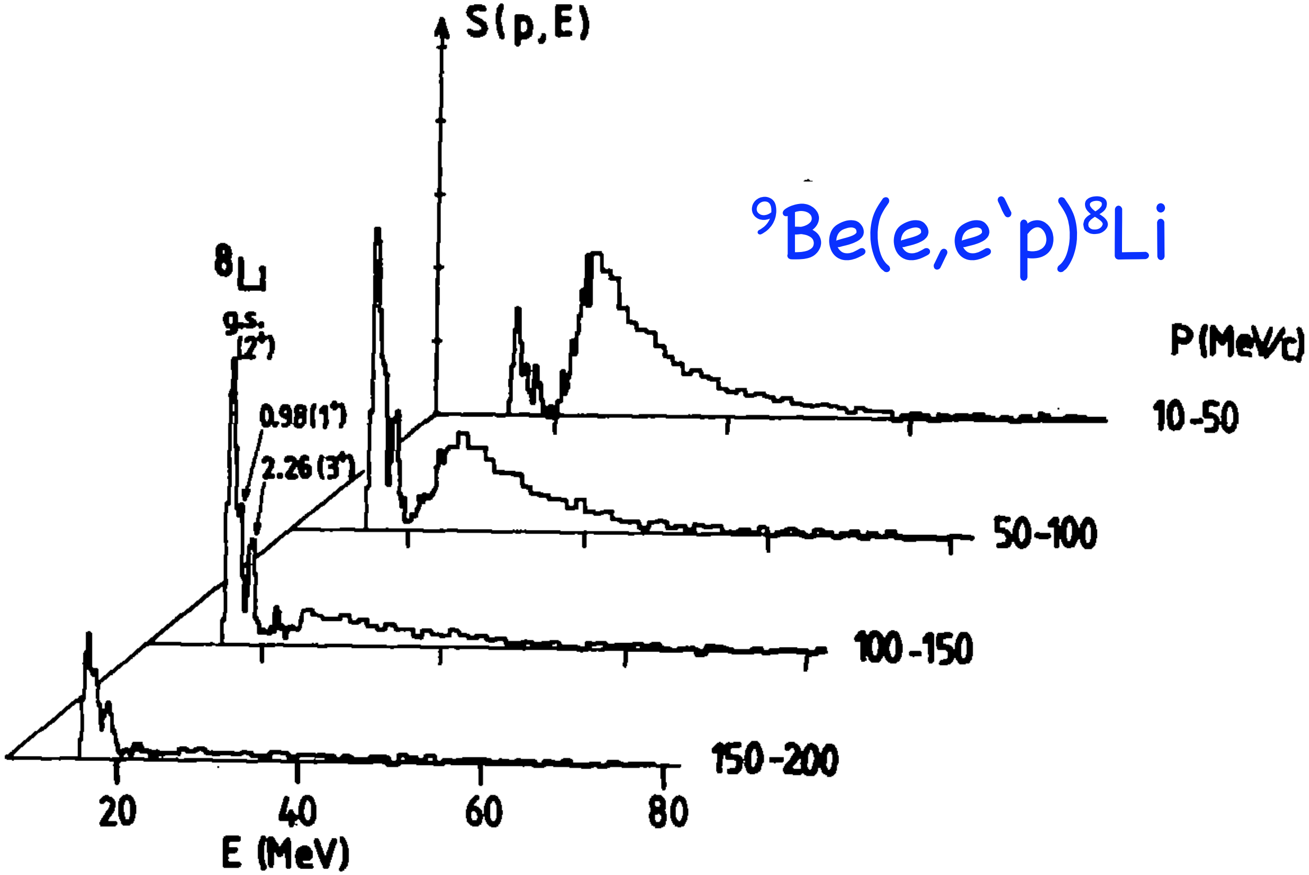
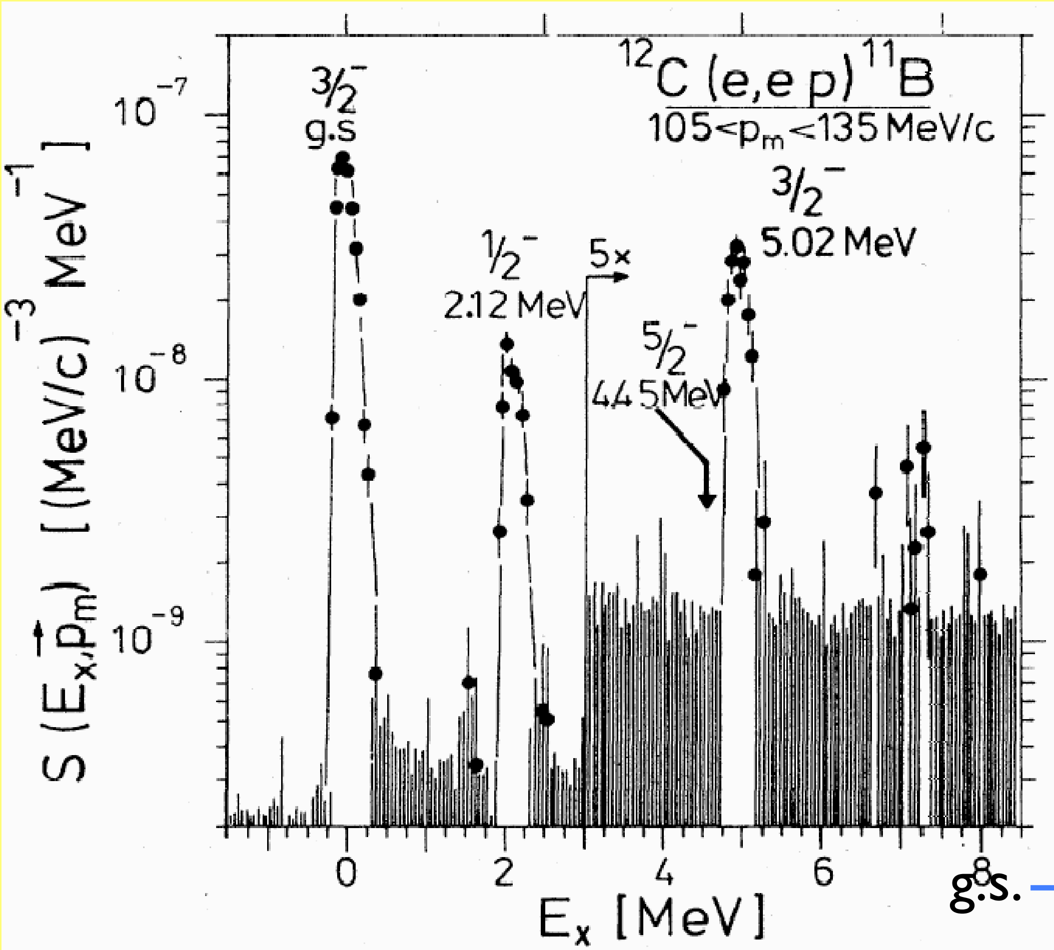


Fig. 10. Proton separation energy spectra for the $^9Be(e,e'p)$ reaction, within different recoil momentum bins. The energy resolution of ~ 0.9 MeV renders visible some different excited states of ^8Li at low separation energy. Data have been corrected for radiative effects, but the overall absolute scale is arbitrary.

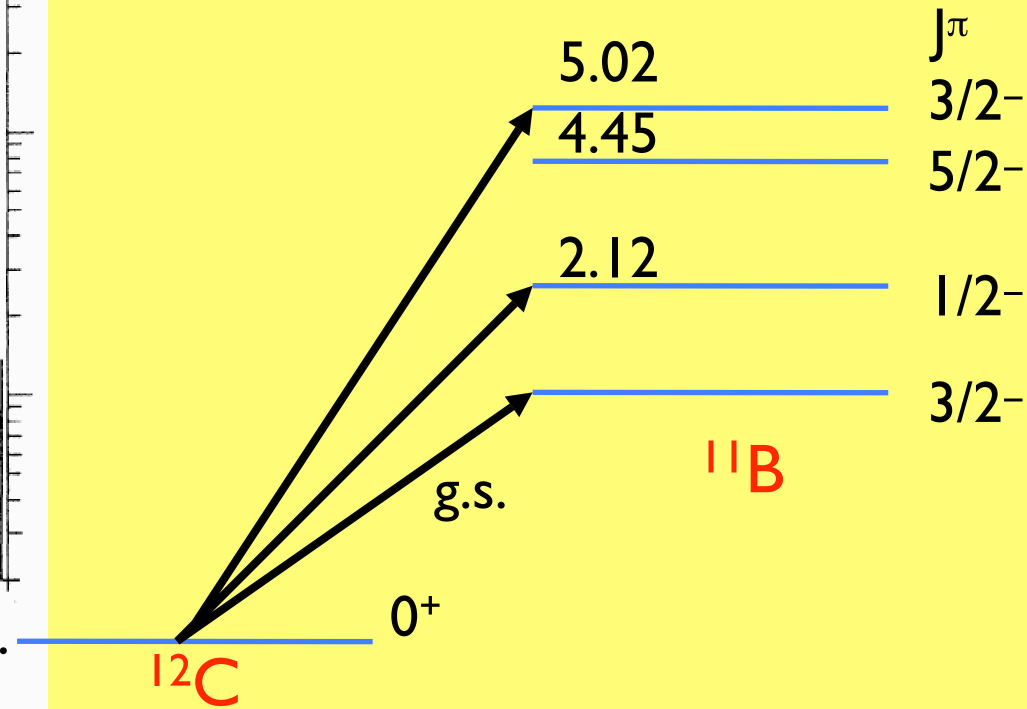
Characteristic momentum behavior of the s and p shells can be clearly seen. J. Mougey ``The (e.e'p) reaction" Nuclear Physics A Volume 335, (1980) 35-53

$$S(\vec{p},E) = \sum_i |\Phi_a(p)|^2 \delta(E+\epsilon_a)$$

NIKHEF: resolution 150 keV



shape described by Lorentz function with central energy E_α + width Γ_α

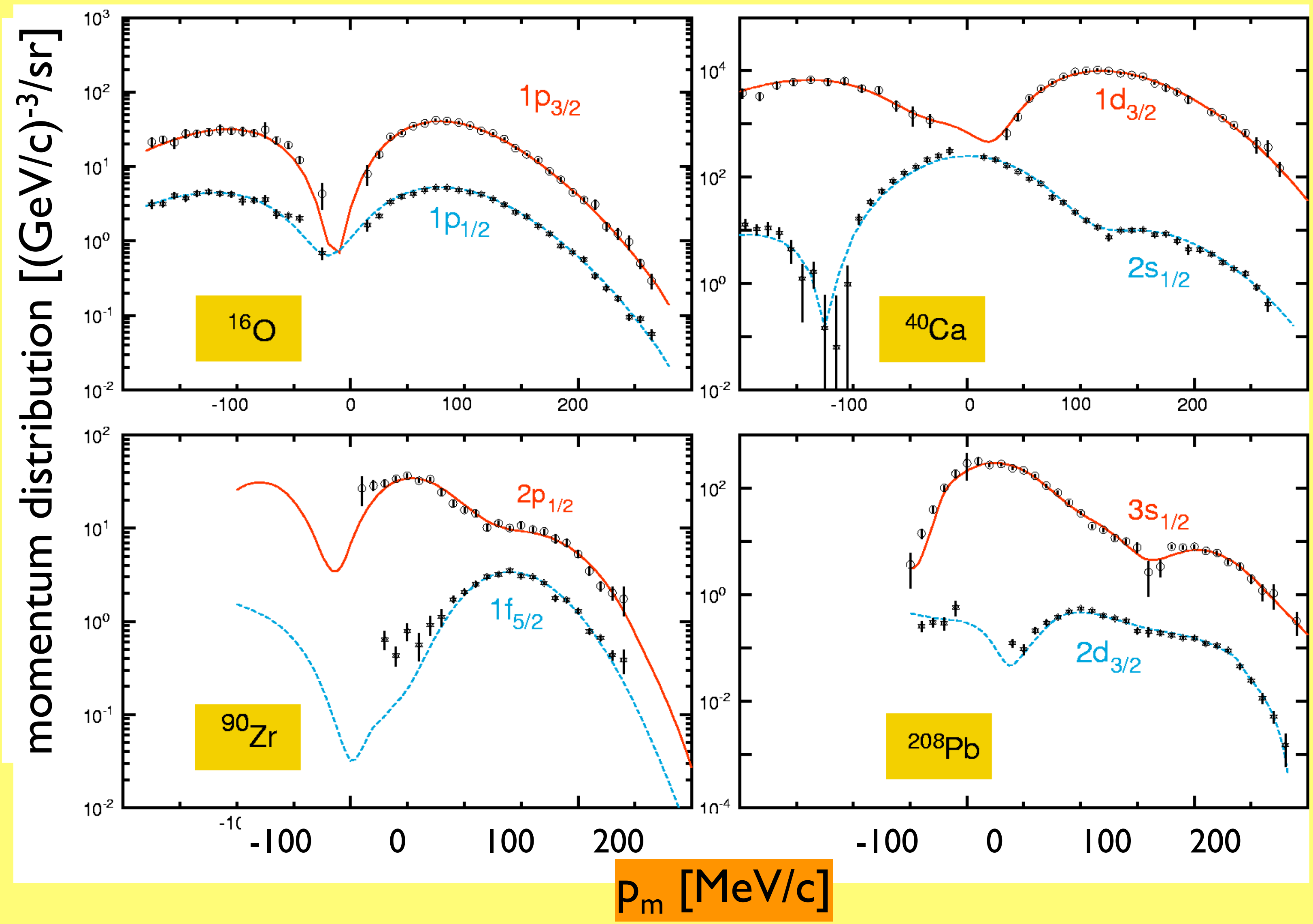


Steenhoven et al., PRC 32, 1787 (1985)

==> electrons are a suitable probe to examine the nucleus

Shell Model: describes basic properties like

spin, parity, magic numbers ...



NIKHEF results

Momentum distribution:

- characteristic for shell (l, j)
- Fourier transformation of Ψ_{lj} (r) ==> info about radial shape

Theory on previous slide (solid line):

Distorted wave impulse approximation (DWIA)

solves the Schrödinger equation using an

optical potential (fixed by p-^{12}C) (Hartree-Fock, self-consistent)

real part: Wood-Saxon potential

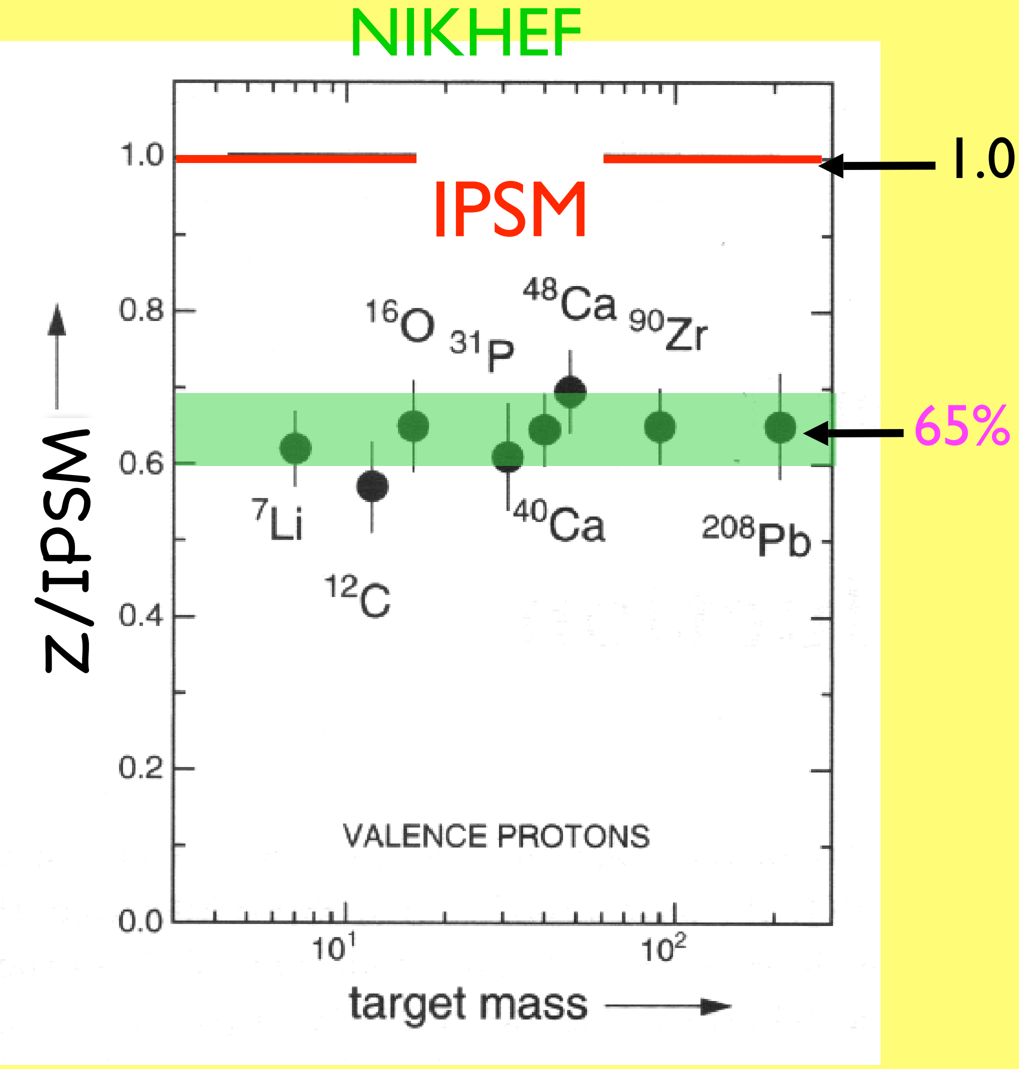
imaginary part: accounts for absorption in the nucleus

Correction for Coulomb distortion

→ well reproduced shape

strength of the transition smaller!

Number of nucleons in each shell (IPSM): = 2j + 1

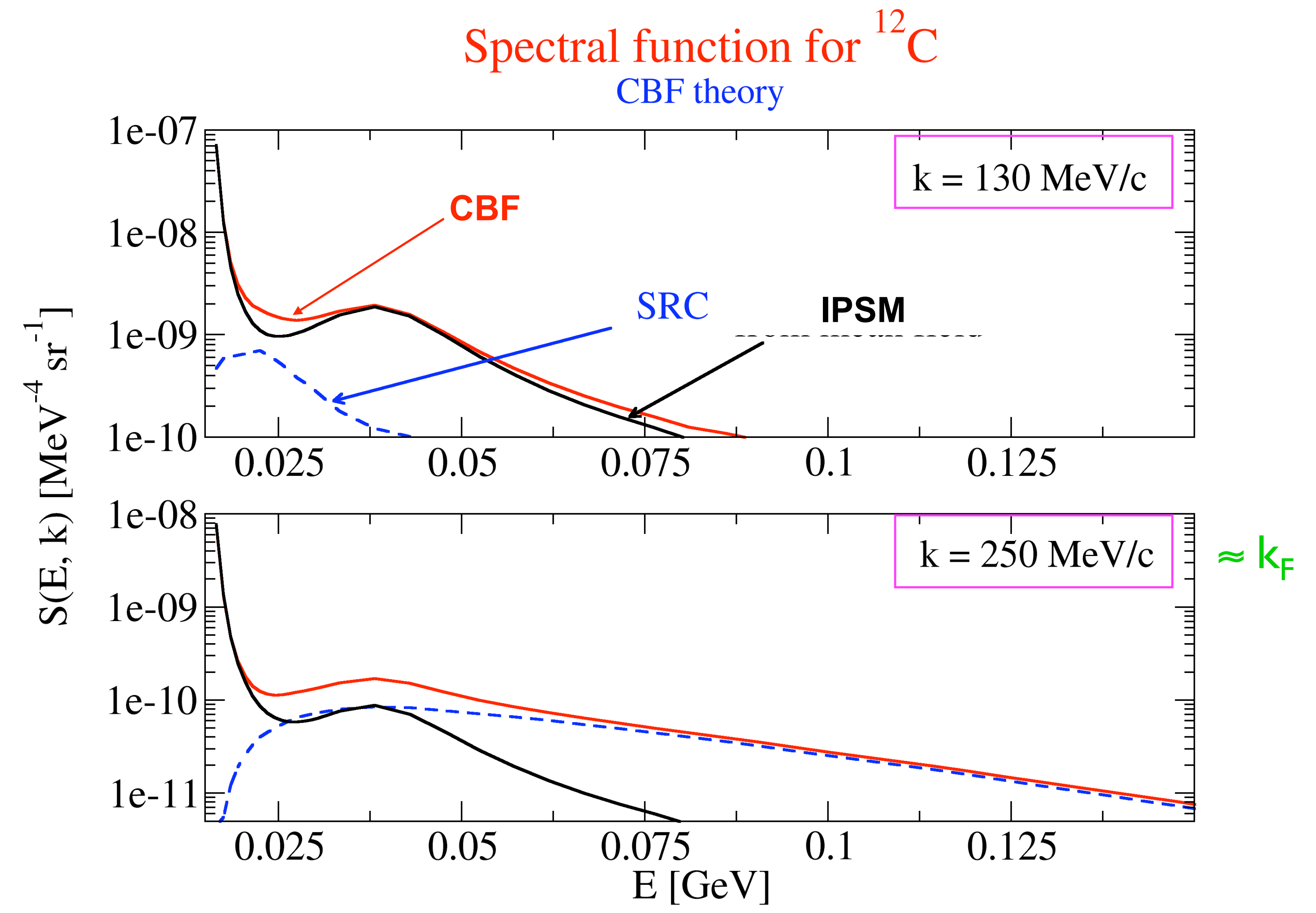


Spectroscopic factor Z_α

$$Z_a = 4\pi \int^{k_f} dE\, dk\, k^2 S(k,E)$$

single particle state α

= number of nucleons in shell



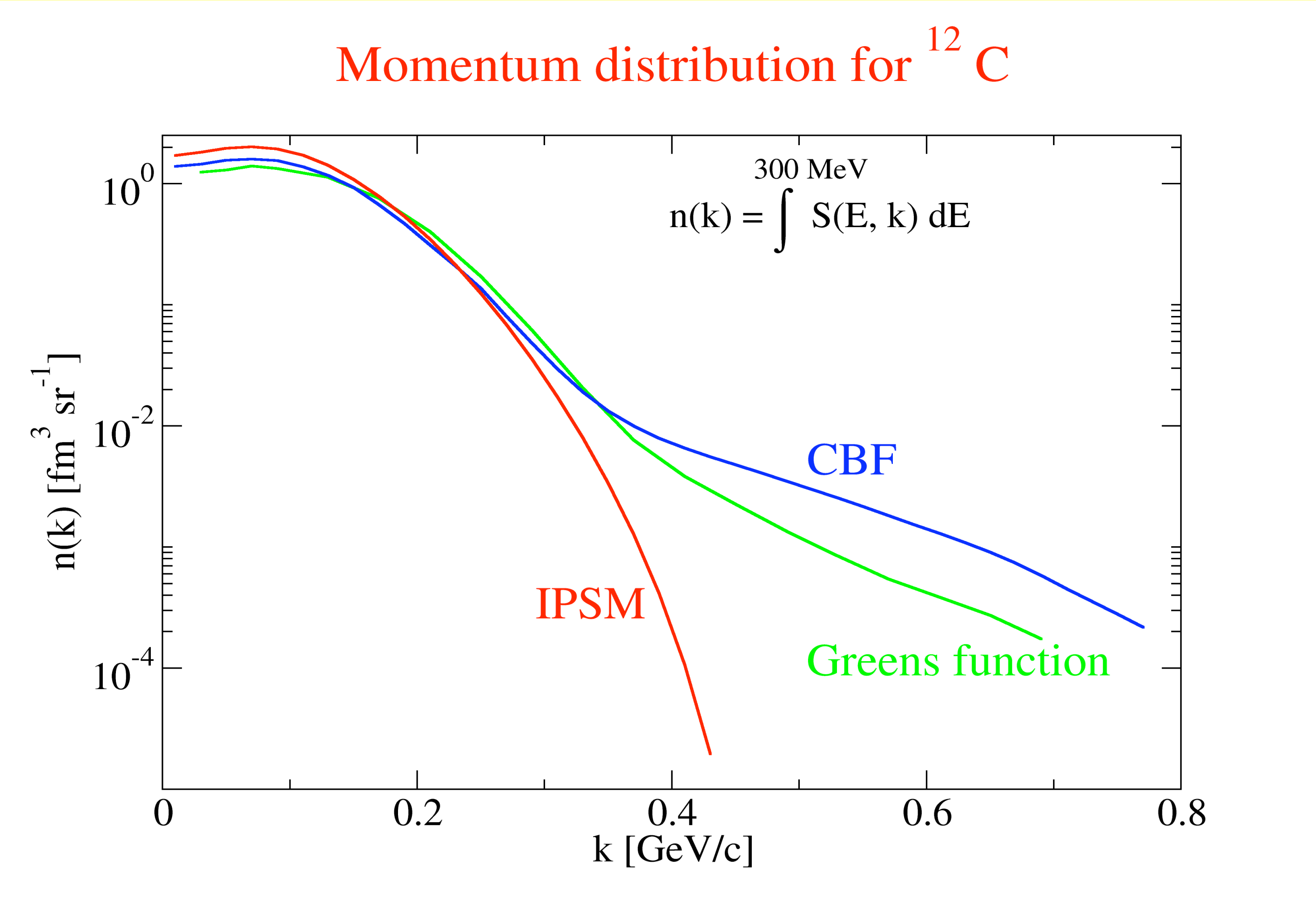
$k < k_F$: single-particle contribution dominates

$k \approx k_F$: SRC already dominates for E > 50 MeV

$k > k_F$: single-particle negligible

consequence: search for SRC at large E, k

method: (e,e′p)-experiment



signature of SRC: additional strength at high momentum

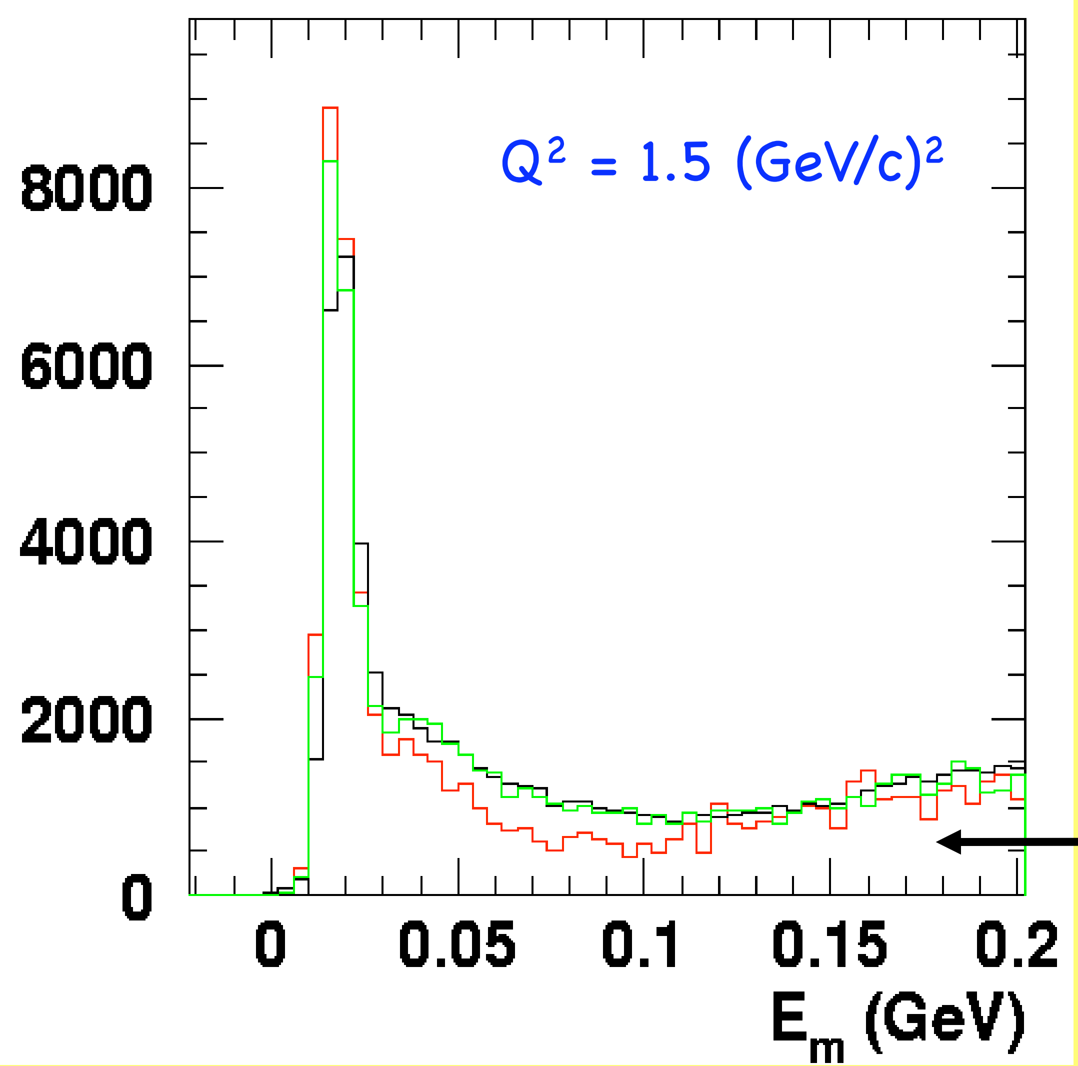
Modern many-body theories:

- Correlated Basis Function theory (CBF)
  O. Benhar, A. Fabrocini, S. Fantoni, Nucl. Phys. A505, 267 (1989)
- Green's function approach (2nd order)
  H. Müther, G. Knehr, A. Polls, Phys. Rev. C52, 2955 (1995)
- Self--consistent Green's function (T = 2 MeV)
  T. Frick, H. Müther, Phys. Rev. C68 (2003) 034310

Missing strength already at moderate p_m

compared to IPSM

200 MeV/c < p_m < 300 MeV/c



data on ^{12}C

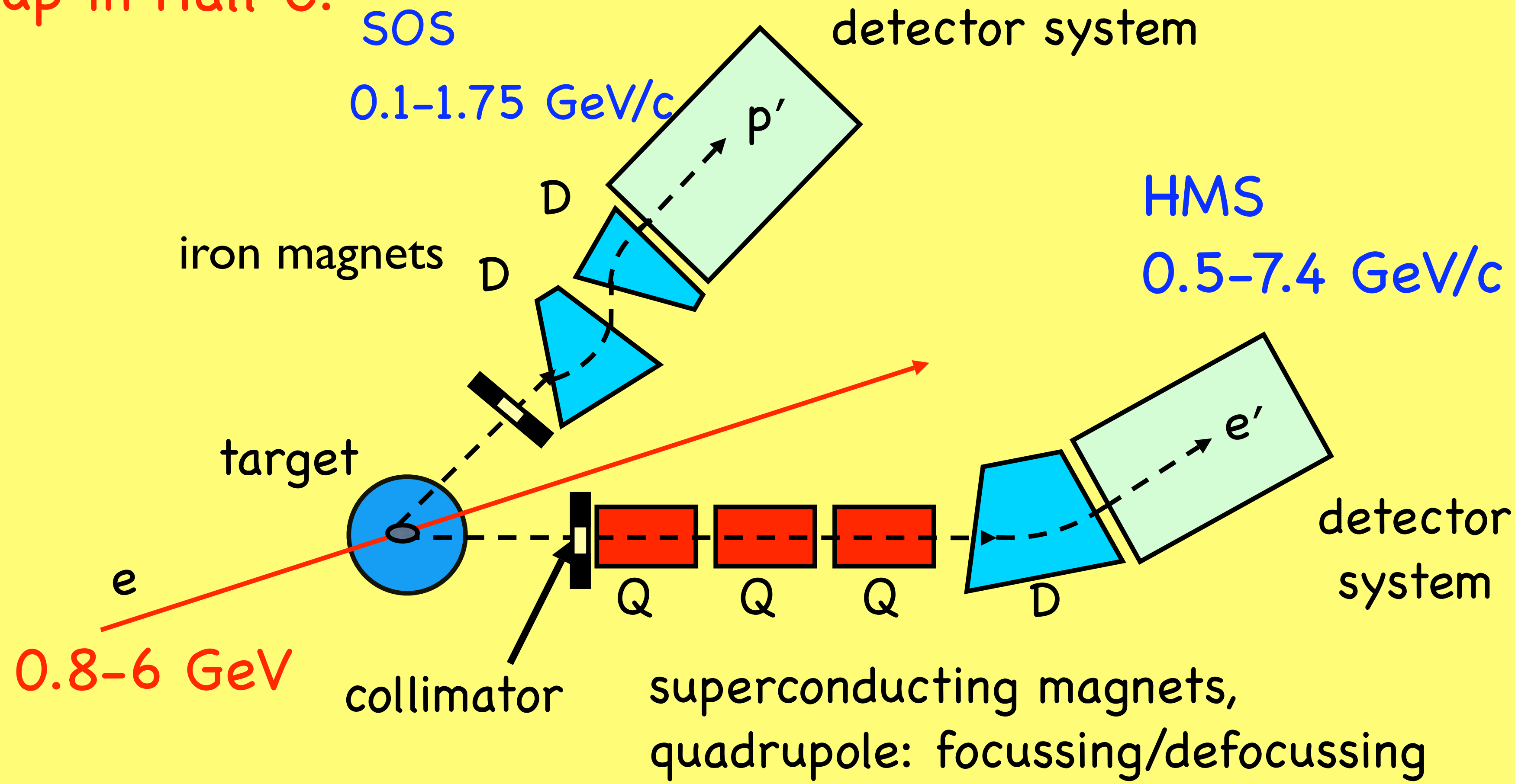
IPSM*0.85

CBF

radiation tail

Spectral function containing SRC: good agreement with data

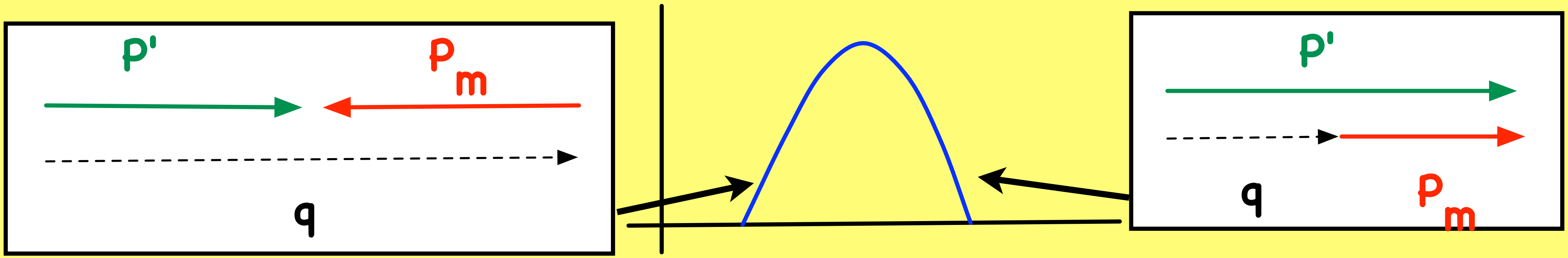
Setup in Hall C:



Performance	HMS	SOS	
momentum range	0.5–7.4	0.1–1.75	
acceptance δ (%)	±10	±15	$p = p_0(1 + \delta)$
solid angle (msr)	6.7	7.5	
target acceptance (cm)	± 7	±1.5	

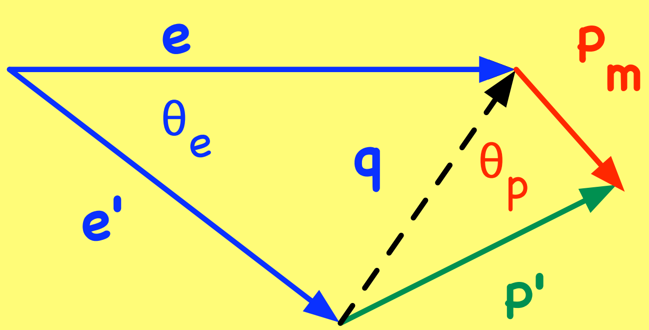
Data at high p_m, E_m measured in Hall C at Jlab:

- targets: C, Al, Fe, Au
- kinematics: 3 parallel **p || q**



To map out $S(E_m,p_m)$ vary **q** keeping **p'** (T_p) constant so that FSI are constant

- kinematics: 2 perpendicular **p** ⊥ **q**

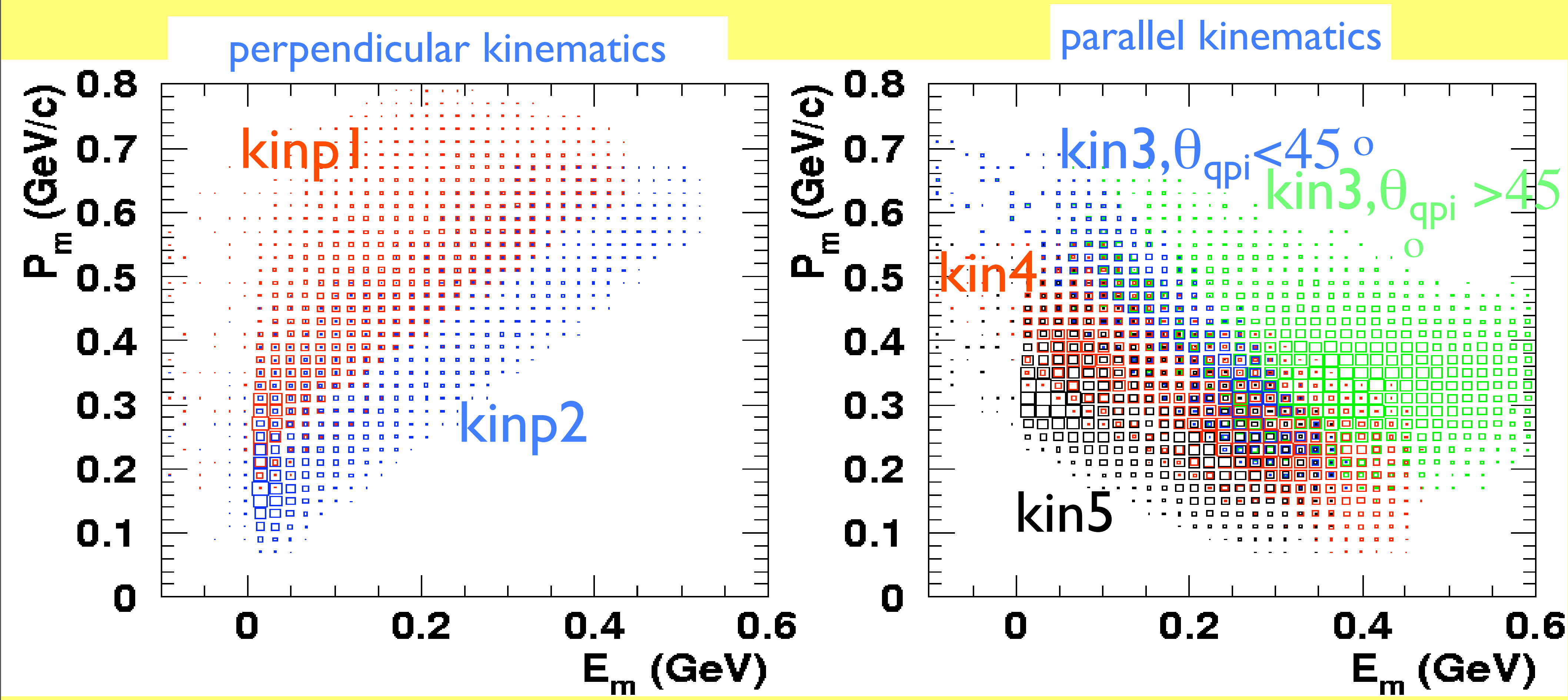


- Fix **e**, θ_e, **p'**
- Vary E_m thru e'
- Vary p_m with proton angle θ_p

Data at high p_m, E_m measured in Hall C at Jlab:

- targets: C, Al, Fe, Au

Covered E_m-p_m range:



high E_m- region: dominated by Δ resonance

Extraction of the spectral function:

only in PWIA possible, care for corrections later

exp. c.s.: $$\frac{d\sigma^{fi}}{dE_e d\Omega_e dE_p d\Omega_p} = K \underbrace{\sigma^{free}}_{e-p\ cs} S(p_m, E_m) T_A$$

Binning of the data $(E_m, p_m)_{ij}$: ΔE_m = 10-50 MeV, Δp_m = 40 MeV/c

$$\left(\frac{d\sigma}{dE_e d\Omega_e dE_p d\Omega_p}\right)_{ij} = \bar{K}\bar{\sigma}^{free} S(\bar{p}_m, \bar{E}_m)_{ij} T_p$$

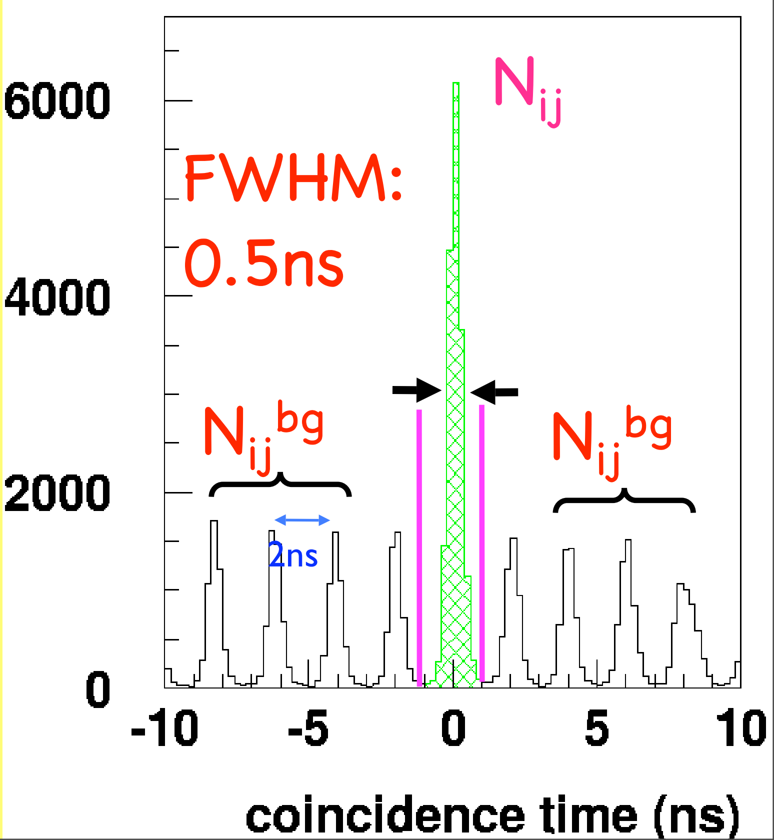
$$= \frac{(N_{ij} - N_{ij}^{bg})/\epsilon}{L P_{i,j}}$$

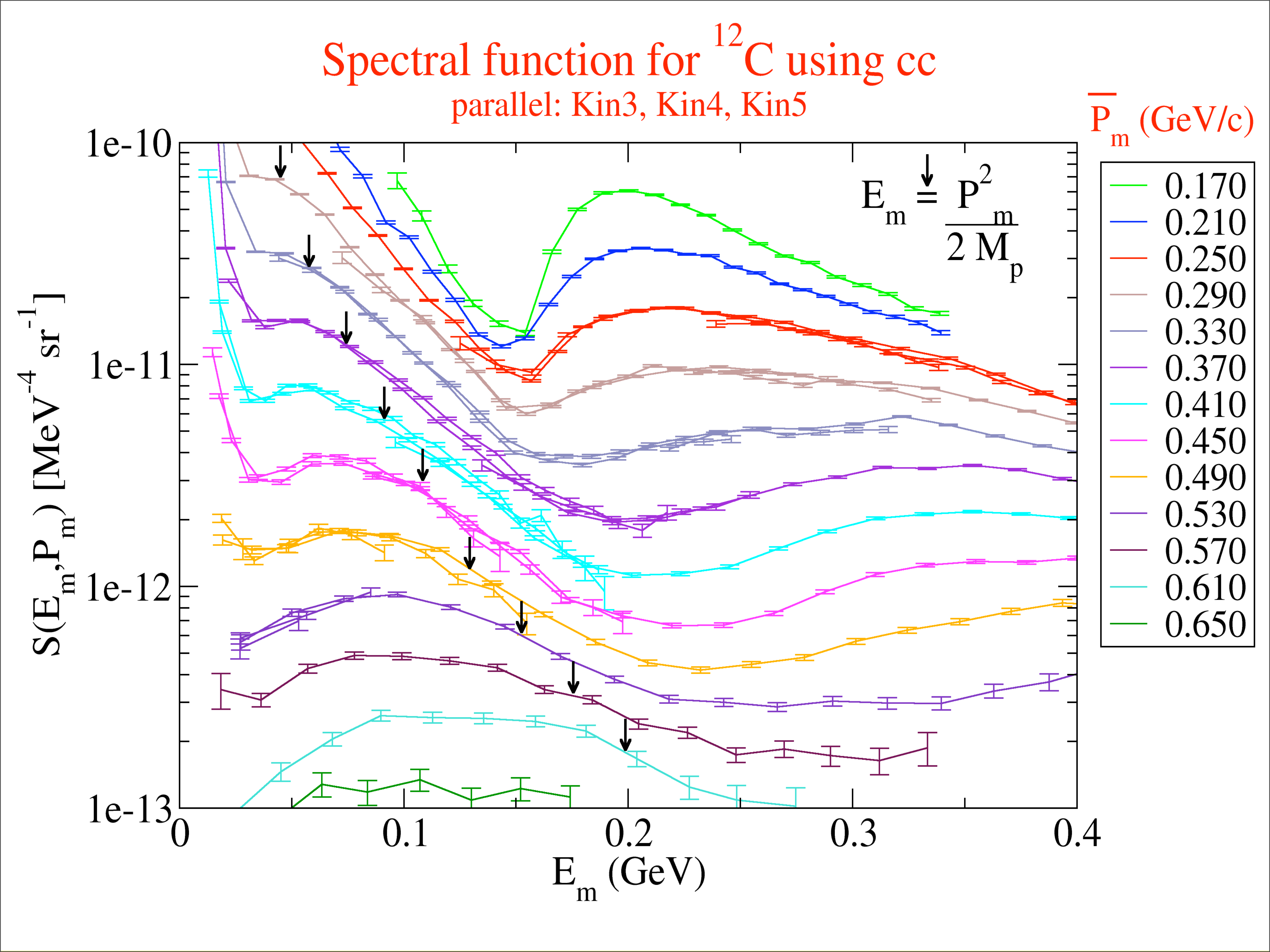
Luminosity

phase space from M.C.

Efficiency, dead time ...

e & p:



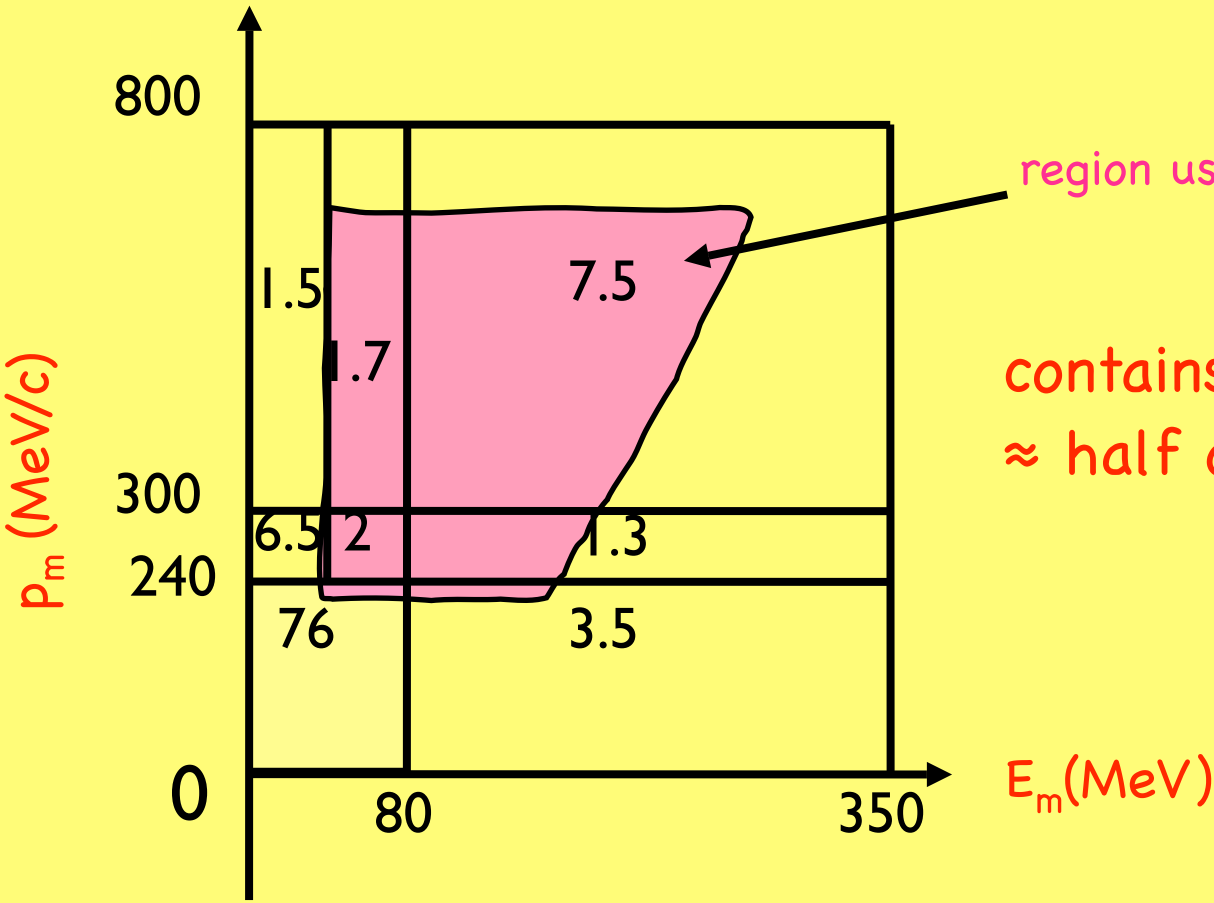


Spectral function for 12C using cc
parallel: Kin3, Kin4, Kin5
Pm (GeV/c)
0.170
0.210
0.250
0.290
0.330
0.370
0.410
0.450
0.490
0.530
0.570
0.610
0.650
Em = P2m / 2 Mp
S(Em,Pm) [MeV-4 sr-1]
Em (GeV)
1e-10
1e-11
1e-12
1e-13
0
0.1
0.2
0.3
0.4

Integrated strength in the covered E_m–p_m region:

$$Z_C = 4\pi \int_{230\,\mathrm{MeV/c}}^{670} dp_m\, p_m^2 \int dE_m\, S(E_m,p_m)$$

Strength distribution in % (from CBF)



contains

≈ half of the total strength

"correlated strength" in the chosen E_m–p_m region:

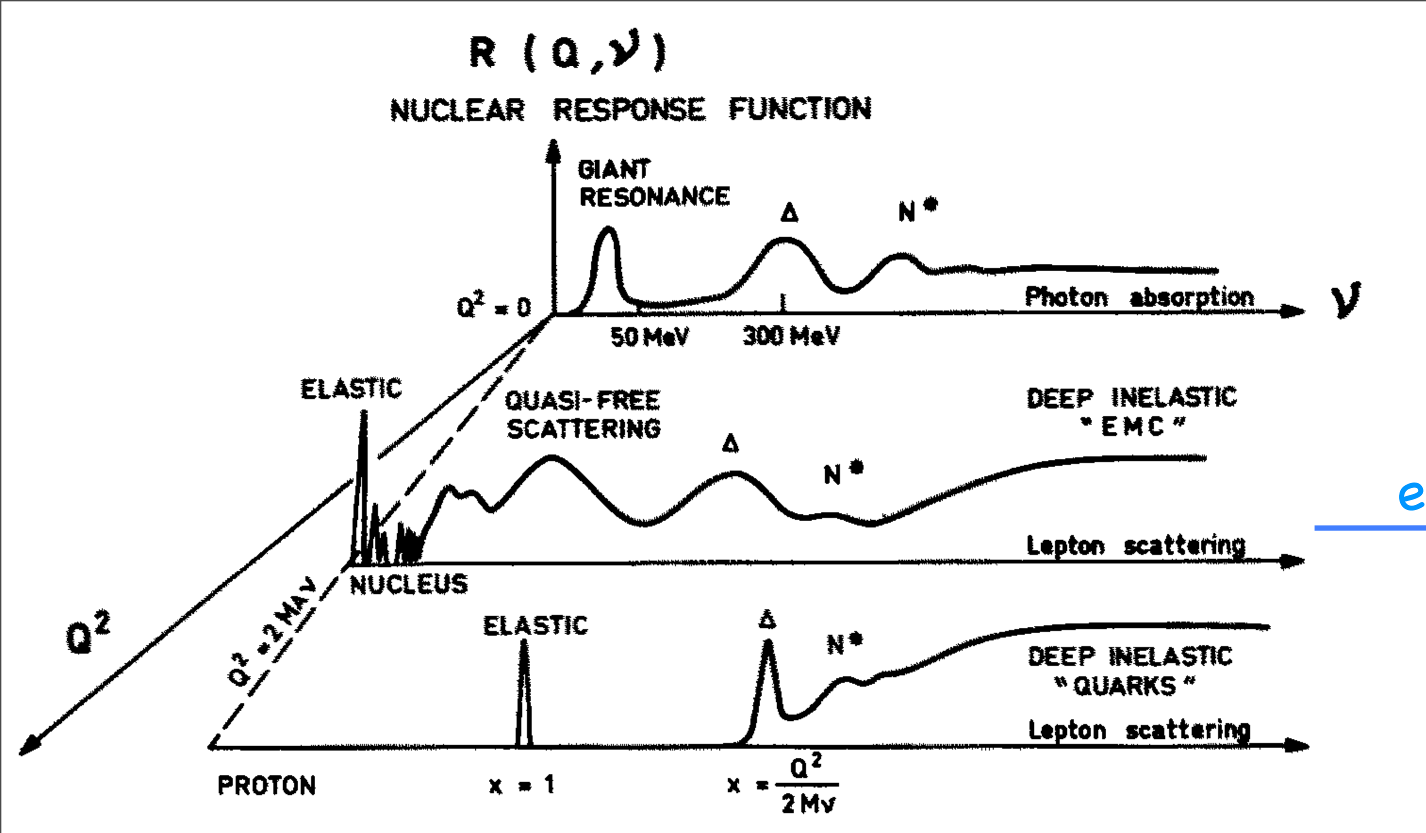
^{12}C	exp.	CBF theory	G.F. 2.order	selfconsistent G.F.
experimental area	0.61	0.64 ≈ 10 %	0.46	0.61
in total (correlated part)		22 %	12%	≈20%

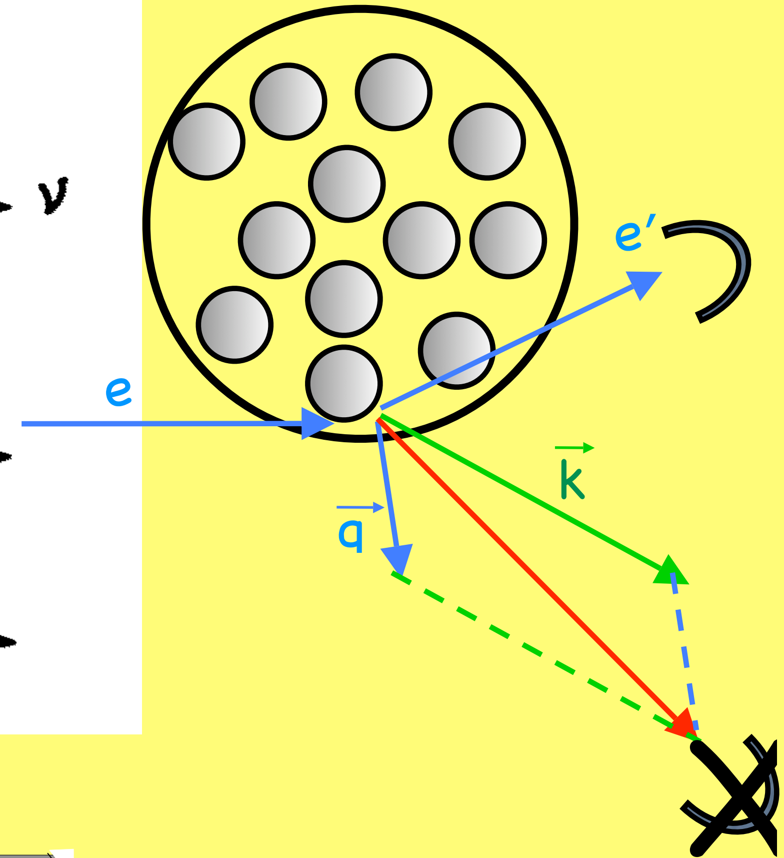
contribution from FSI: -4 %

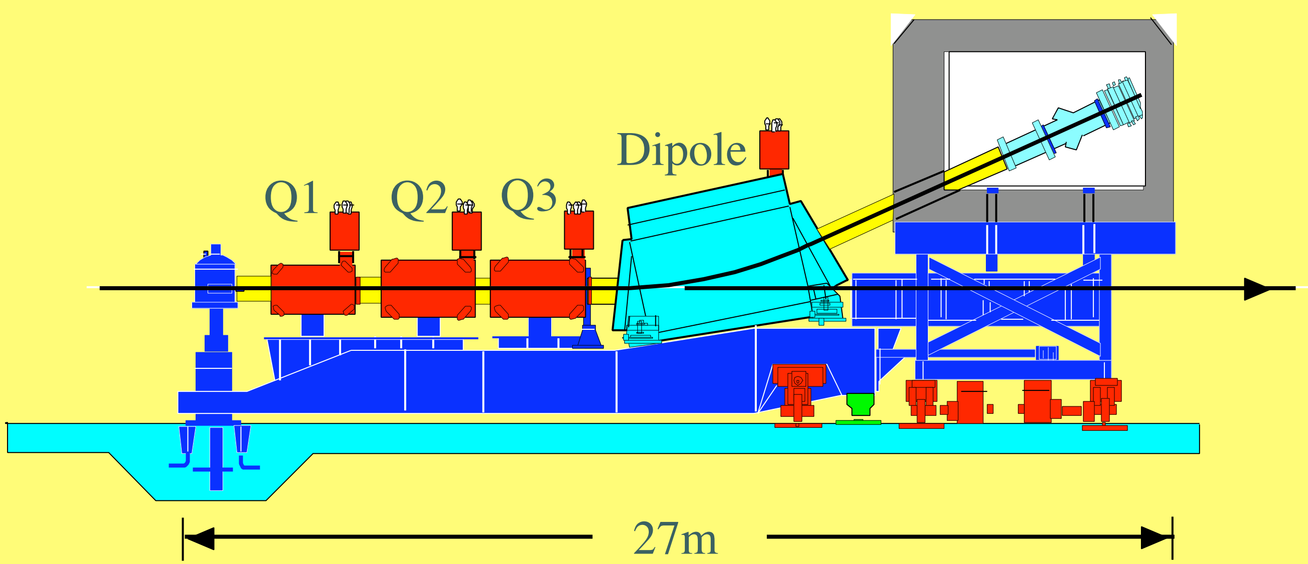
- ≈ 10% of the protons in ^{12}C at high p_m, E_m found
- first time directly measured

comparing to theory leads to conclusion that
≈ 20% of the protons in Carbon are beyond the IPSM region

Rohe et al.,
Phys. Rev. Lett. 93, 182501 (2004)


R (Q, ν)
NUCLEAR RESPONSE FUNCTION
GIANT RESONANCE
Δ
N*
$Q^2 = 0$
50 MeV
300 MeV
Photon absorption
ν
ELASTIC
QUASI-FREE SCATTERING
Δ
N*
DEEP INELASTIC "EMC"
Lepton scattering
NUCLEUS
Q^2
$Q^2 = 2M\nu$
ELASTIC
Δ
N*
DEEP INELASTIC "QUARKS"
Lepton scattering
PROTON
x = 1
$x = \frac{Q^2}{2M\nu}$

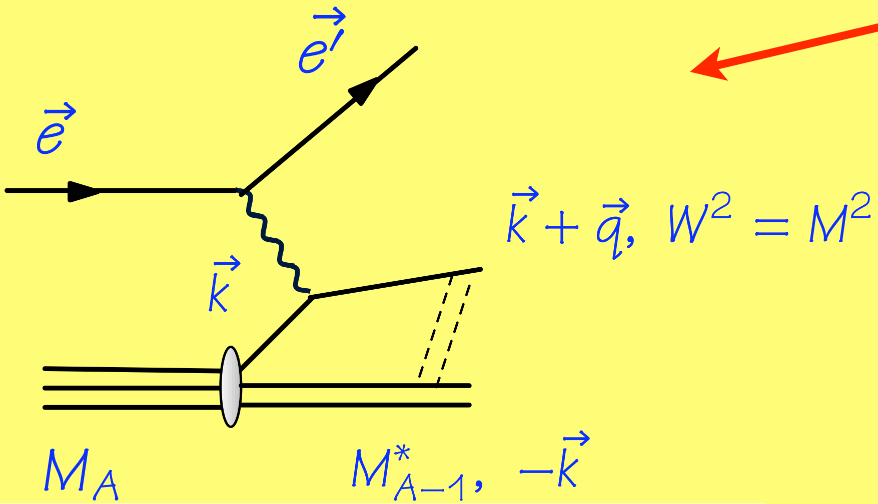

e
e′
$\vec{q}$
$\vec{k}$

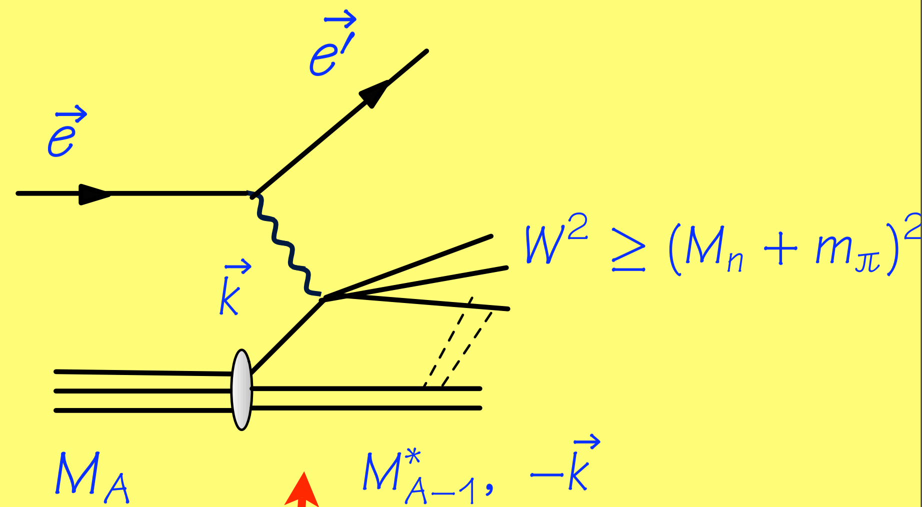

Q1
Q2
Q3
Dipole
27m

Inclusive Electron Scattering from Nuclei

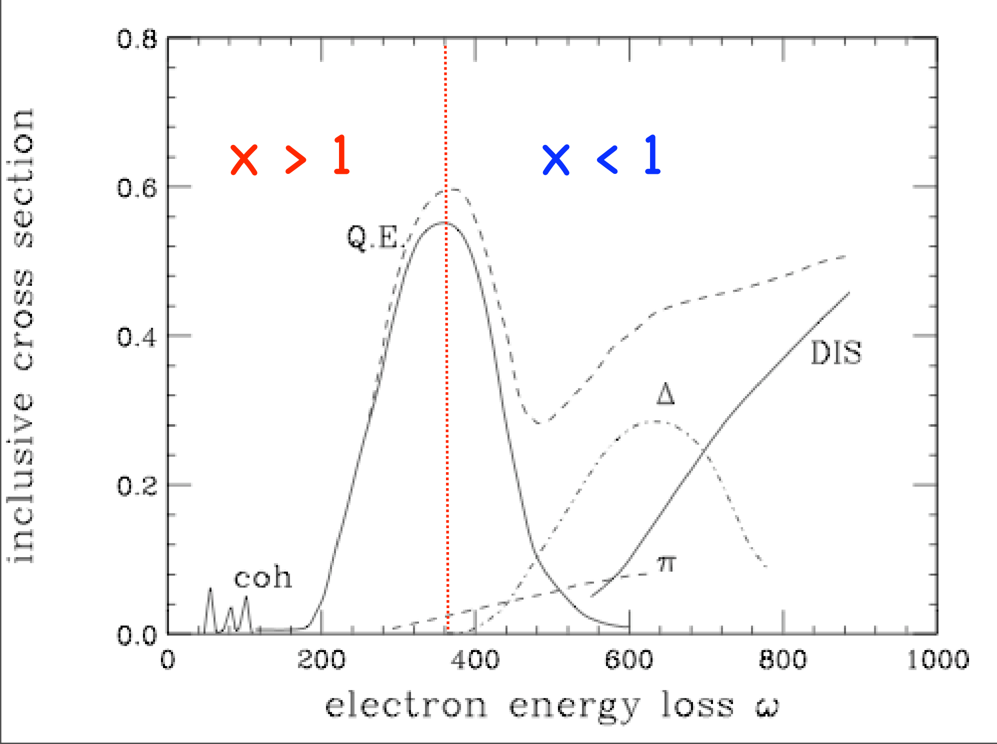
Two distinct processes

Quasielastic from the nucleons in the nucleus





Inelastic and DIS from the quark constituents of the nucleon.



Inclusive final state means no separation of two dominant processes

$x = Q^2/(2m\upsilon)$

υ,ω=energy loss

Formalism

$$\frac{d\sigma^2}{d\Omega_{e'} dE_{e'}} = \frac{\alpha^2}{Q^4}\frac{E'_e}{E_e} L_{\mu\nu}W^{\mu\nu}$$

$$L_{\mu\nu} = 2\left[k_e^\mu k_{e'}^\nu + k_e^\nu k_{e'}^\mu - g^{\mu\nu}(k_e k_{e'})\right] \quad W^{\mu\nu} = \sum_X \langle 0|J^\mu|X\rangle \langle X|J^\nu|0\rangle \delta^{(4)}(p_0 + q - p_X)$$

Currents can be written as sum of one-body currents which (eventually) allows (See O. Benhar)

$$W^{\mu\nu}(\mathbf{q},\omega) = \int d^3k \;\; dE \;\; \left(\frac{m}{E_{\mathbf{k}}}\right)\left[ZS_p(\mathbf{k},E)w_p^{\mu\nu}(\tilde{q}) + (A-Z)S_n(\mathbf{k},E)w_n^{\mu\nu}(\tilde{q})\right]$$

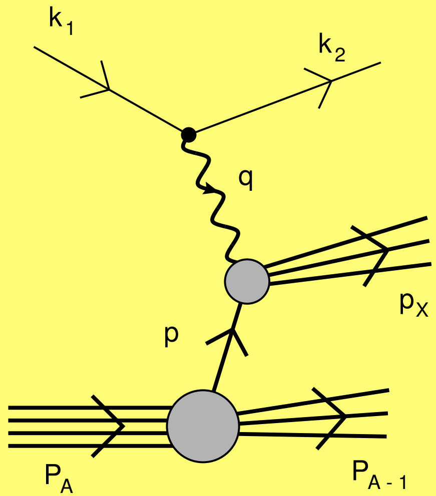
where $w^{\mu\nu}$ describes the e/m response of a bound nucleon with momentum **k**

which consists of an elastic and inelastic component.

QES in IA $$\frac{d^2\sigma}{d\Omega d\nu} \propto \int d\vec{k}\int dE\sigma_{ei} \underbrace{S_i(k,E)}_{Spectral\ function}\delta()$$ $G_E^{p,n}(Q^2)$ and $G_M^{p,n}(Q^2)$

DIS $$\frac{d^2\sigma}{d\Omega d\nu} \propto \int d\vec{k}\int dE\; W_{1,2}^{(p,n)} \underbrace{S_i(k,E)}_{Spectral\ function}$$ $W_{1,2}^{p,n}(Q^2,\nu) \rightarrow W_{1,2}^{p,n}(x)$

+ log(Q^2) corrections



There is a rich, if complicated, blend of nuclear and fundamental QCD interactions available for study from these types of experiments.

The limits on the integrals are determined by the kinematics. Specific (x, Q^2) select specific pieces of the spectral function.

The two processes share the same initial state

QES in IA $$\frac{d^2\sigma}{d\Omega d\nu} \propto \int d\vec{k} \int dE \sigma_{ei} \underbrace{S_i(k,E)}_{\text{Spectral function}} \delta()$$

$$n(k) = \int dE\ S(k,E)$$

DIS $$\frac{d^2\sigma}{d\Omega d\nu} \propto \int d\vec{k} \int dE\ W_{1,2}^{(p,n)} \underbrace{S_i(k,E)}_{\text{Spectral function}}$$

However they have very different Q^2 dependencies

$\sigma_{ei} \propto$ elastic (form factor)^2 $W_{1,2}$ scale with <u>ln Q^2</u> dependence

Exploit this dissimilar Q^2 dependence

Relation to charged current neutrino-nucleus scattering

$$e + A \longrightarrow e' + X \qquad \frac{d\sigma^2}{d\Omega_{e'} dE_{e'}} = \frac{\alpha^2}{Q^4} \frac{E'_e}{E_e} L_{\mu\nu} W^{\mu\nu}$$

$$\nu_l + A \longrightarrow l^- + X \qquad \frac{d\sigma^2}{d\Omega_l dE_l} = \frac{G^2}{32\pi^2} \frac{|\vec{k'}|}{|\vec{k}|} L_{\mu\nu} W^{\mu\nu}$$

Both can be cast in the same form

$$\frac{d^2\sigma}{d\Omega d\nu} \propto \int d\vec{k} \int dE \sigma_{ei} \underbrace{S_i(k,E)}_{\text{Spectral function}} \delta()$$

$\sigma_{ei} \longrightarrow \sigma_{\nu i}$ weak charged current interaction with a nucleon

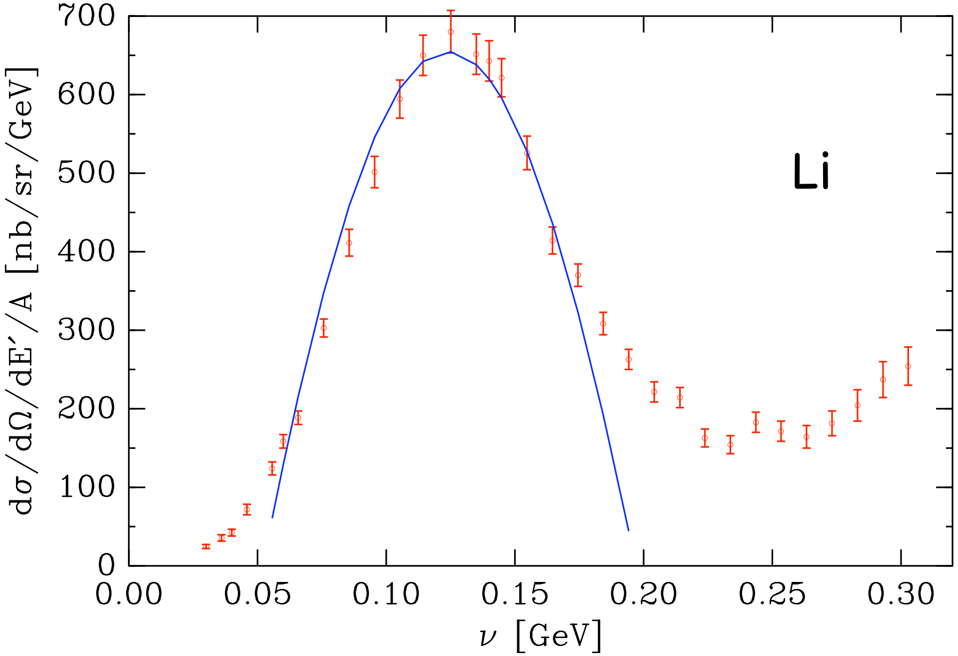
Early 1970's Quasielastic Data

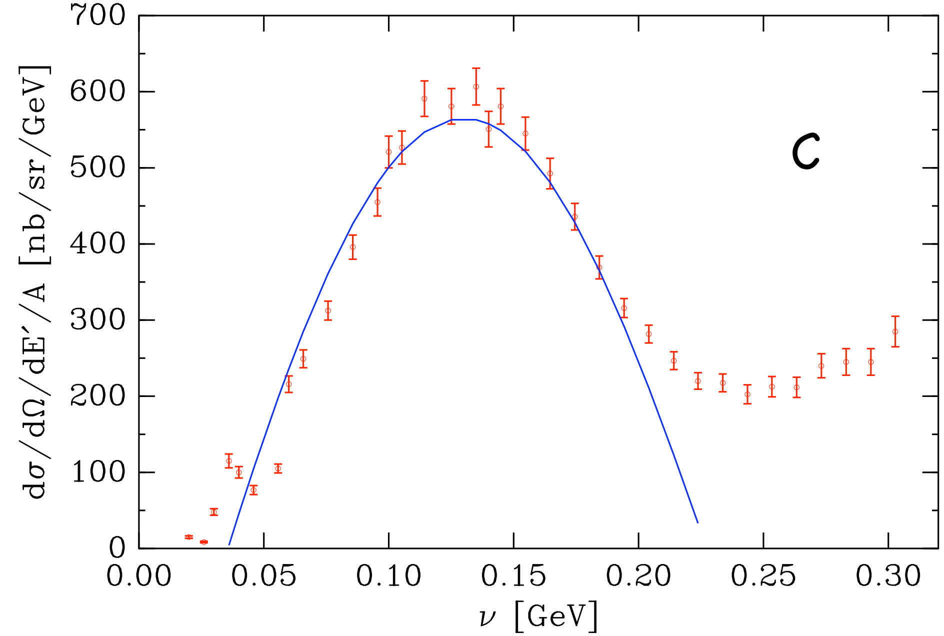
-> getting the bulk features

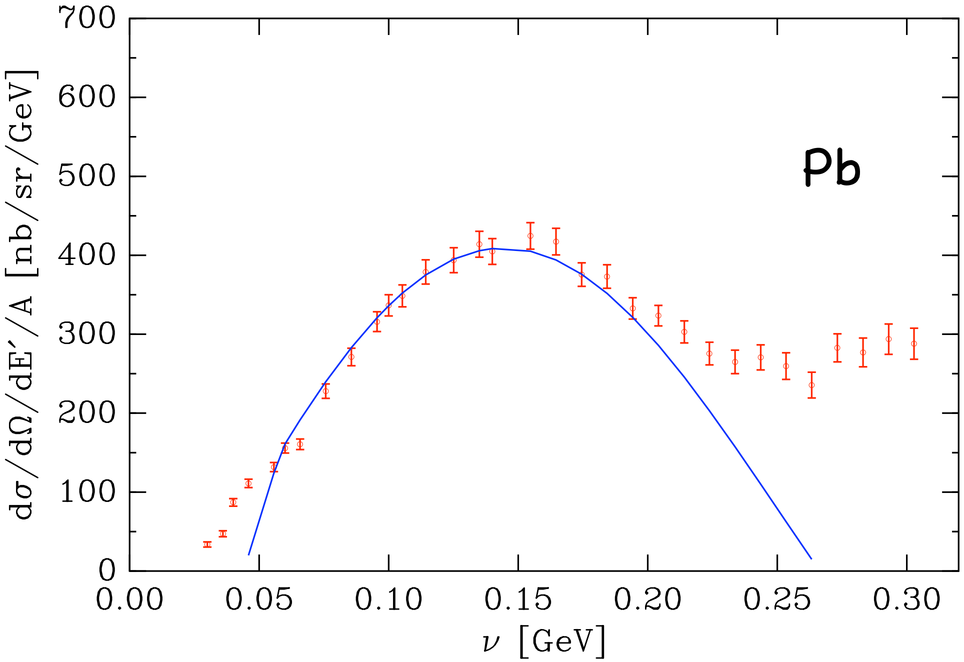
500 MeV, 60 degrees

$\vec{q} \simeq 500 MeV/c$

R.R. Whitney et al., Phys. Rev. C 9, 2230 (1974).

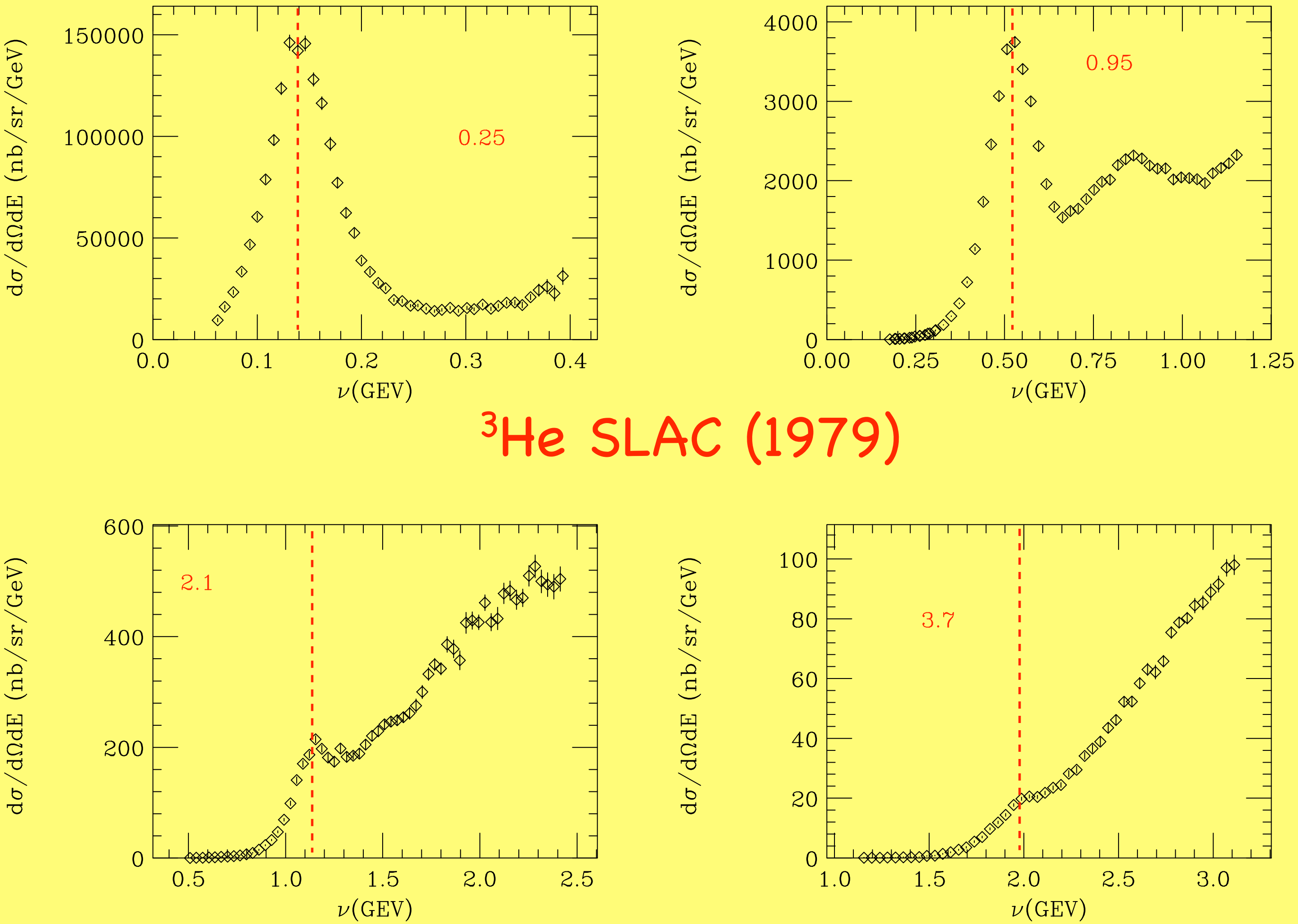






Nucleus	k_F	$\bar{\epsilon}$
^{6}Li	169	17
^{12}C	221	25
^{24}Mg	235	32
^{40}Ca	251	28
^{nat}Ni	260	36
^{89}Y	254	39
^{nat}Sn	260	42
^{181}Ta	265	42
^{208}Pb	265	44

compared to Fermi model: fit parameter k_F and ϵ

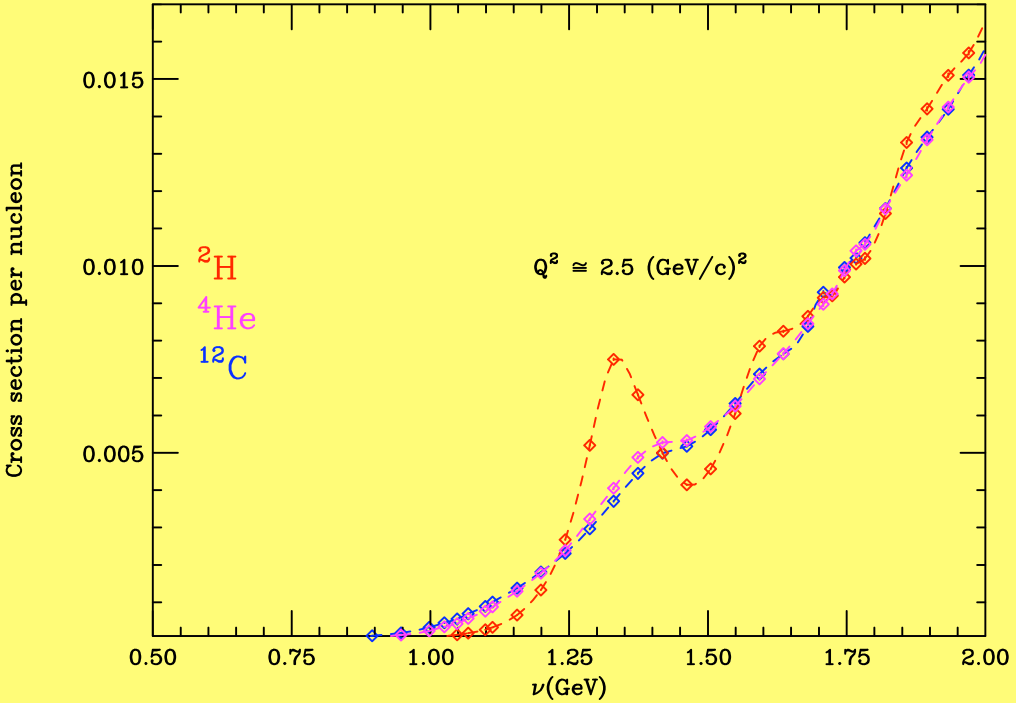


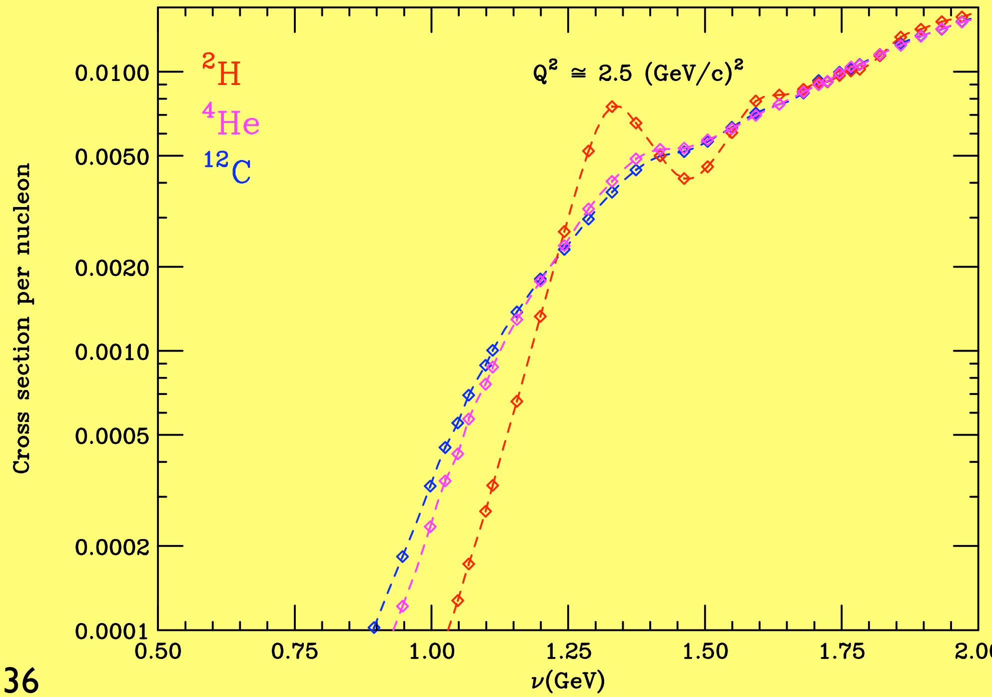
The quasielastic peak (QE) is broadened by the Fermi-motion of the struck nucleon.

The quasielastic contribution dominates the cross section at low energy loss (ν) even at moderate to high Q^2.

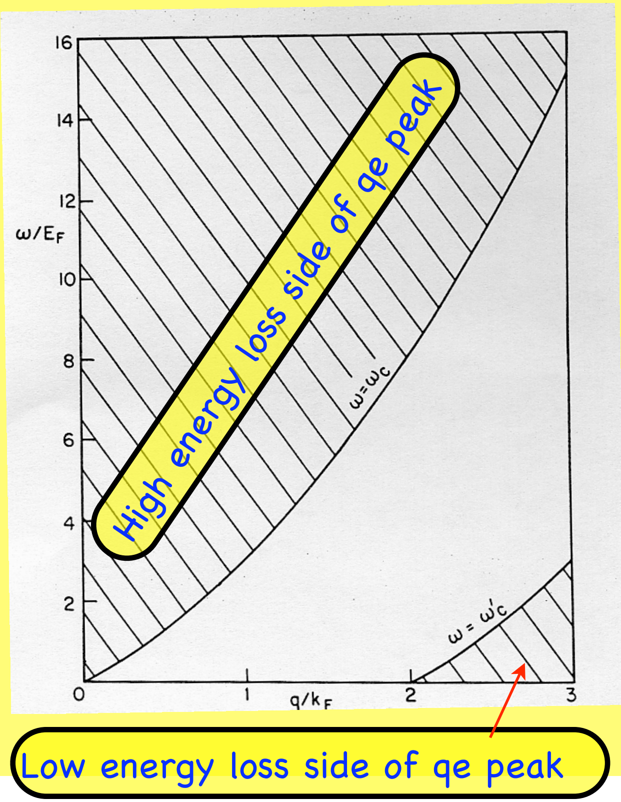
- The shape of the low ν cross section is determined by the momentum distribution of the nucleons.
- As Q^2 >> inelastic scattering from the nucleons begins to dominate
- We can use x and Q^2 as knobs to dial the relative contribution of QES and DIS.

A dependence: higher internal momenta broadens the peak





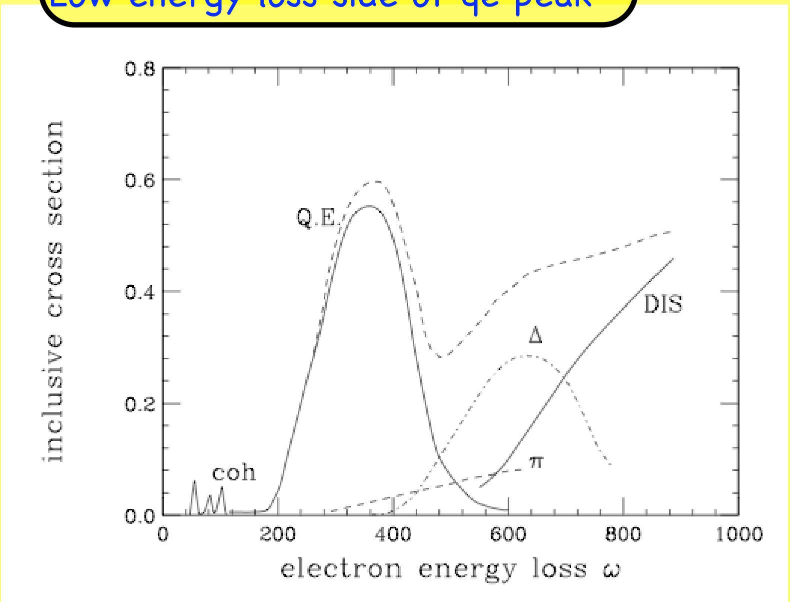
Correlations and Inclusive Electron Scattering

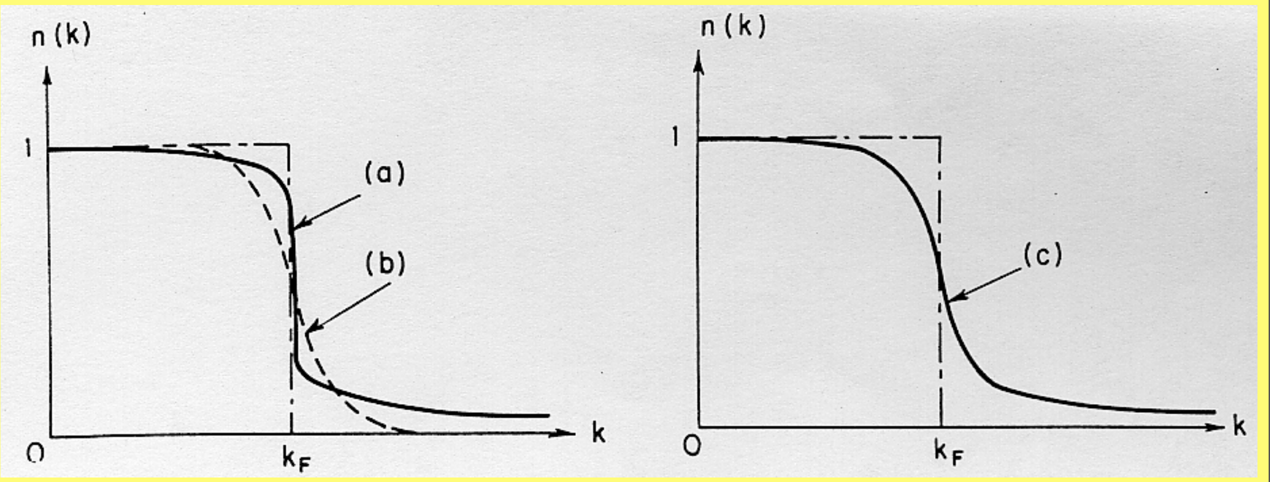


Shaded domain where scattering is restricted solely to correlations. Czyz and Gottfried (1963)

$$\omega_c = \frac{(k+q)^2}{2m} + \frac{q^2}{2m} \qquad \omega_c' = \frac{q^2}{2m} - \frac{qk_f}{2m}$$

Czyz and Gottfried proposed to replace the Fermi n(k) with that of an actual nucleus. (a) hard core gas; (b) finite system of noninteracting fermions; (c) actual large nucleus.





Studying Superfast Quarks

- In the nucleus we can have $0<x<A$
- In the Bjorken limit, $x > 1$ DIS tells us the virtual photon scatters incoherently from quarks
- Quarks can obtain momenta $x>1$ by abandoning confines of the nucleon
  - deconfinement, color conductivity, parton recombination multiquark configurations
  - correlations with a nucleon of high momentum (short range interaction)
- DIS at $x > 1$ is a filter that selects out those nuclear configurations in which the nucleon wave functions overlap. We are studying the dynamics of partons that have abandoned the confines of the nucleon.

$$\langle r_{NN} \rangle \approx 1.7 \text{ fm} \approx 2 \times r_n = 1.6 \text{ fm}$$

The probability that nucleons overlap is large and at $x > 1$ we are kinematically selecting those configurations.

Short Range Correlations (SRCs)

Mean field contributions: $k < k_F$
Well understood, Spectroscopic Factors ≈ 0.65

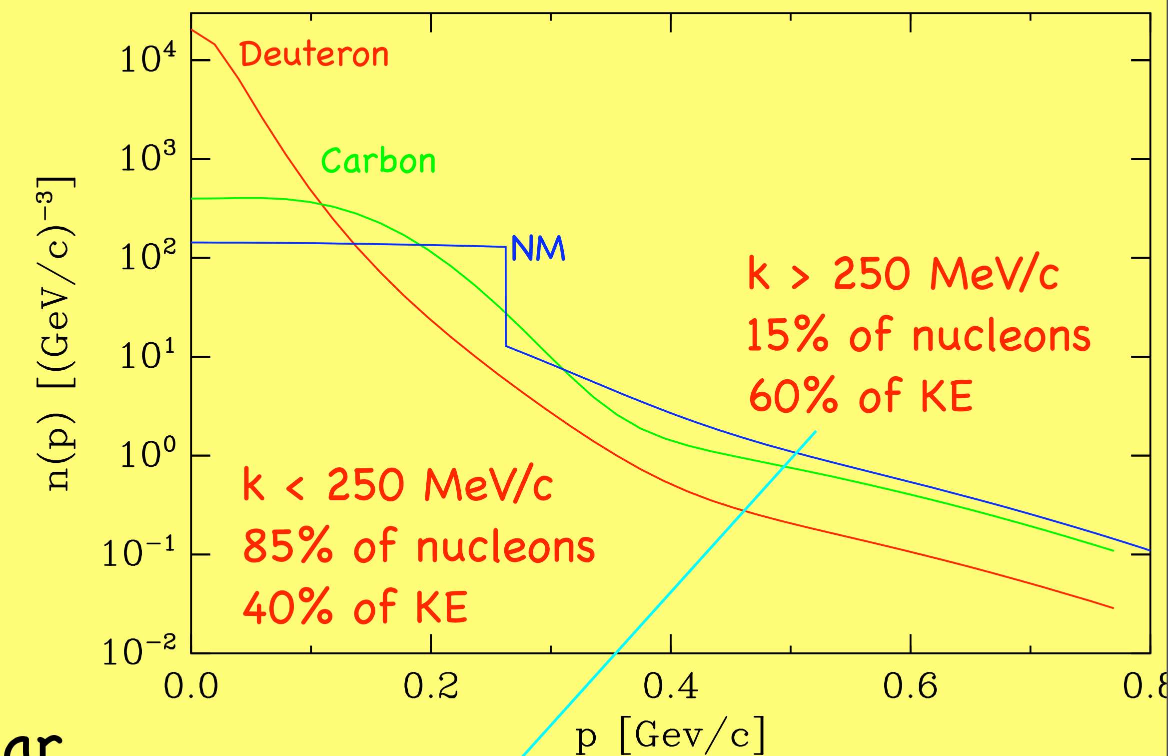
High momentum tails: $k > k_F$
Calculable for few-body nuclei, nuclear matter.
Dominated by two-nucleon short range correlations

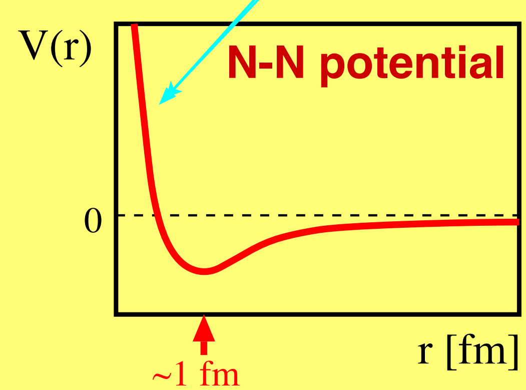
Isolate short range interactions (and SRC's) by probing at high p_m: (e,e'p) and (e,e')

Poorly understood part of nuclear structure

Sign. fraction have $k > k_F$

Uncertainty in SR interaction leads to uncertainty at k>>, even for simplest systems





Calculations of SRC

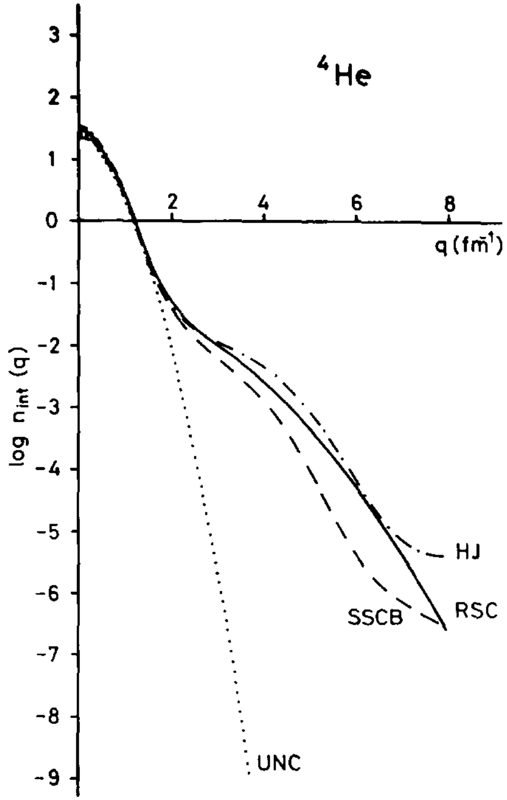


Fig. 2. Momentum distributions for ^{4}He, HJ: Hamada–Johnston potential, RSC: Reid soft core potential, SSCB: de Tourreil–Sprung super soft core potential B, UNC: uncorrelated, for the RSC potential. The other uncorrelated distributions do not differ appreciably for $q > 2\ fm^{-1}$.

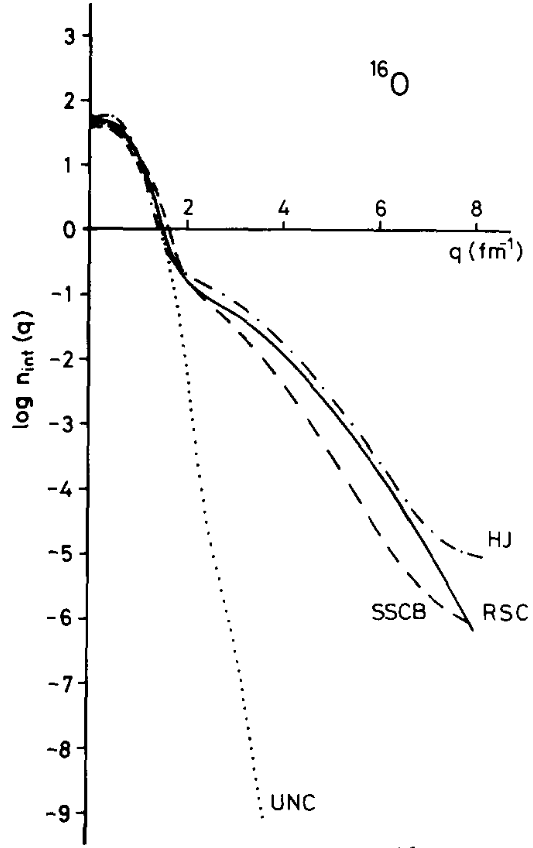
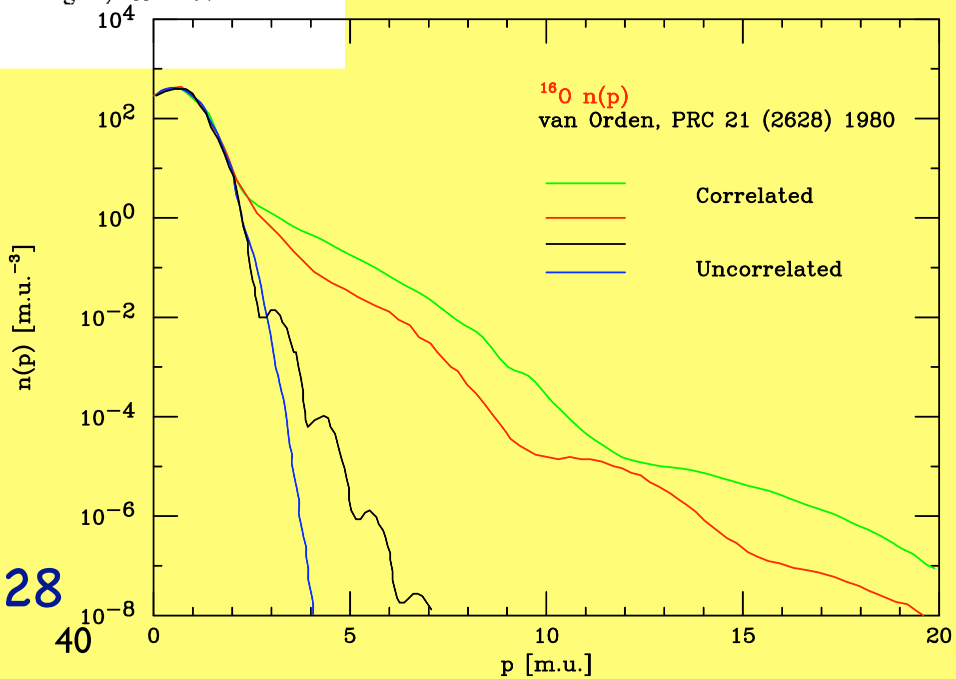


Fig. 3. Same as fig. 2, for ^{16}O.

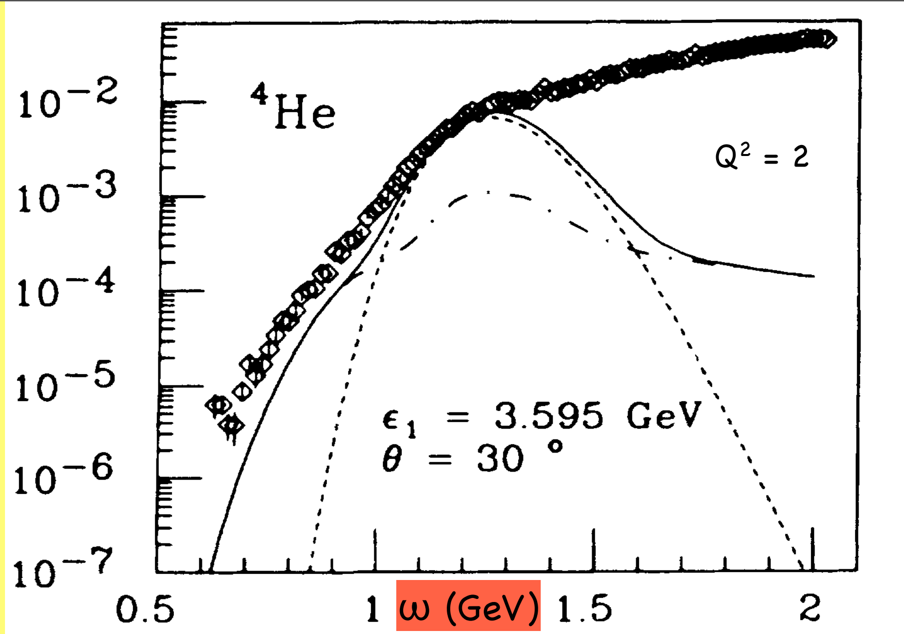
Show up at large momentum

Zabolitzky and Ey, PLB 76, 527



Van Orden et al., PRC21, 2628

Correlations are accessible in QES and DIS at large x (small energy loss)



CdA, Day, Liuti, PRC 46 (1045) 1992