# HUGS Lecture Notes

## 2007

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University of Virginia

# Lecture Caveats

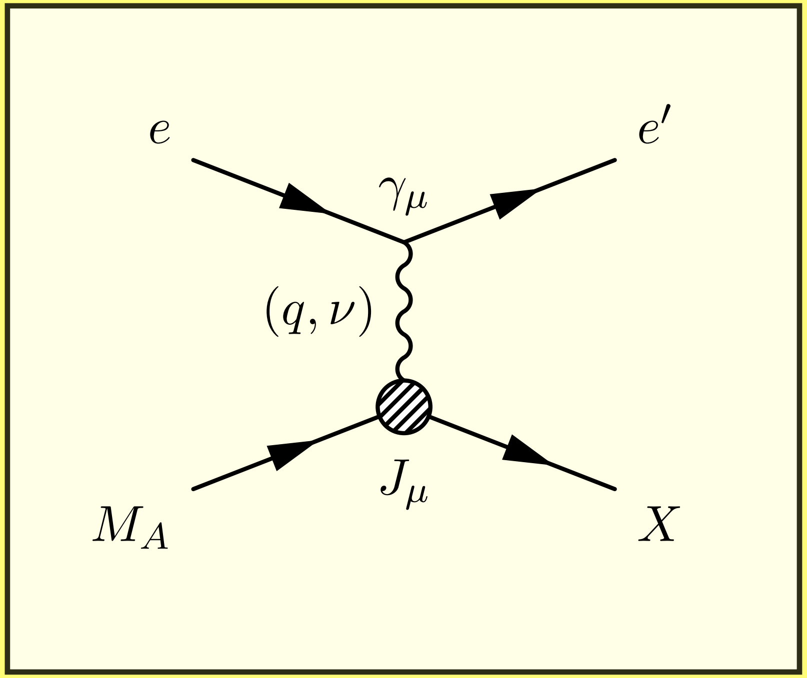
- Given by an experimentalist
  - little formalism
  - provide some experimental details
  - should be pretty basic for Ph.D students
- Style
  - Informal - ask questions at any time

- General Introduction
- Nuclear Inelastic Response
- Nucleon Elastic Form Factors
- Polarized Targets

# Lecture Outline

- Topics
    - Why electron scattering?
    - Experimental Techniques
    - Elastic electron scattering
        - nuclear charge and magnetization densities
        - nucleon form factors ←
    - Quasi-elastic scattering
        - A(e,e′)X ←
        - A(e,e′p)X
    - Deep Inelastic Scattering
        - Unpolarized
        - Polarized
    - Polarized Targets ←

# Why use electrons?




- Electron-nucleus interaction is well known
  - QED: exact theory, point-like probe

    If using a strongly interaction probe (p, π); both the interaction and the system are unknown; further the probe can have internal d.o.f
- Interaction is weak, $\alpha$ = 1/137
  - perturbation of nucleus small
  - reaction mechanism simple
- Interaction is known (QED): E/M interaction of electron with the charge $\rho$ and current J densities of the nucleus
  - $\sigma$ is calculable

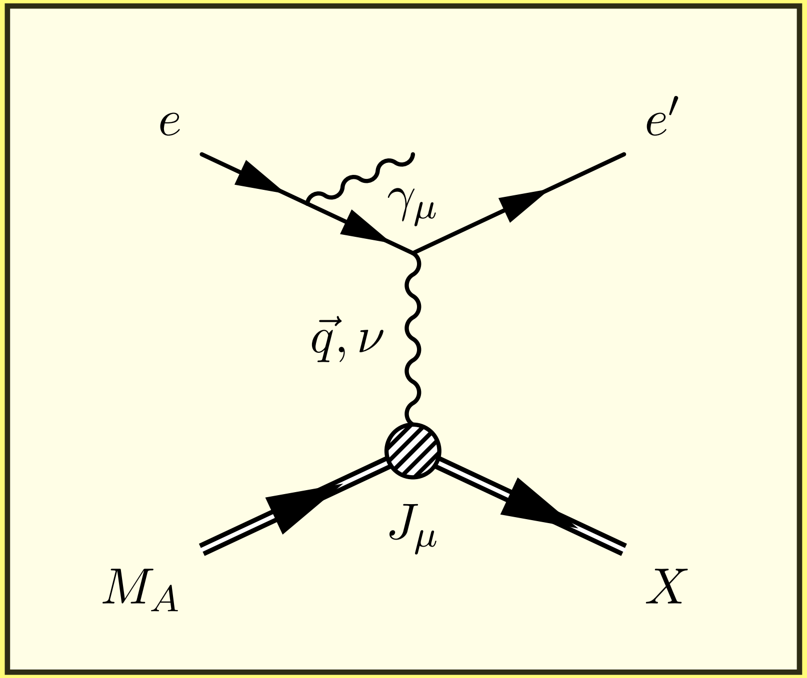
    solution of Dirac equation

    quantitative

    confidence

# There are disadvantages

- $\sigma \propto \alpha^2 \sim 10^{-4}$, small
  - need high intensity
  - need thick targets
  - need large solid angles
- Electron mass small
  - for small $\lambda$ need large accelerators
    - in the past they had poor duty factor and poor $\Delta E/E$ (resolution)
  - Radiative effects



# Experimental aims:

1) elastic scattering

- (spin-) structure of the nucleus

  → form factors, charge distribution

  analyzing power $T_{20}$

2) quasielastic scattering (exclusive, inclusive)

- structure of the nucleon → form factors
- medium modification
- momentum distribution, occupancies
- shell structure in the nucleus
- transparency factor, color transparency
- x>1 on light to heavy nuclei, scaling

examine the N-N interaction

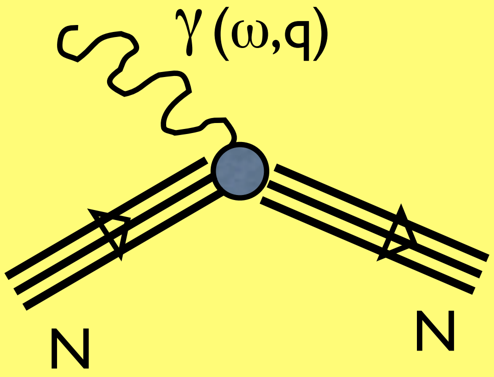
3) (deep) inelastic scattering

- excitation of resonances
- x-scaling of cross sections measured on different nuclei
- composition of the nucleon (gluons, quarks, spin)

very few of these topics are covered by these lectures

# Different energy regimes,

depending on the momentum q and energy ω carried by the photon




3 cases:

1. low q, ω

- photon wavelength λ is long compared with the size of the nucleon.
- nucleon is seen as a point (probably a nucleus can be resolved)

2. higher q, ω , E ~100 MeV to ~1 GeV

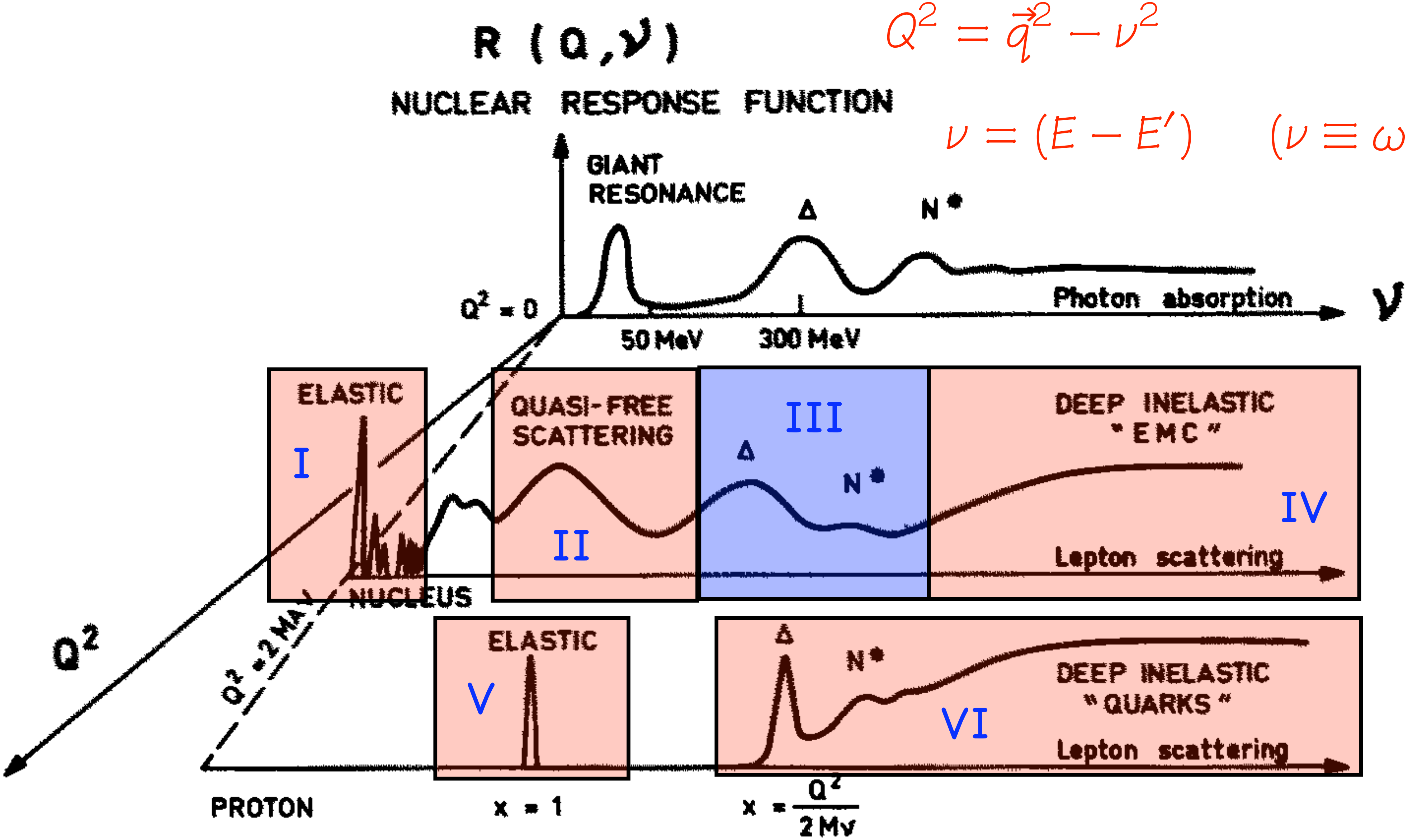
- wavelength is comparable to the nucleon size.
- can resolve the finite size of the nucleon.

3. very high q, ω

- wavelength is much shorter than the nucleon size
- photon can resolve the internal structure of the nucleon.

Determines resolution: $\simeq \hbar c/q$

# Nuclear Response Function




$Q^2 = \vec{q}^{\,2} - \nu^2$

$\nu = (E - E') \qquad (\nu \equiv \omega)$

# Electron beams

- need high energy
  - $q \sim 2E \sin(\theta/2)$
  - E ~ 0.5 -> 1 GeV for resolution 1.5/q ~ 0.2 fm
- need high duty cycle for coincidence reactions
  - for coincidence expts: accidentals ~ $I^2$
  - reduces rates in detectors, multiple hits, tracking
- need high beam intensity to compensate for $\alpha^2$
- need small $\Delta E/E$ to separate nuclear levels
- need polarized electrons
  - statistical error $\propto \frac{1}{P_e P_t} \frac{1}{\sqrt{t}}$

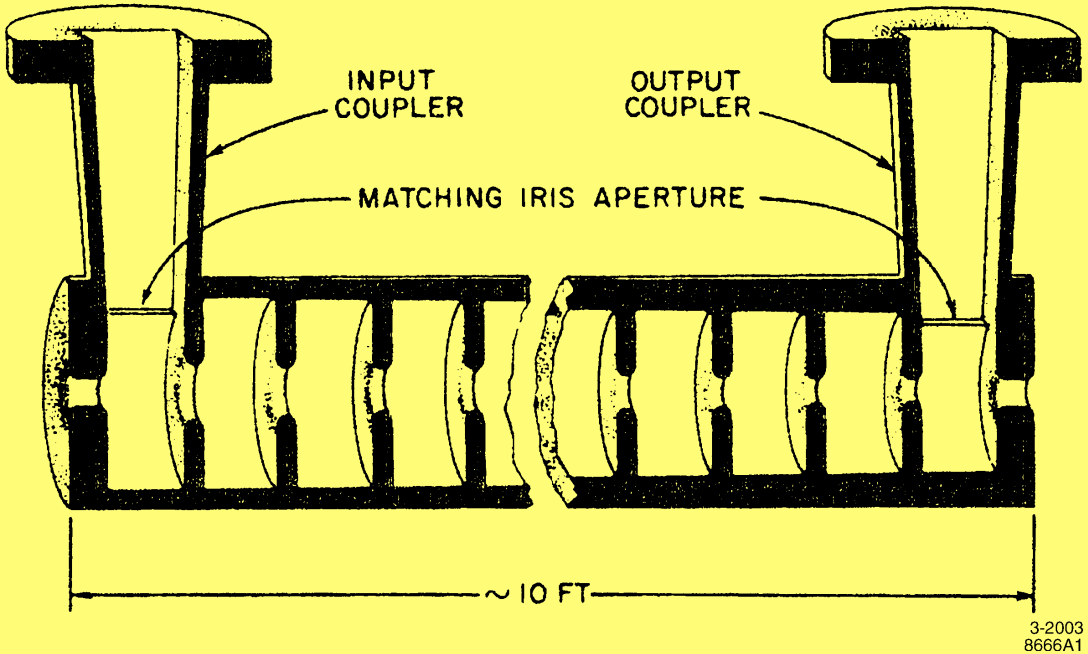
# Electron beams

Free space solution to Maxwell's equations

$$\vec{\nabla} \cdot \vec{E} = \frac{\partial E_z}{\partial z} = -kE_{0,z} exp[i(\omega t - kz)] = 0$$

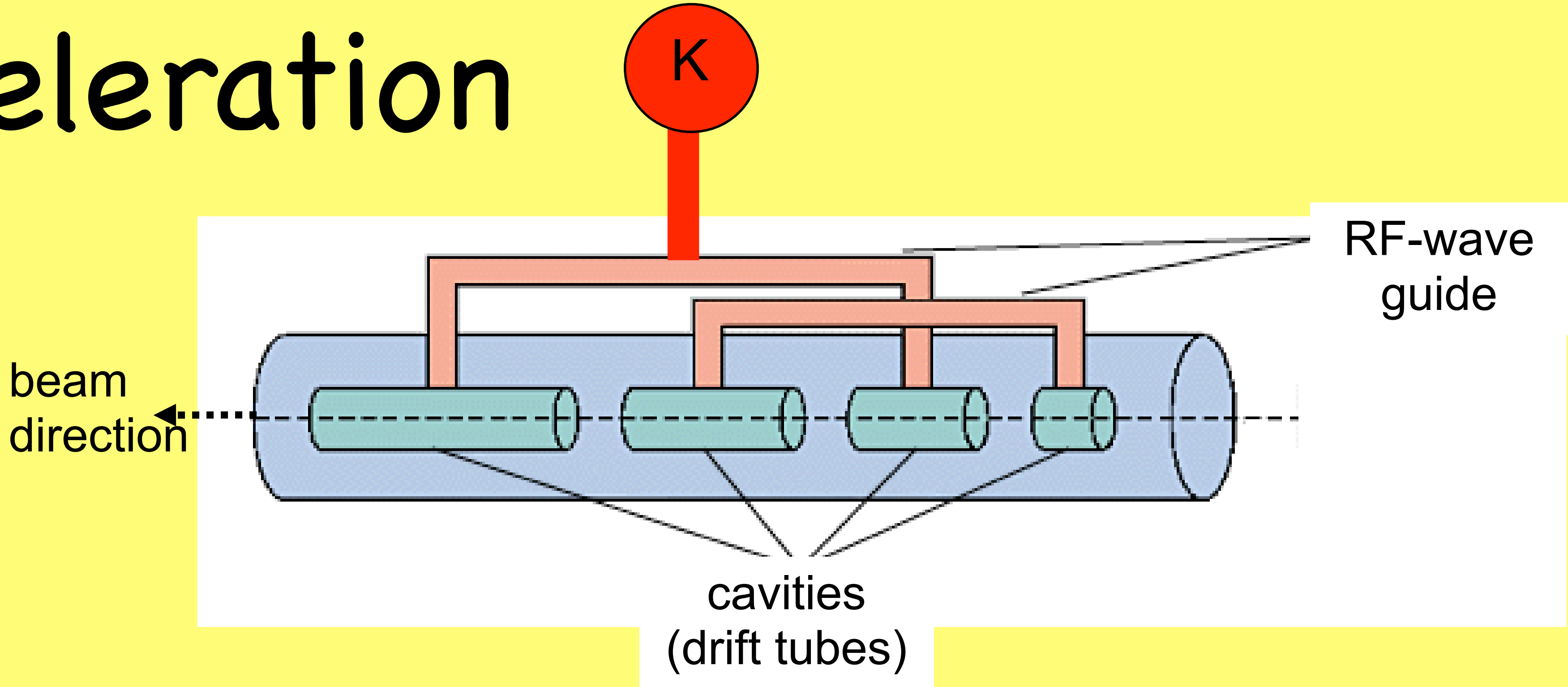
No good for our purpose.

Exploit B.C.s => disk loaded cylindrical wave guide




http://www.desy.de/~njwalker/uspas/

# Acceleration




- electrons are riding on a microwave (standing or travelling wave)
- source: klystron
- resonator: cavity

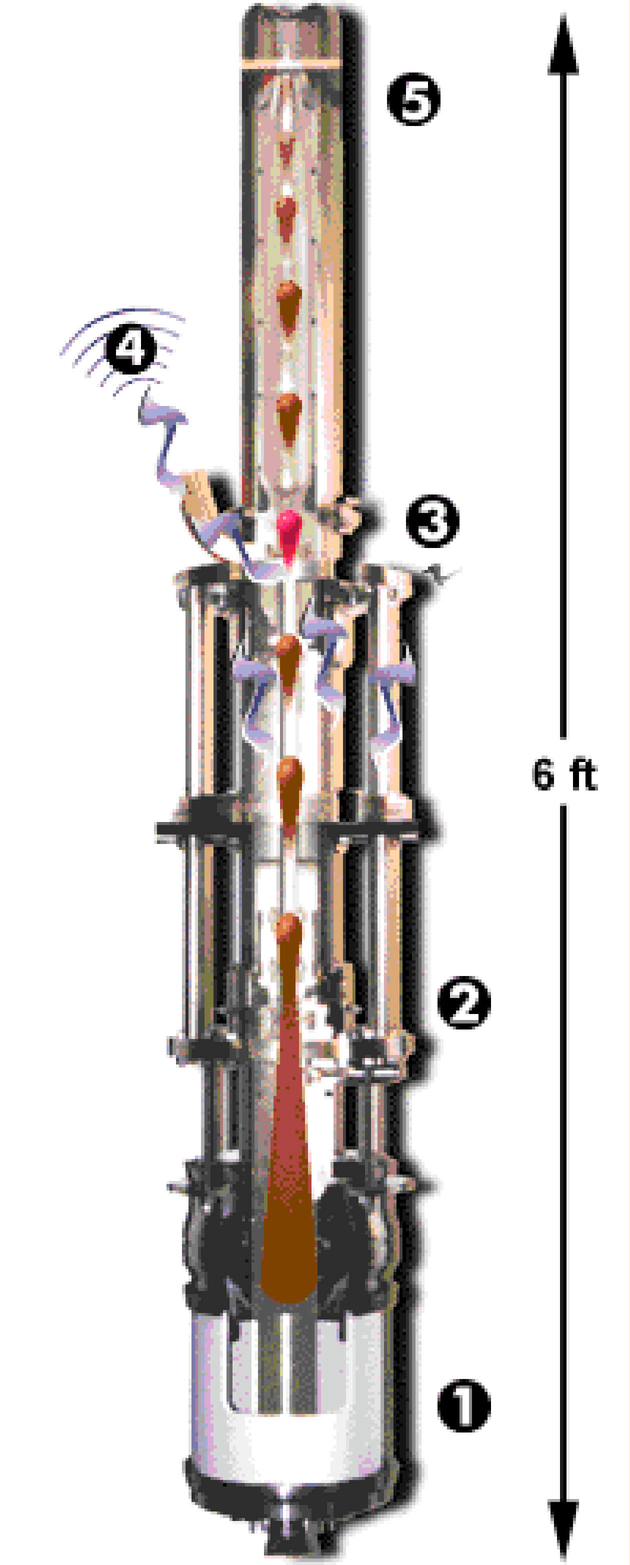
$TM_{01}$ mode: transverse magnetic field
longitudinal electric field

acceleration only if phase velocity = particle velocity and it arrives at the right time (phase)

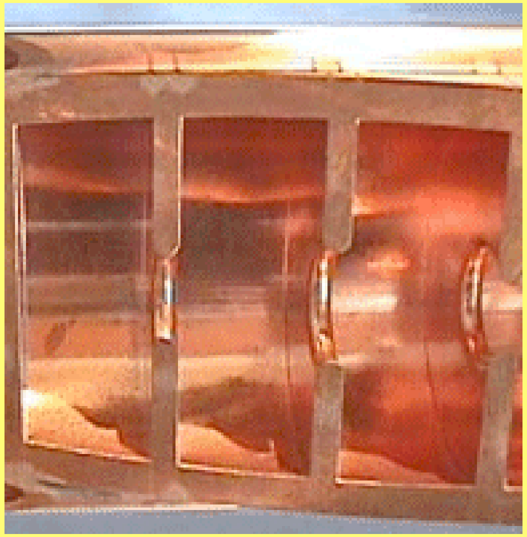
in a (hollow) wave guide: $v_\varphi > c$

# Klystron

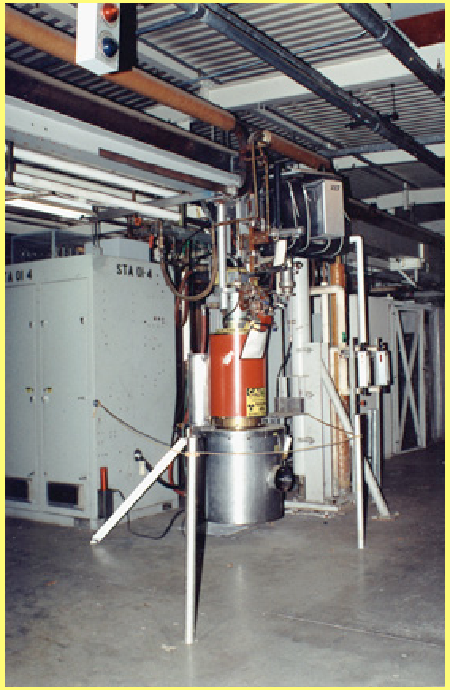
1. The electron gun produces a flow of electrons.
2. The bunching cavities regulate the speed of the electrons so that they arrive in bunches at the output cavity.
3. The bunches of electrons excite microwaves in the output cavity of the klystron.
4. The microwaves flow into the waveguide , which transports them to the accelerator.
5. The electrons are absorbed in the beam stop.

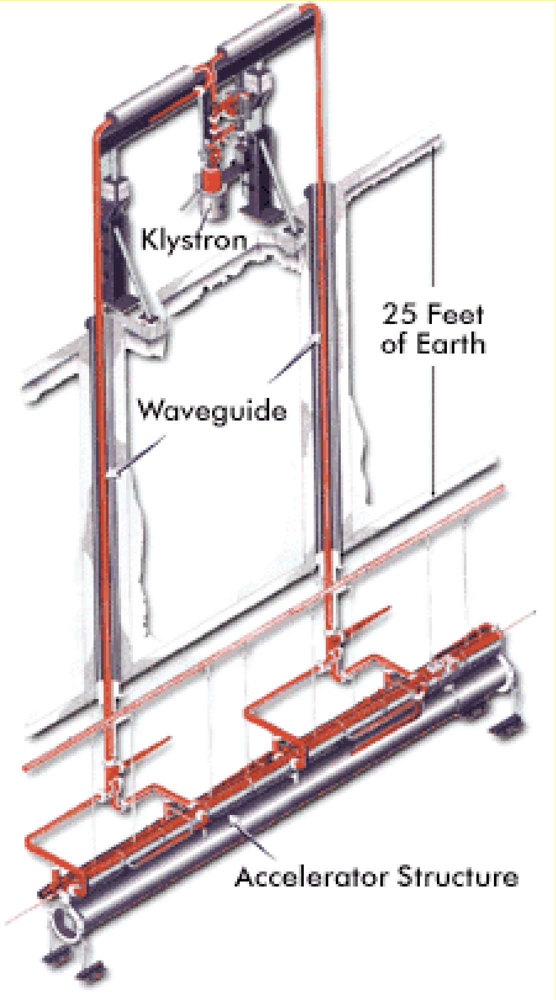


# Linear Accelerators



- Cu-cavities
- high field gradient -> large power losses
- consequences
  - pulsed machines
  - poor duty factor, $10^{-4}$ -> $10^{-2}$
  - poor energy resolution of beam, $10^{-2}$ -> $10^{-3}$
- Stanford, SLAC, Bates, NIKHEF, Saclay






http://www.desy.de/~njwalker/uspas/

# Linac Stretchers

- Add stretchers ring to linac
  - inject pulses
  - extract during time between pulses
- get duty factor of ~0.8
- get intensities of ~20 μA
- energy resolution still a problem

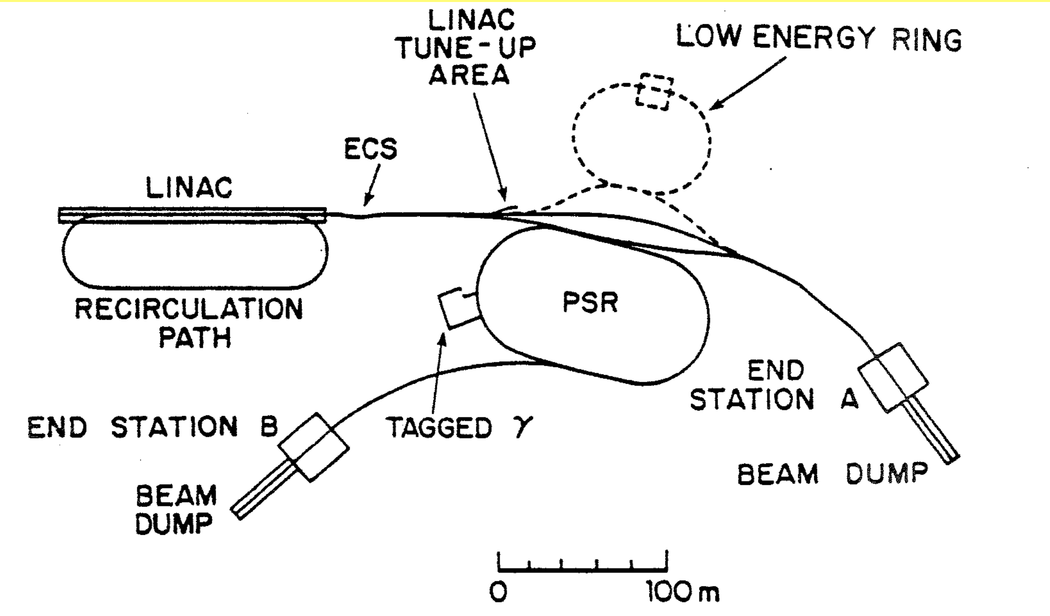
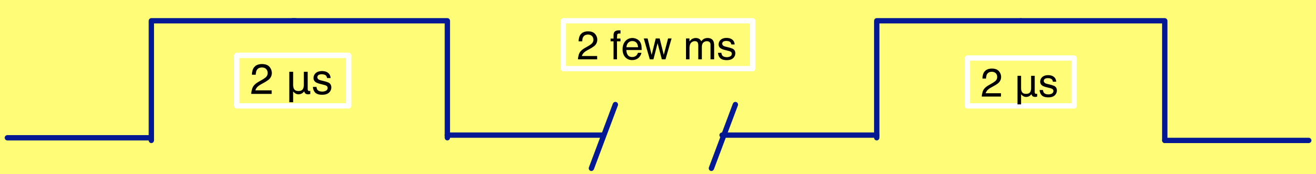
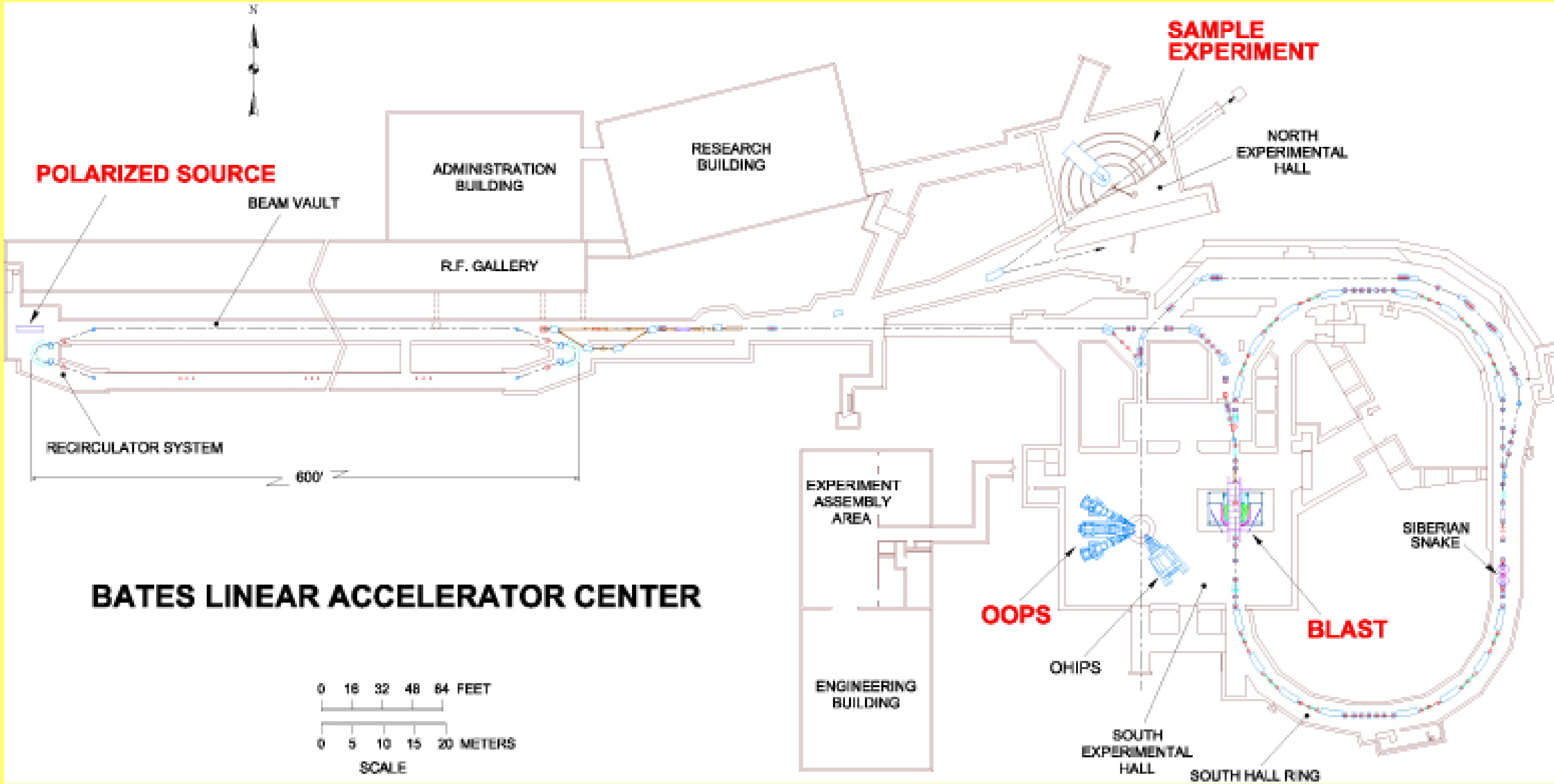



Fig. II-1. Accelerator Layout

Original design for Jefferson Lab



# Linac & Storage Ring



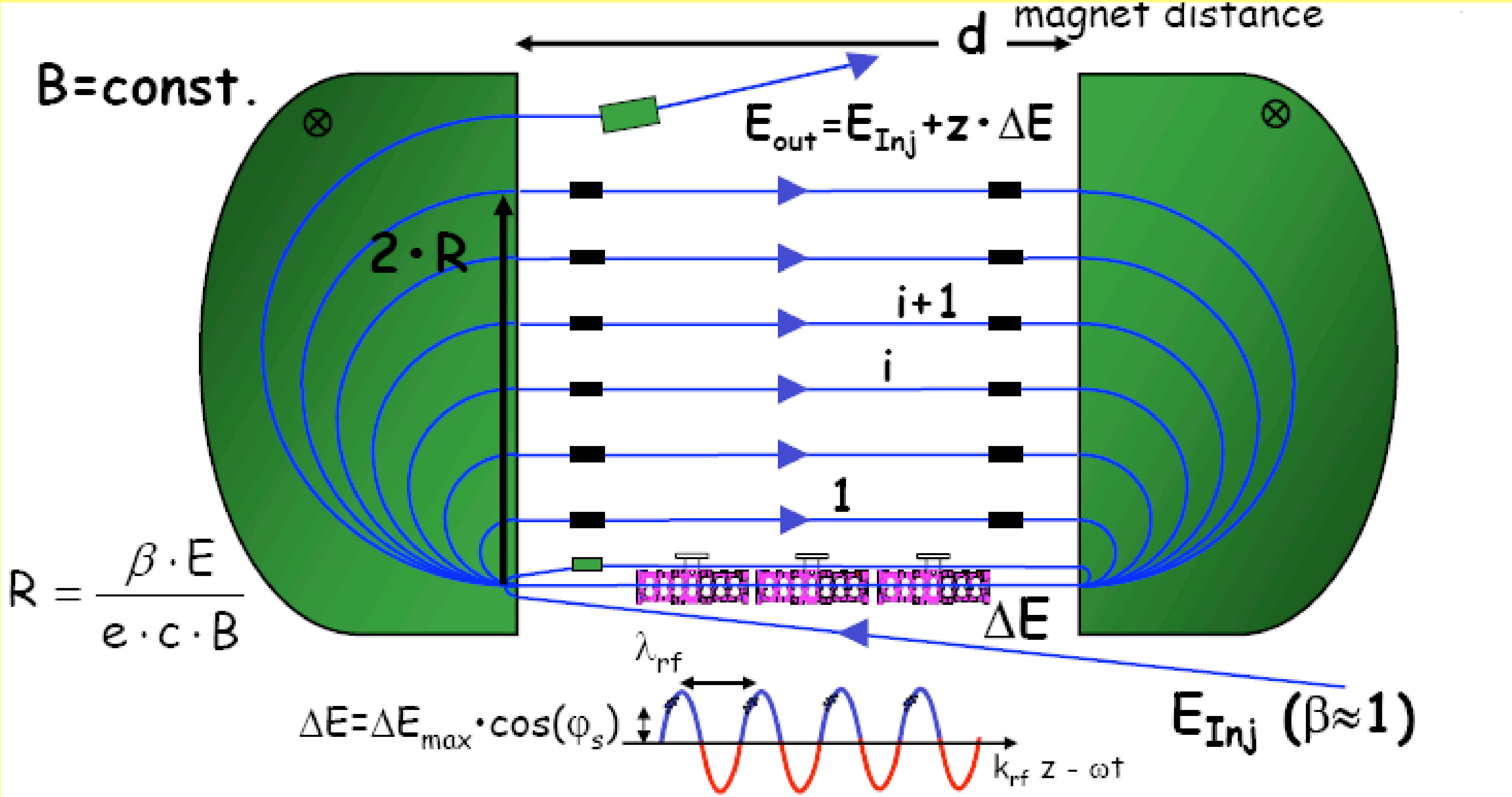

- Accumulate many pulses of linac
- internal beam of 200 mA
- use with internal targets
- good duty factor
- acceptable luminosity
- large acceptance detectors

# Modern Accelerators

- Race-track microtrons
  - Room temperature Cu cavities
  - low gradient allows for CW operation
  - recirculate many times

- Superconducting cavities
  - use Nb-cavities at 2K
  - Q-values of $10^{13}$ -> low losses
  - CW and a gradient of 5 - 10 MeV/m
  - JLAB, Darmstadt

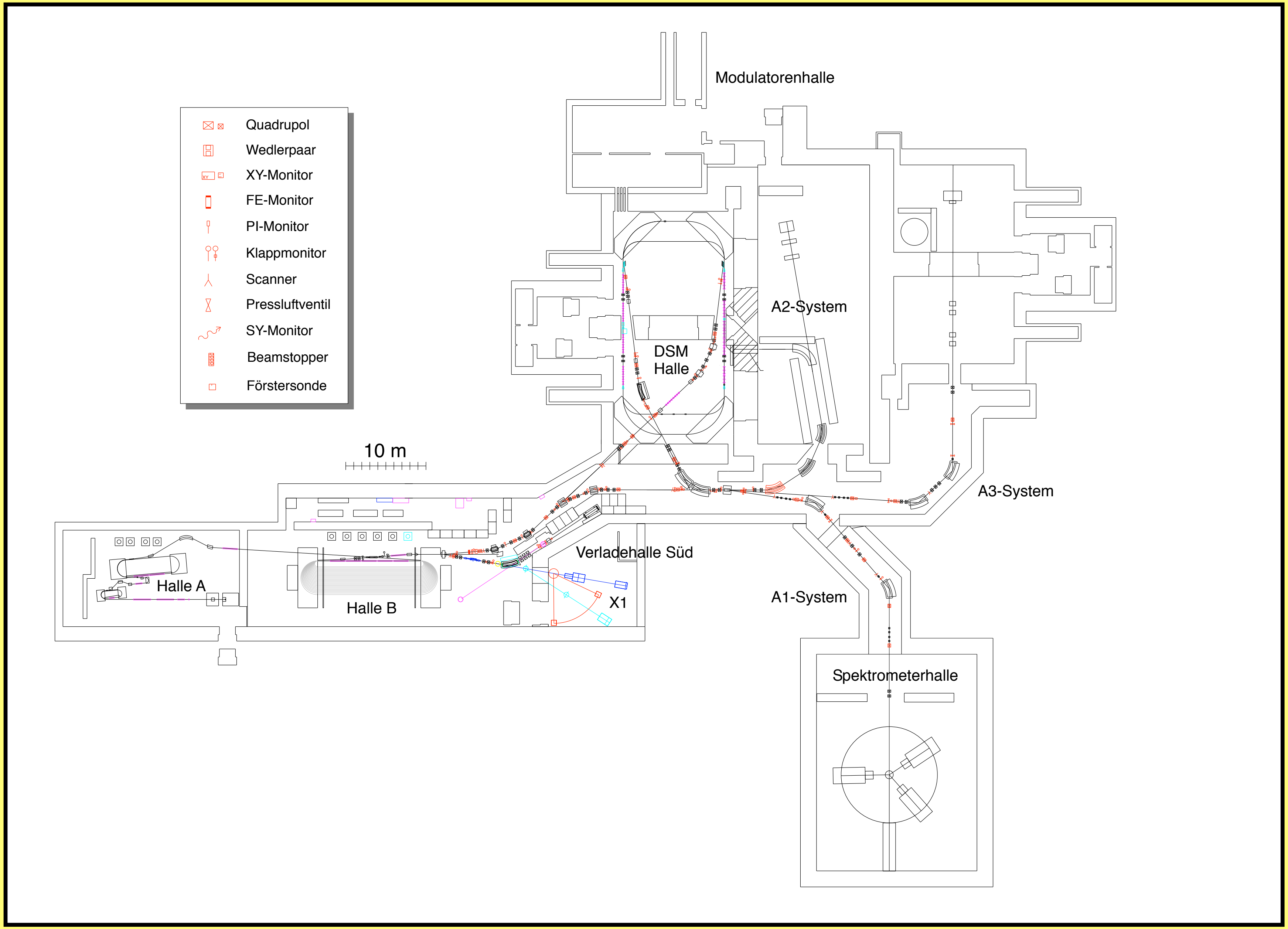
# Racetrack Microtron




- neighboring cavities have opposite polarity
- travel time of e to next cavity = 1/2f
- one accelerator
- focusing on only one path
- corrector magnets on every orbit

# MAMI

3 consecutive microtrons (200-800 MeV)
4th stage: Double sided microtron -> 1.6 GeV



# Superconducting cavity:

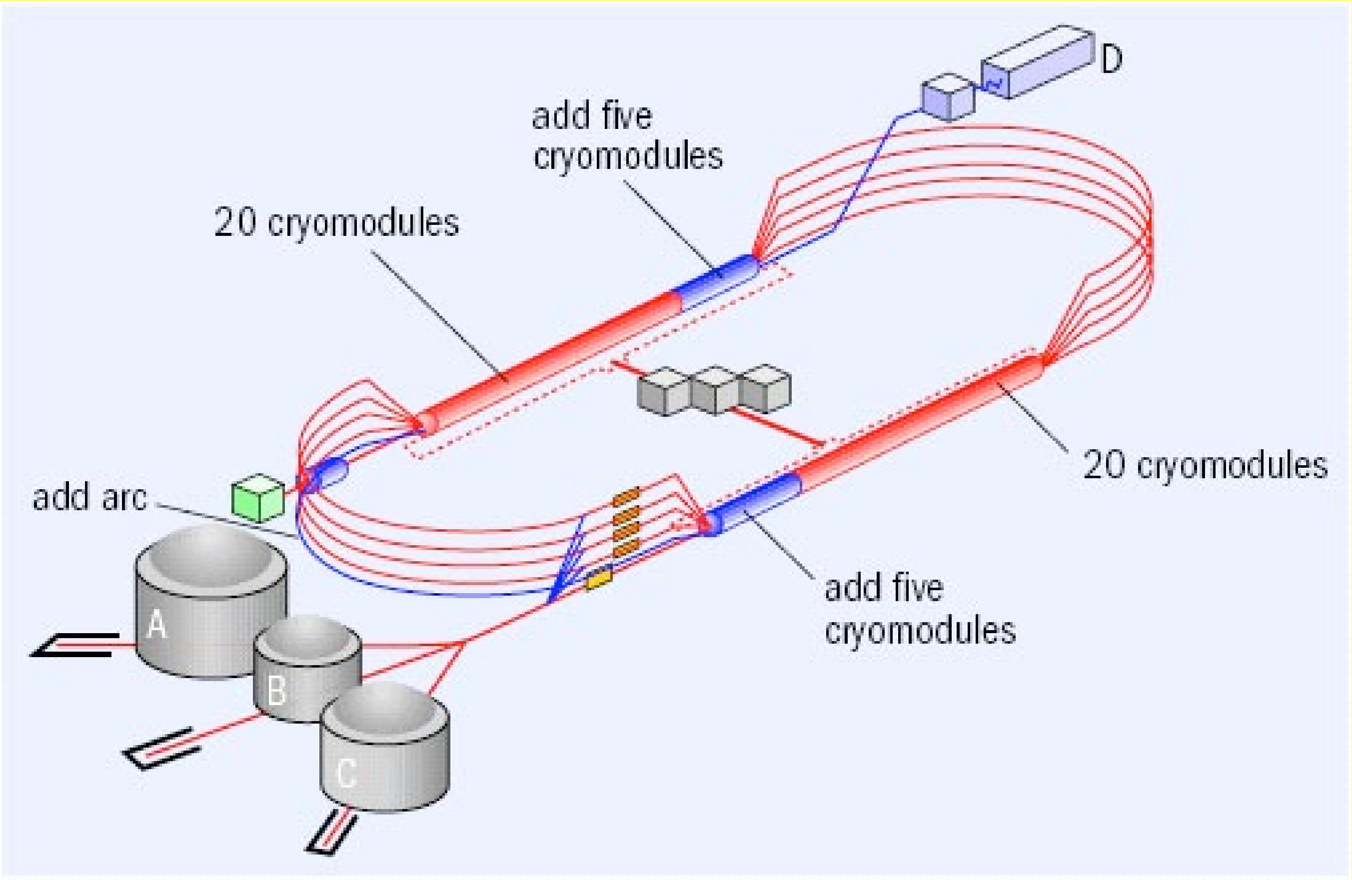
2 niobium (multi-) cavity

CEBAF: 2x160 cavities, 1.497 GHz,
2 K (liquid He), 7.7 MV/m

high gradients needed!
record: 51 MV/m KEK (single cell)
standard: 35 MV/m
problem:
magnetic field at the surface heats cavity up
cleaning room:
surface ultra-smooth, scrubbed with
100 bar water beam
high Q: loaded $\sim 10^7$
intrinsic $\sim 10^{10}$

for ILC:
20000 Nb cavities needed,
= 500 t Nb
= 4 years production
= 300 M$

# CEBAF



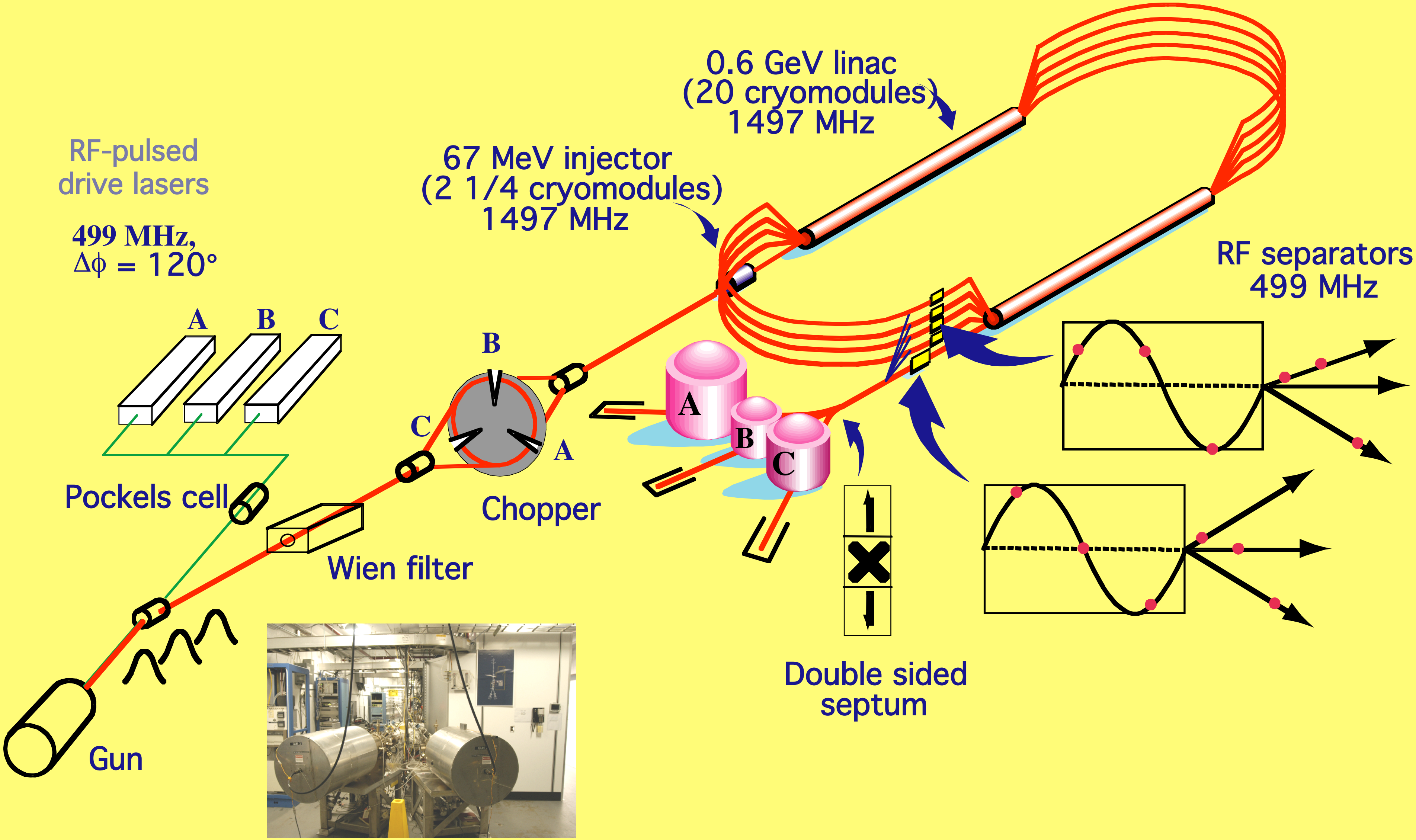

- double-sided microtron, superconducting Nb cavities at 2K
- 7.7 MeV/m
- high Q ~$10^6$
  - low losses
- energy: 0.8 – 6 GeV
  - spread: 5 10-5
- current: 1 – 120 µA (A & C), 1nA – 1µA (B)

spot at target: > 50 µm
divergence : < 100 µrad

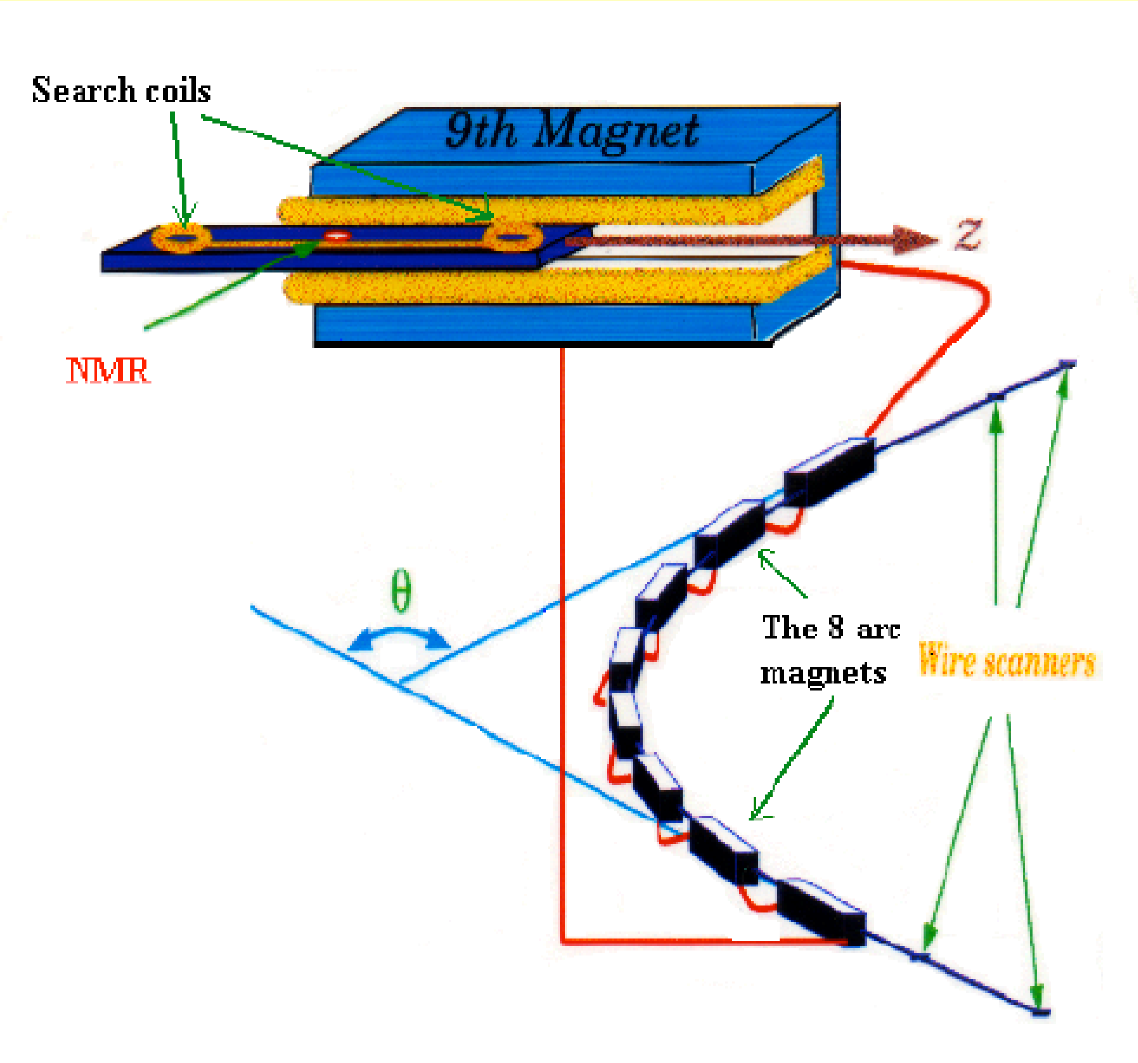
RF beam splitter -> 3 simultaneous beams

correlated energies
independent currents

# Continuous Electron Beam Accelerator Facility



# Energy Measurement




energy: $E = \frac{c}{\theta} \int B dl$

needed:

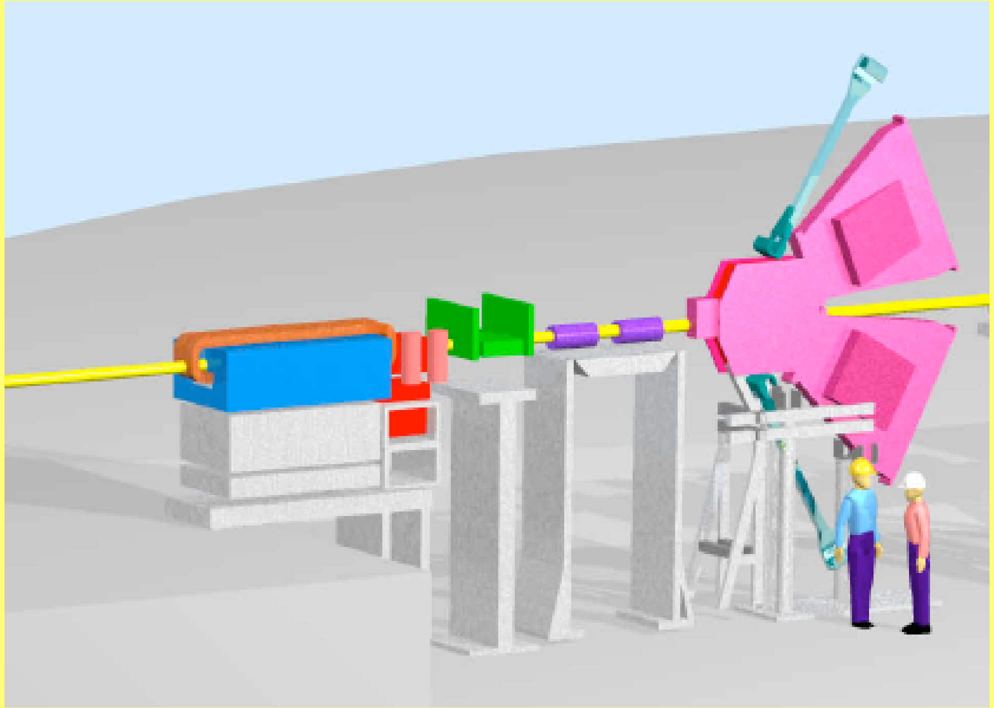
- measurement of angle
  wire scanners survey

- field integral
  reference magnet/NMR
  measured to $10^{-5}$

accuracy of energy measurement: $2\ 10^{-4}$

# Another method: e-p elastic (Jlab, Hall A)

Measure electron and recoil angle of the electron and proton scattered from H



target: 10-30 µm $CH_2$

detectors: Si strip

proton: fixed at 60°,
TOF measurement

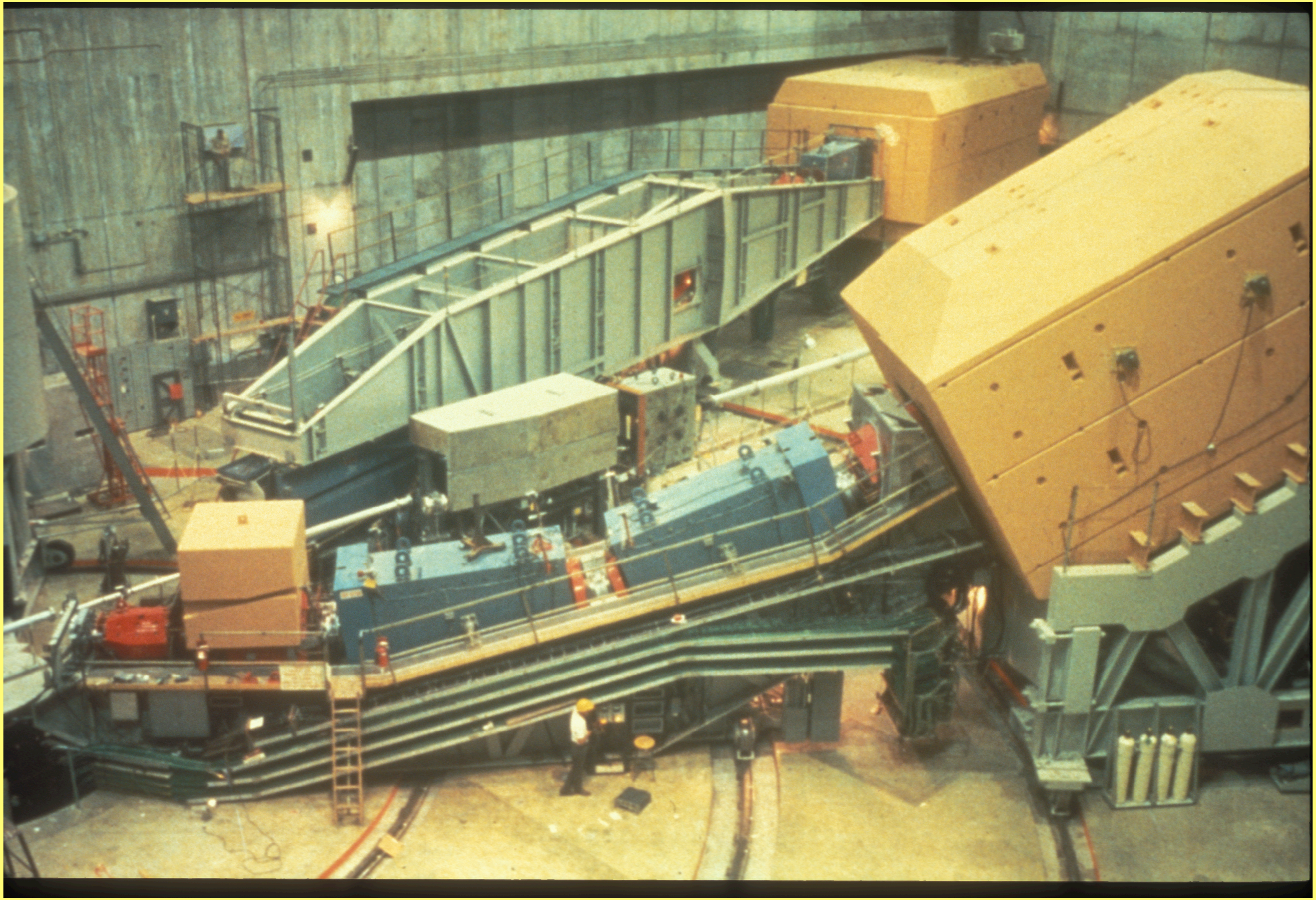
electron: 9° – 41°
Cerenkov

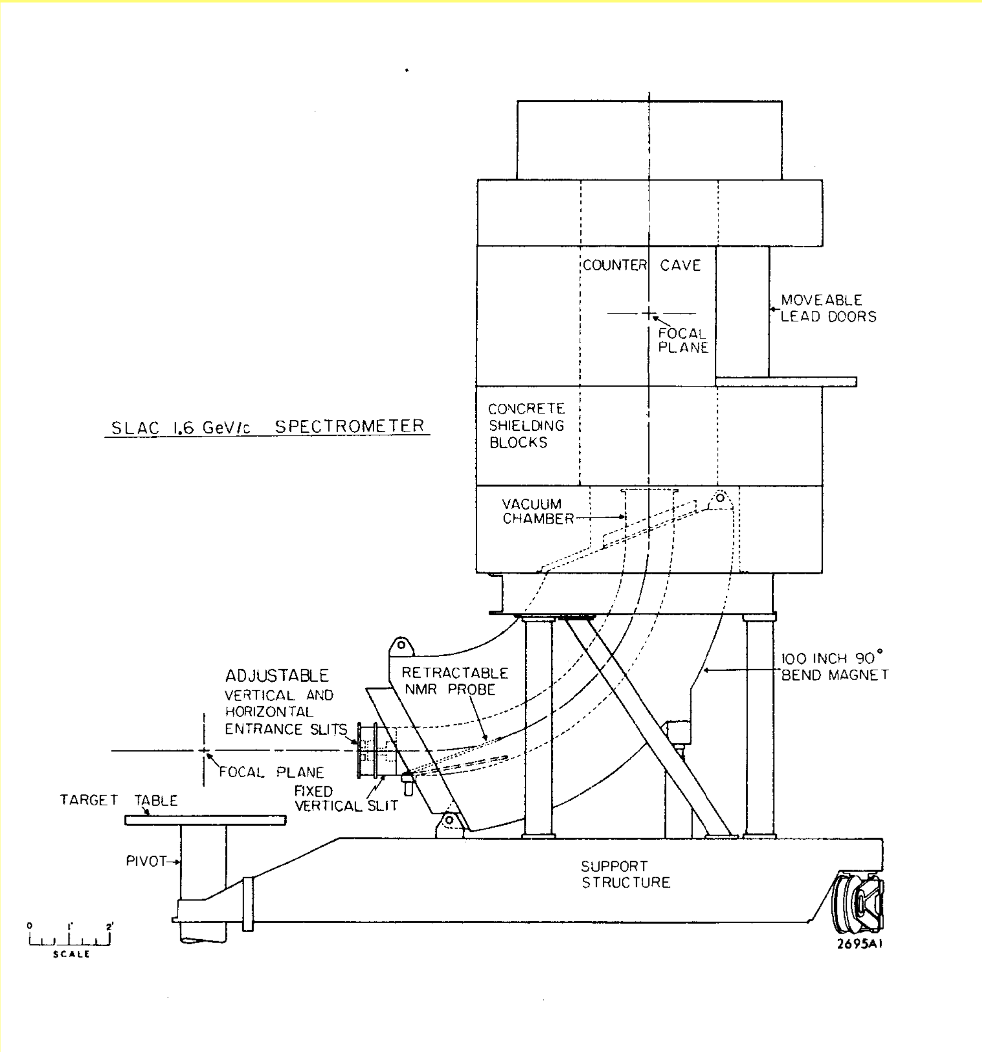
beam energy
range: 0.5 – 6 GeV
accuracy: $\Delta p/p < 2\ 10^{-4}$

All three measurements agree!

arc in hall A&C, ep-method

# Spectrometers/Detectors






Counter array used to detect electrons in the 1.6 counter cave. The Pb glass hosodscope, pre-radiator, TA1 and TA2 formed the total absorption counter.

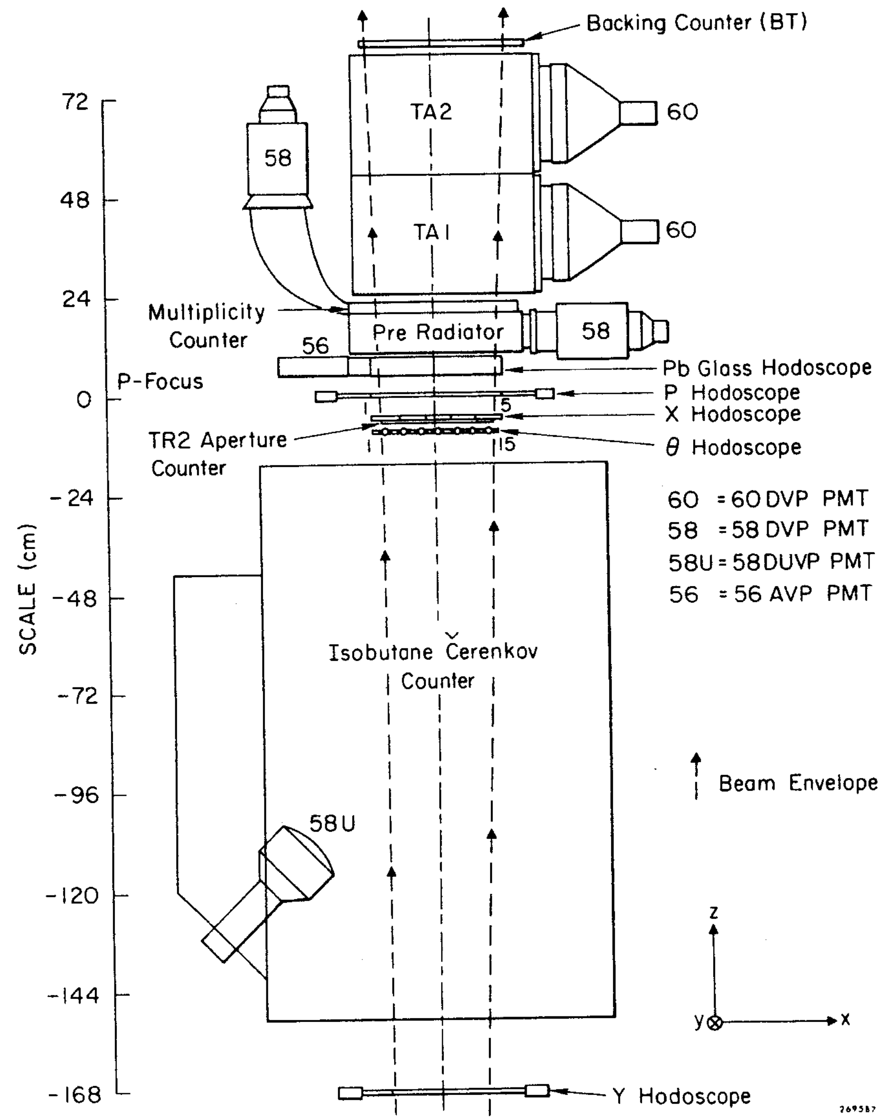
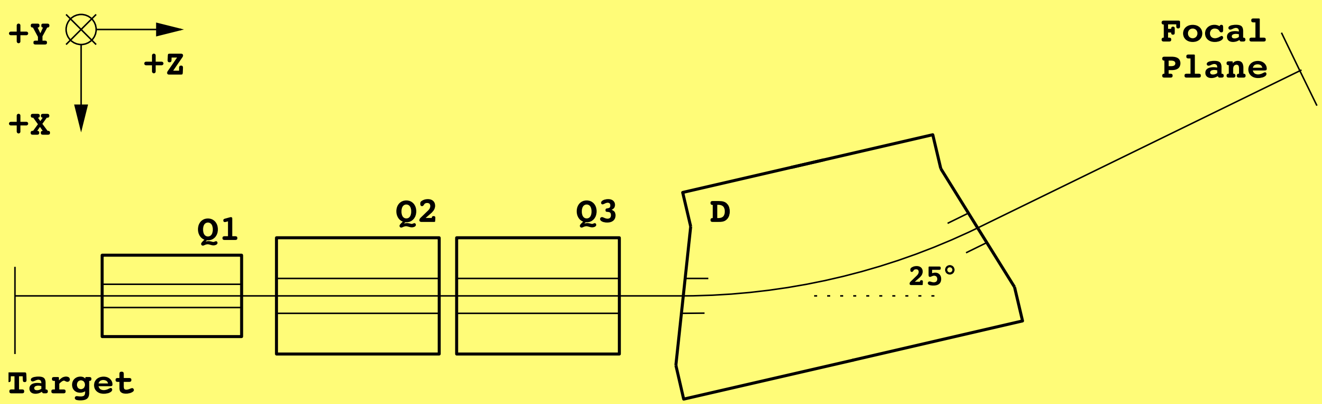
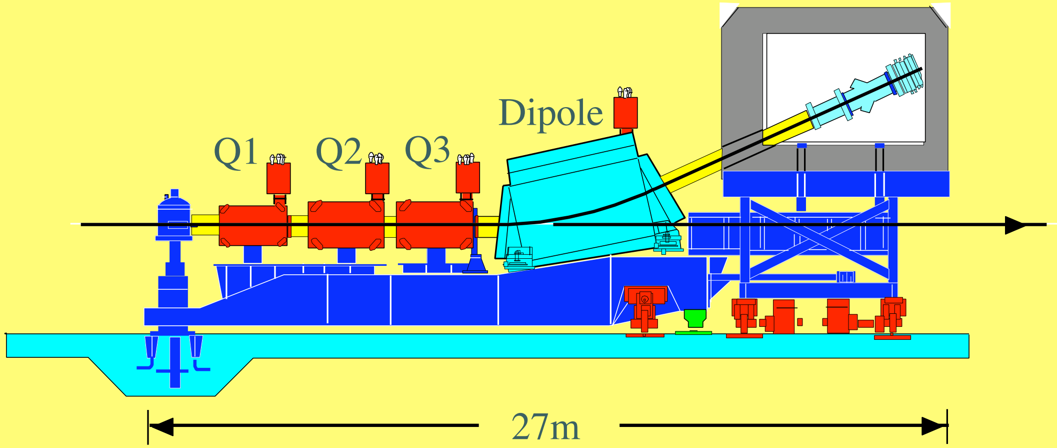
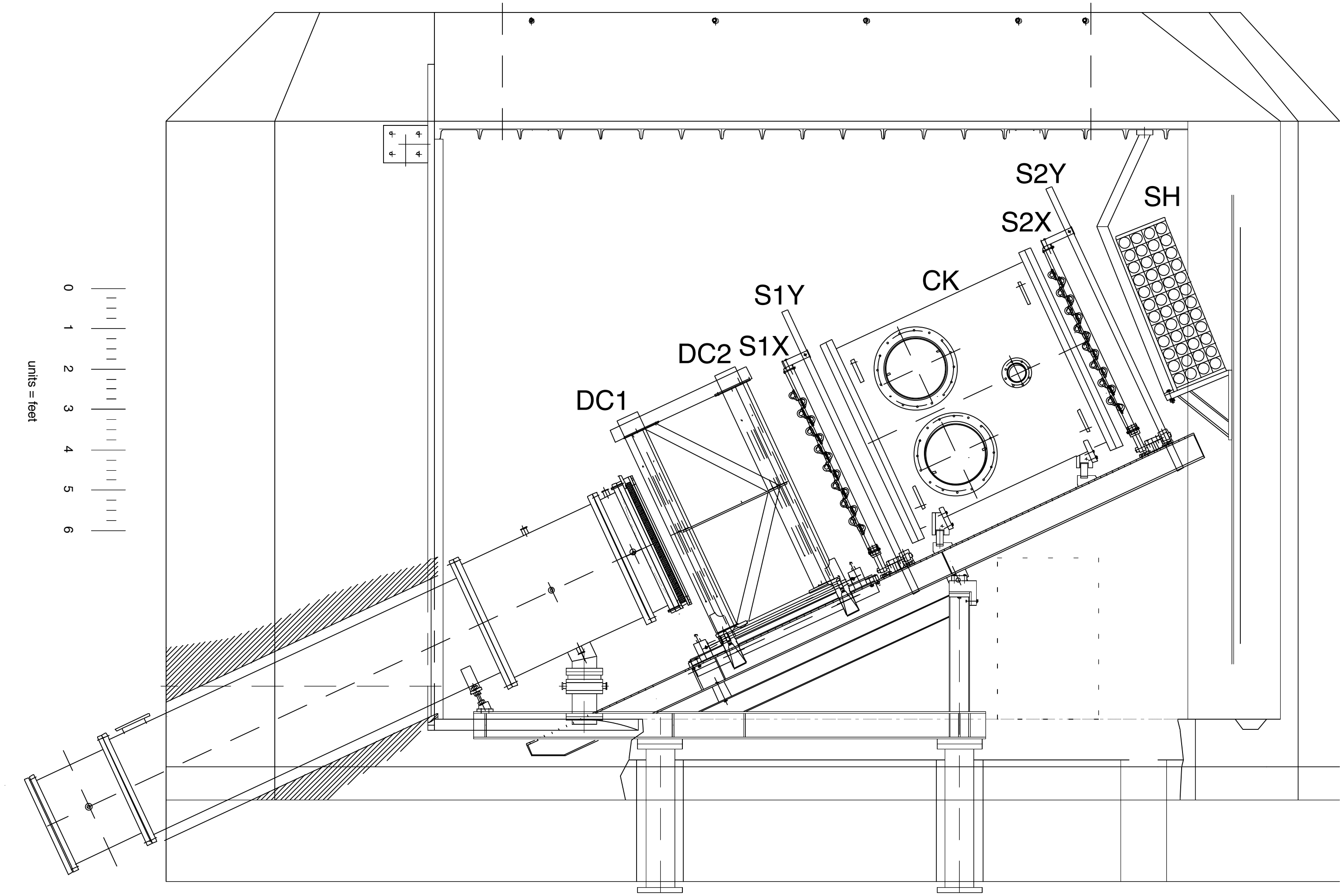



FIG.II-8a

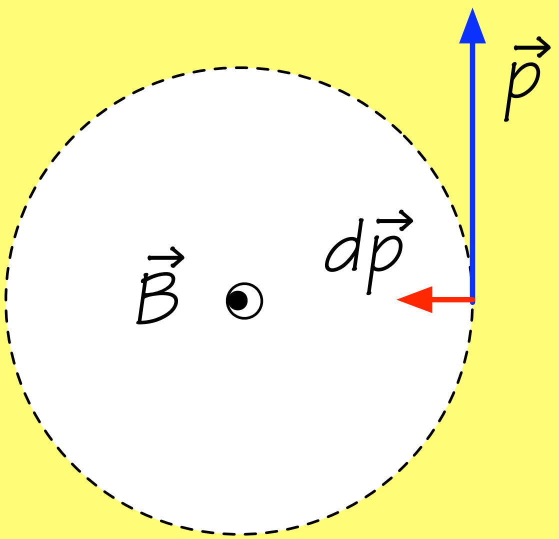
# Hall C
# HMS




Dipole
Q1
Q2
Q3
27m


S2Y
SH
S2X
CK
S1Y
DC2
S1X
DC1
0 1 2 3 4 5 6
units = feet

# Deflection of electrons in magnetic field



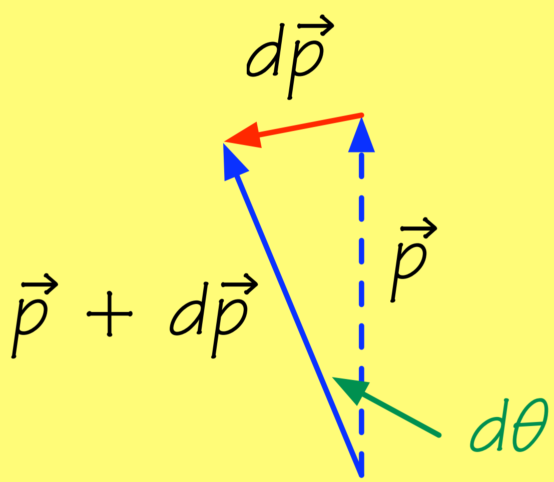

**Lorentz Force**

$$\boxed{\vec{F} = q\vec{v} \times \vec{B} = \frac{d\,\vec{p}}{dt}}$$

$$d\vec{p} = q(\vec{v}dt) \times \vec{B} = q d\vec{l} \times \vec{B}$$

$$d\theta = \frac{dp}{p} = \frac{q}{p} B dl$$

$$\boxed{\Rightarrow \Delta\theta = \frac{q}{p} \int B dl}$$



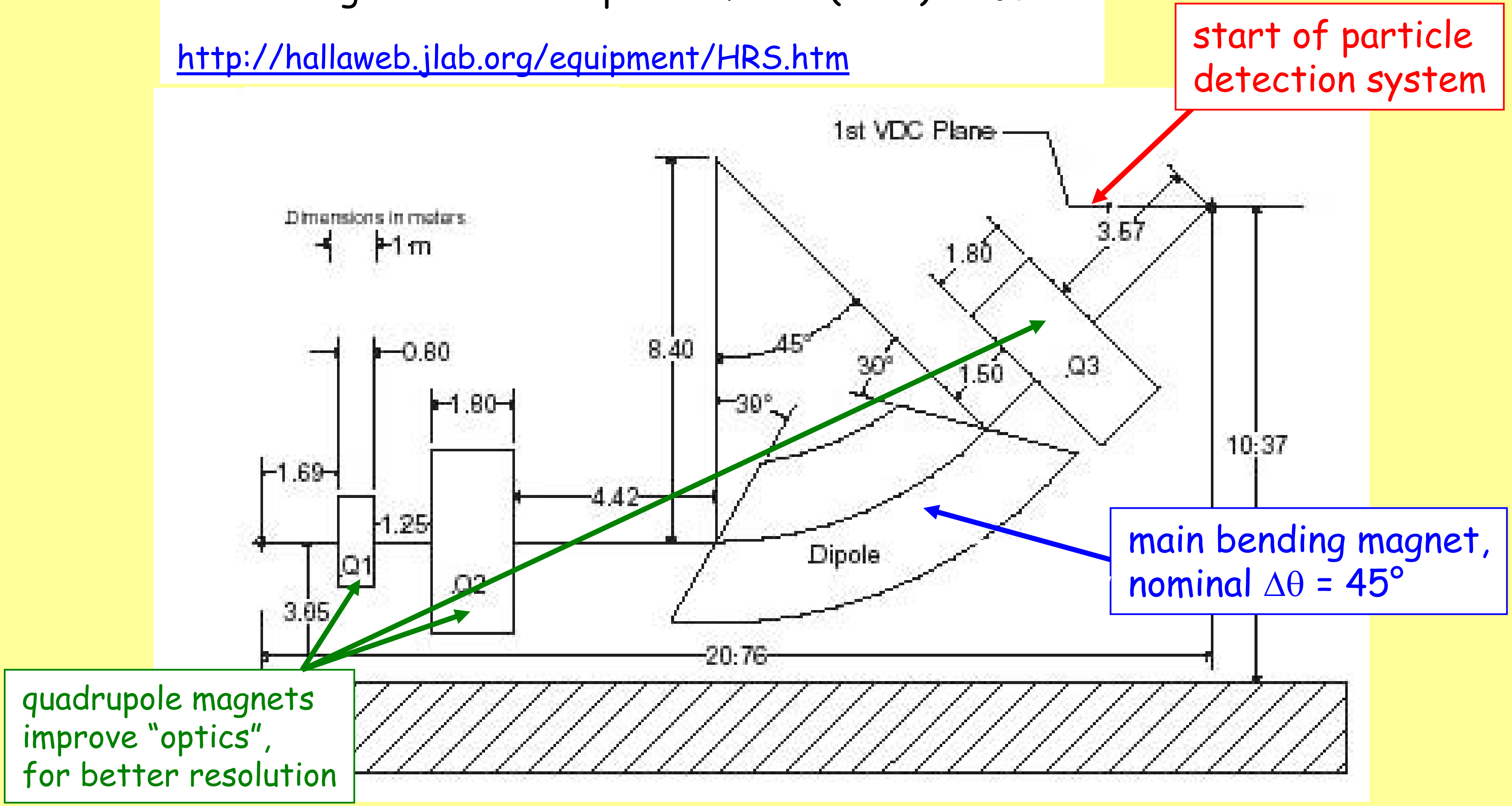

Deflection $\Delta\theta$, even if B field is not uniform

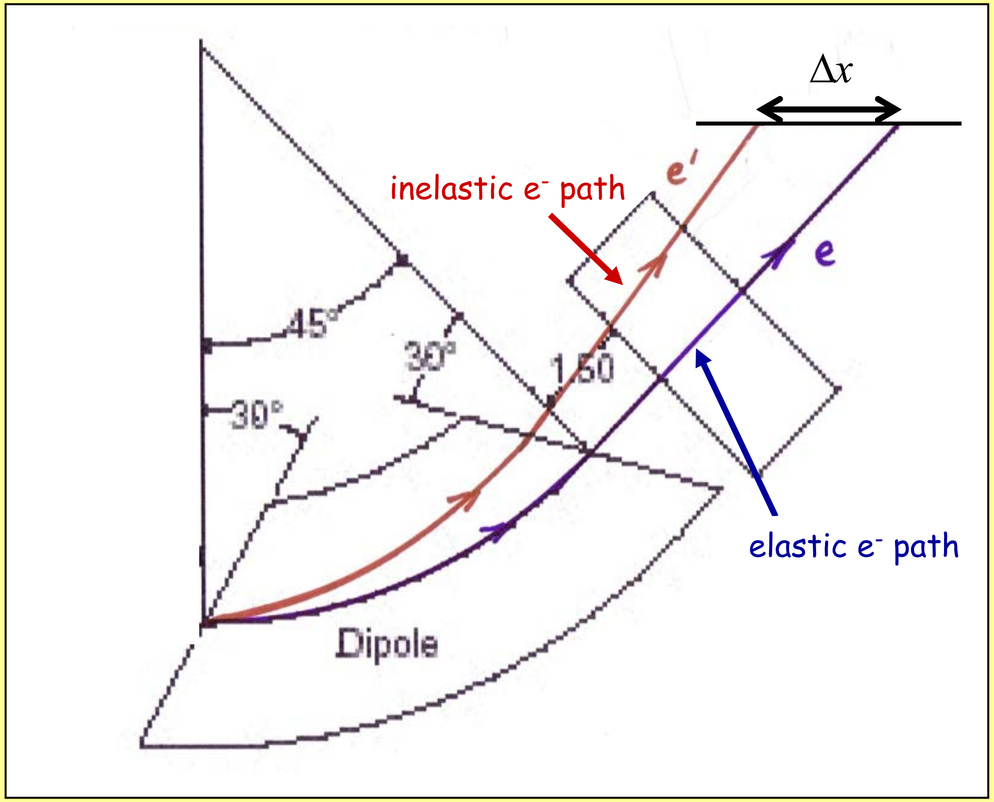
# Deflection in magnetic field measures the momentum

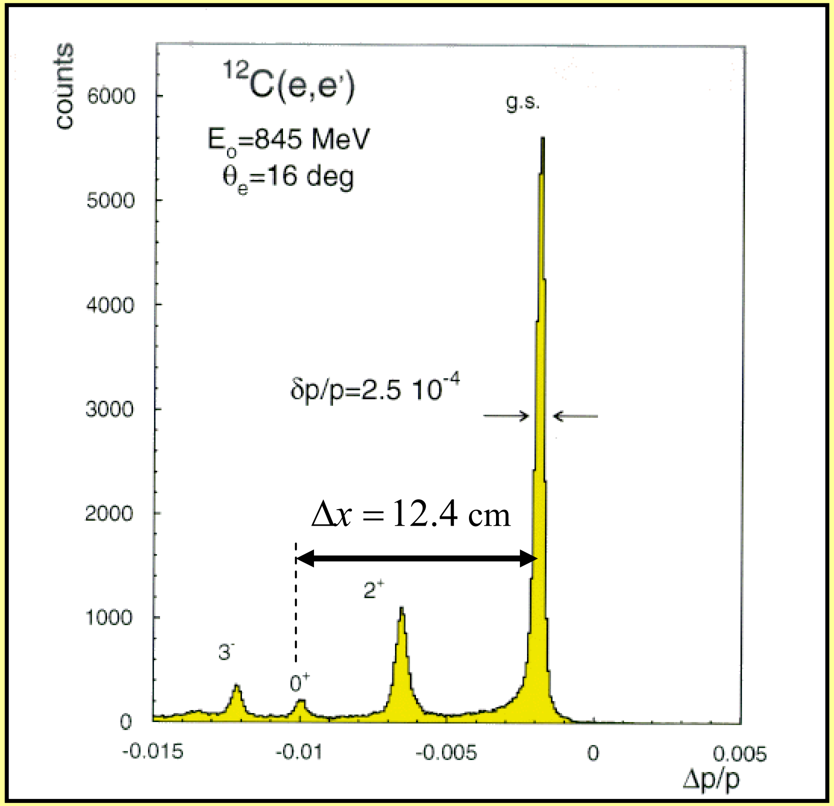
$$\Delta\theta = \frac{e}{p}\int B\,d\ell = \frac{const.}{p} \quad \text{for central trajectories}$$

Hall A High Resolution Spectrometer (HRS) at JLab:

http://hallaweb.jlab.org/equipment/HRS.htm





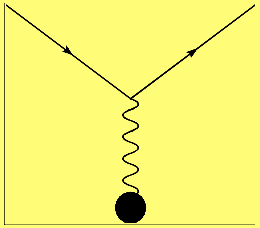




**Dispersion:** D = 12.4 cm/ %

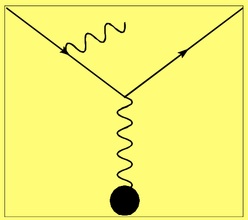
→ A 1% shift from the central momentum corresponds to a deflection at the focal plane of 12.4 cm **more** than the elastic peak.

# Basics of QED radiative corrections

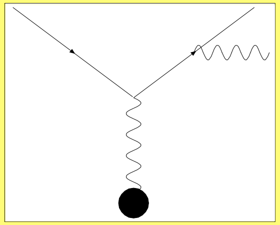
(internal bremsstrahlung: Mo,Tsai, Maximon)



(First) Born approximation



Initial-state radiation

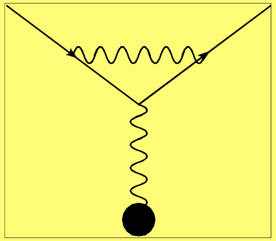


Final-state radiation

- photon emission during reaction,
- changes cross section

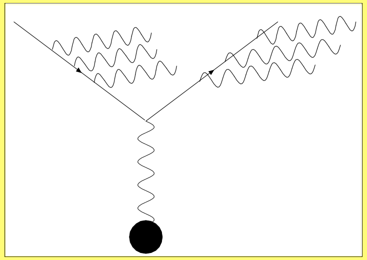
Cross section for photon emission ~ $d\omega/\omega$

=> integral diverges logarithmically: IR catastrophe



Vertex correction => cancels divergent terms; Schwinger (1949)

$$\sigma_{exp} = (1+\delta)\sigma_{Born}, \ \delta = \frac{-2\alpha}{\pi}\left[(\ln\frac{E}{\Delta E} - \frac{13}{12})(\ln\frac{Q^2}{m_e^2} - 1) + \frac{17}{16} + \frac{1}{2}f(\theta)\right]$$



Multiple soft-photon emission: solved by exponentiation,

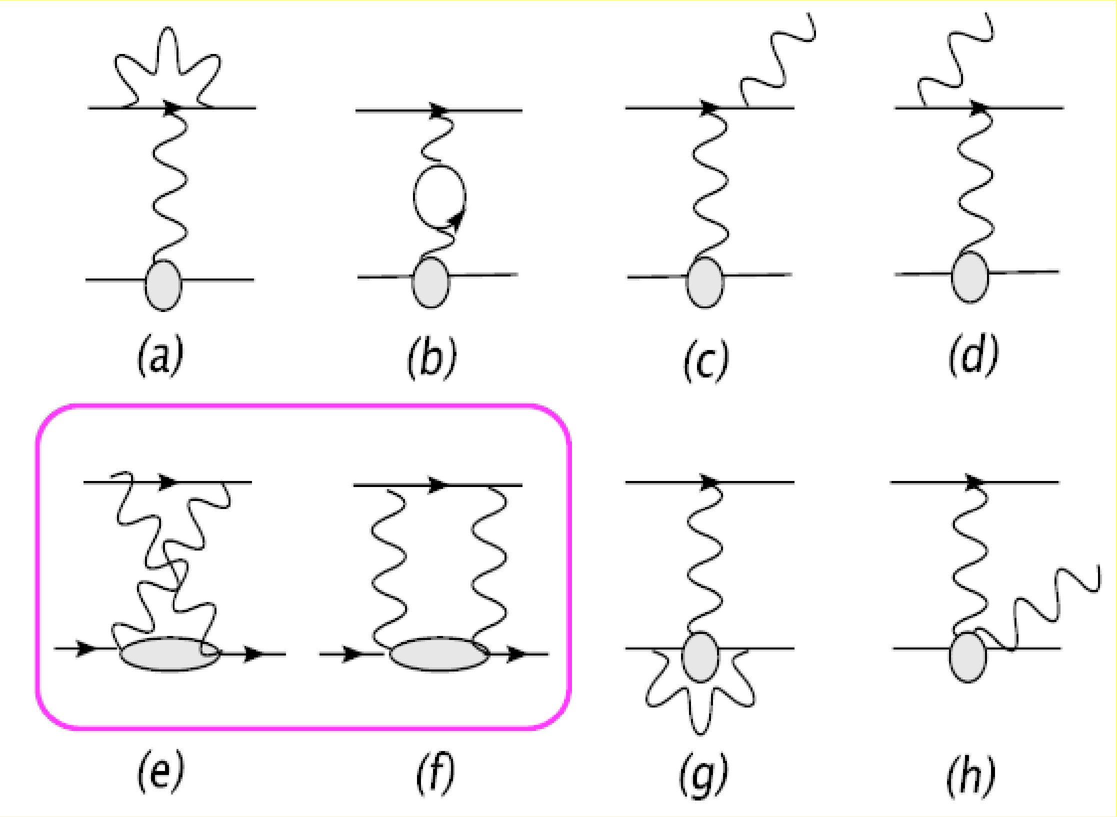
Yennie-Frautschi-Suura (YFS), 1961

$$(1+\delta) \Rightarrow e^{\delta}$$

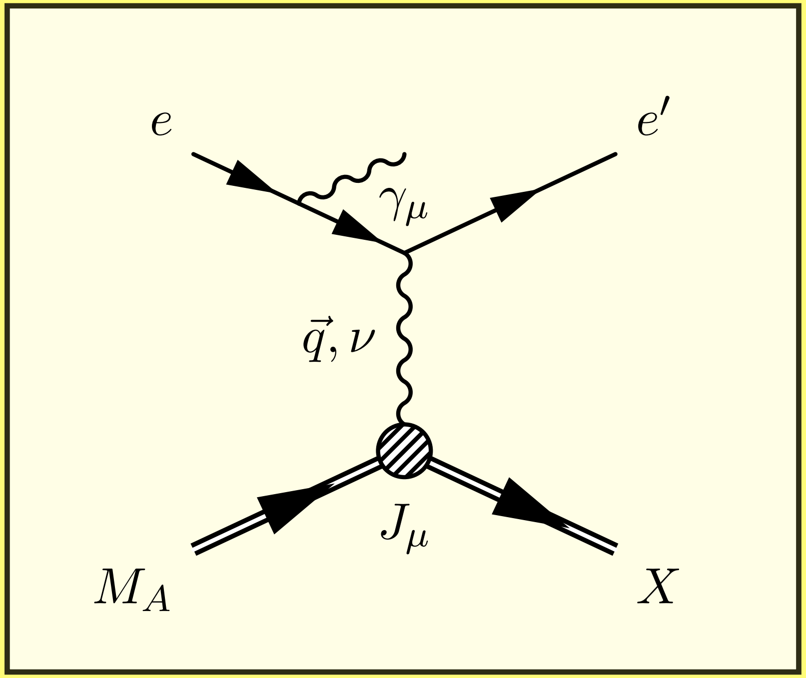
# Radiative corrections

Radiative Corrections:

- Electron vertex correction (a)
- Vacuum polarization (b)
- Electron bremsstrahlung (c,d)
- Two-photon exchange (e,f)
- Proton vertex and VCS (g,h)
- Corrections (e-h) depend on the nucleon structure

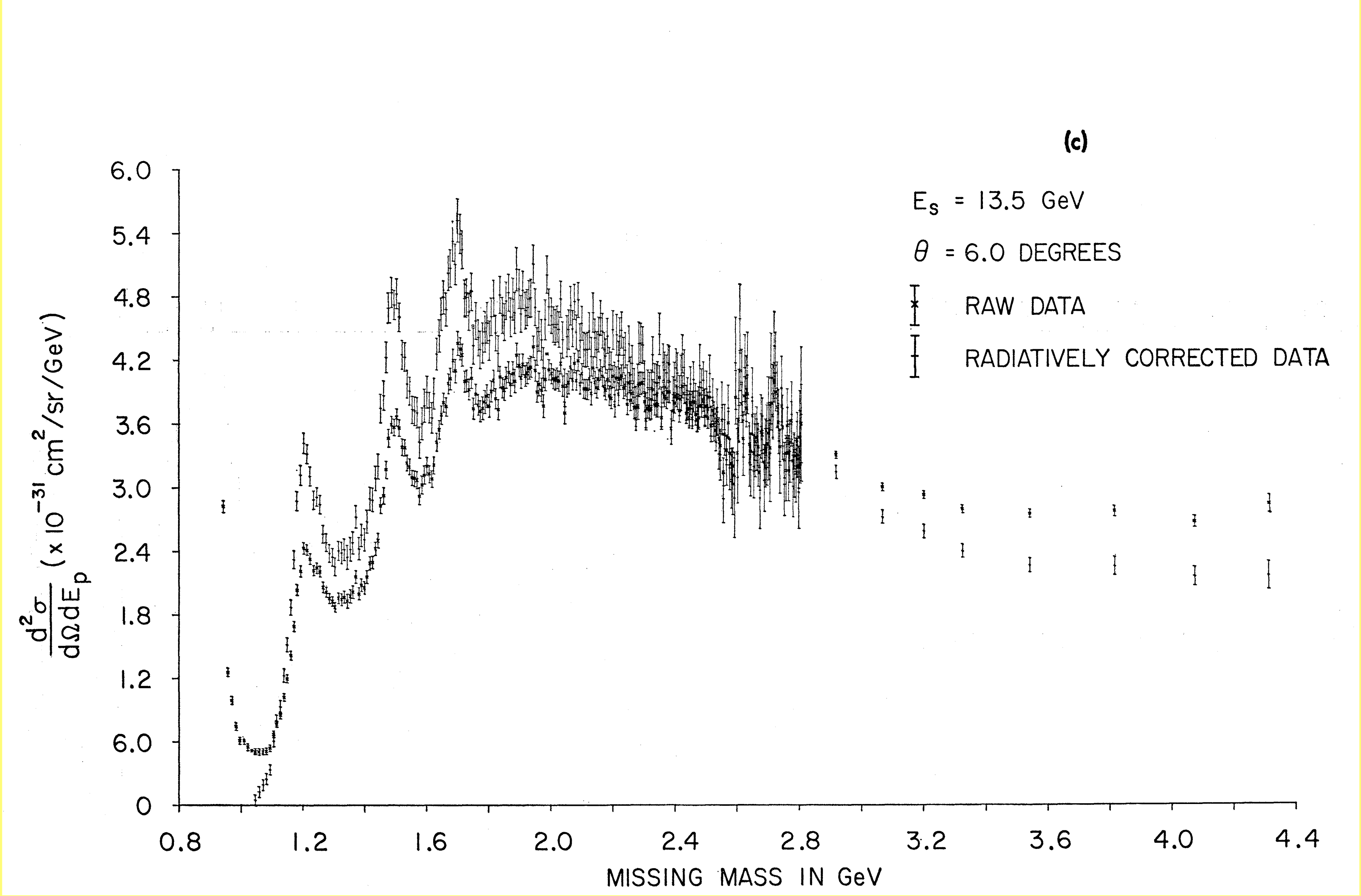


# How are corrections done?

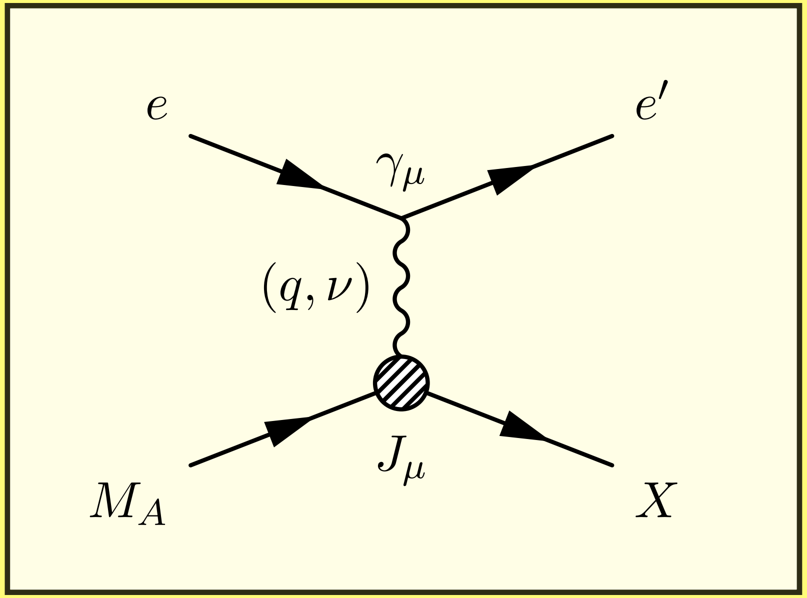


# Radiative corrections

## Inelastic Electron-Proton Scattering



# Some kinematics




4-momentum transfer:
$q^2$ = (energy change)$^2$ - (momentum change)$^2$

$$e_\mu = (e, \vec{e}) \qquad e'_\mu = (e', \vec{e}')$$

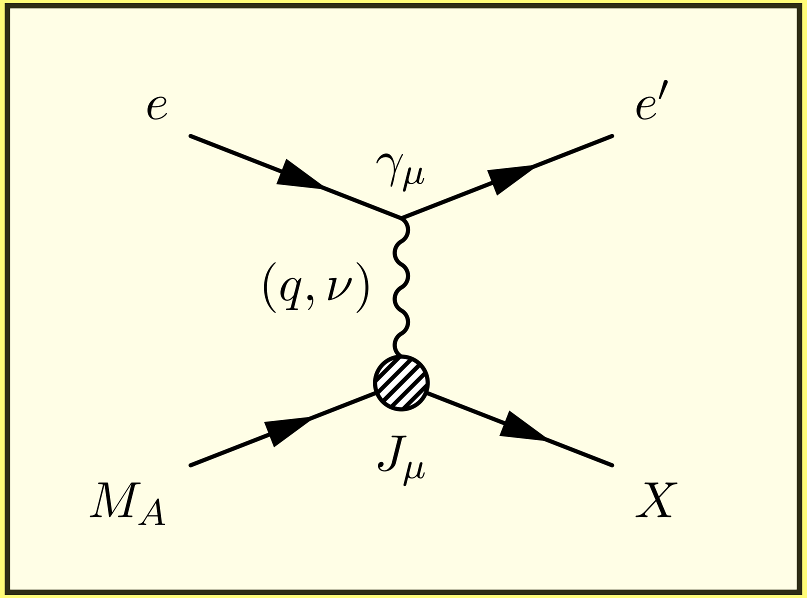
$$\begin{aligned} q^2 &= (e - e')^2 - (\vec{e} - \vec{e}')^2 \\ &= e^2 + e'^2 - 2ee' - (e^2 + e'^2 - 2ee' \cos(\theta)) \\ &= -2ee'(1 - \cos(\theta)) \\ &= -2ee'(2 \sin^2 \frac{\theta}{2}) \\ &= -4ee' \sin^2 \frac{\theta}{2} \equiv -Q^2 \end{aligned}$$

$$\begin{aligned} (e - e' + P)^2 &= X^2 \\ q^2 + P^2 + 2Pq &= X^2 \\ -Q^2 + M^2 + 2M\nu &= X^2 \end{aligned}$$

For elastic scattering $X^2 = M^2$ and $Q^2 = 2M\upsilon$

Usually X is called W and referred to as the mass of the final hadronic state.

# Some kinematics




$$W^2 = q^2 + M_T^2 + 2\nu M_T$$

$$W^2 = -2ee'(1-\cos\theta) + 2(e-e')M_T + M_T^2$$

$$W^2 - M_T^2 - 2eM_T = -e'(2e(1-\cos\theta) + 2M_T)$$

$$e' = \frac{M_T^2 - W^2 + 2eM_T^2}{2e(1-\cos\theta) - 2M_T}$$

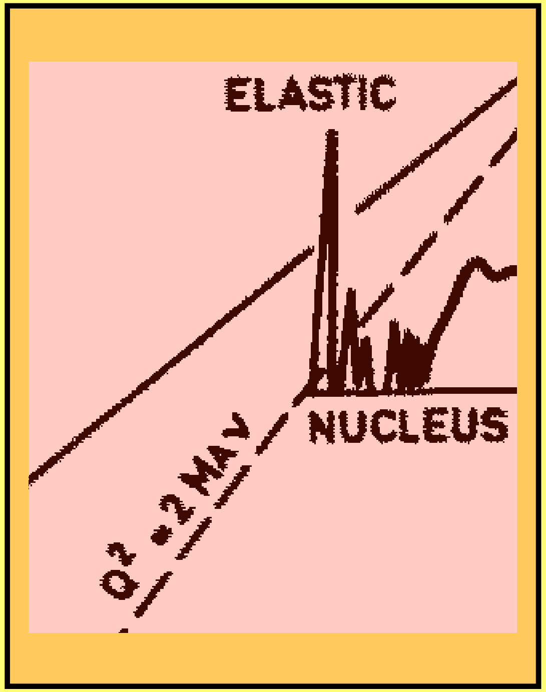
Elastic scattering, W = M,

$$e' = \frac{e}{1 + \frac{2e}{M}\sin^2\theta/2}$$

What if X = W = ($M_T$ + $m_\pi$),

$$e' = \frac{M_T^2 - (M_T + m_\pi)^2 + 2eM_T^2}{2e(1-\cos\theta) - 2M_T}$$

# I. Elastic Electron Scattering from Nuclei



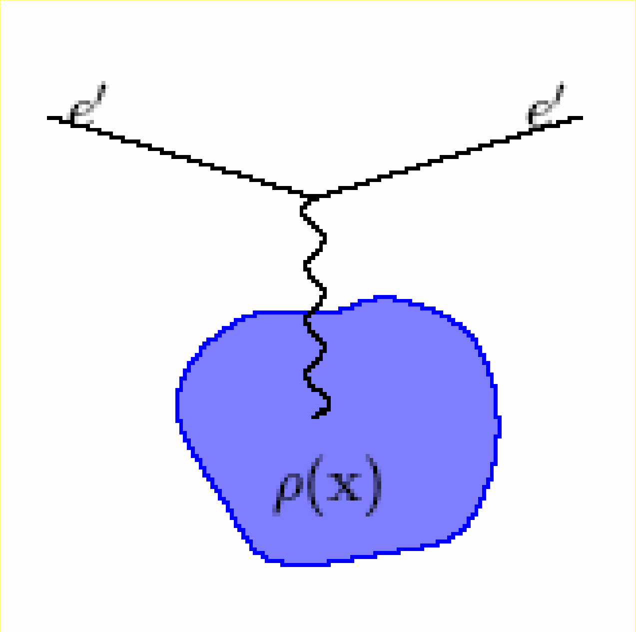

Fermi's Golden Rule

$$\frac{d\sigma}{d\Omega} = \frac{2\pi}{\hbar} \mid M_{fi} \mid^2 D_f$$

$M_{fi}$: scattering amplitude

$D_f$: density of the final states (or phase factor)

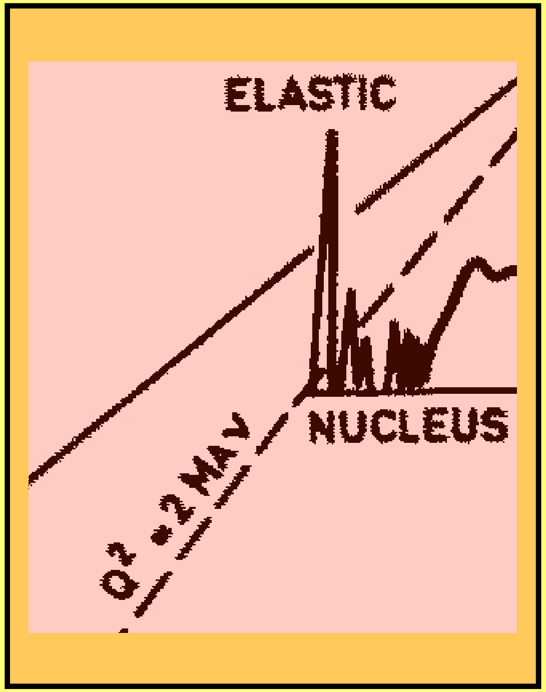
$$\begin{aligned} M_{fi} &= \int \Psi_f^* V(x) \Psi_i d^3x \\ &= \int e^{-k_f \cdot x} V(x) e^{-k_i \cdot x} d^3x \\ &= \int e^{iq \cdot x} V(x) d^3x \end{aligned}$$




Plane wave approximation for incoming and outgoing electrons

Born approximation (interact only once)

# I. Elastic Electron Scattering from Nuclei




## Form Factor and Charge Distribution

Using Coulomb potential from a charge distribution, ρ(x),

$$V(x) = -\frac{Ze^2}{4\pi\epsilon_o}\int \frac{\rho(x')}{|x-x'|}d^3x'$$

$$M_{fi} = -\frac{Ze^2}{4\pi\epsilon_o}\int e^{iq\cdot x}\int \frac{\rho(x')}{|x-x'|}d^3x'd^3x$$

$$= -\frac{Ze^2}{4\pi\epsilon_o}\int e^{iqR}\left[\int \frac{e^{iq\cdot x'}\rho(x')}{|R|}d^3x'\right]d^3R$$

$$= -\frac{Ze^2}{4\pi\epsilon_o}\int \frac{e^{iqR}}{R}d^3R\int e^{iq\cdot x'}\rho(x')d^3x'$$

$$F(q) = \int e^{iq\cdot x'}\rho(x')d^3x'$$

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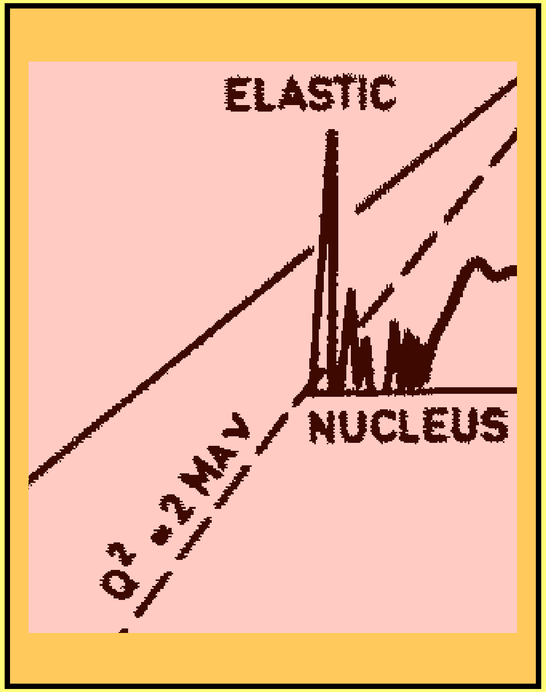



## Form factor and cross section

- For point-like particle, $\rho(\mathbf{x}') = \delta(\mathbf{x}')$ and $F(\mathbf{q}) = 1 \rightarrow$ Rutherford-*like* scattering

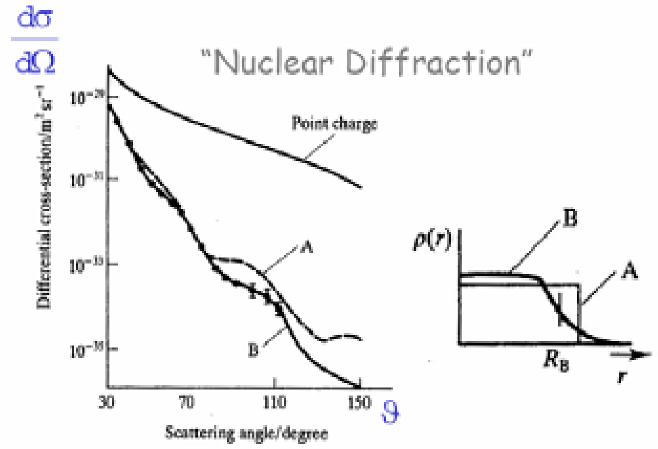
$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{point}} \equiv \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{(Z\alpha)^2 \cos^2\frac{\theta}{2}}{4E^2 \sin^4\frac{\theta}{2}}$$

Cos² term only difference
From Rutherford formula
Arises from Dirac theory
For spin ½ particle

- Scattering from a charge distribution

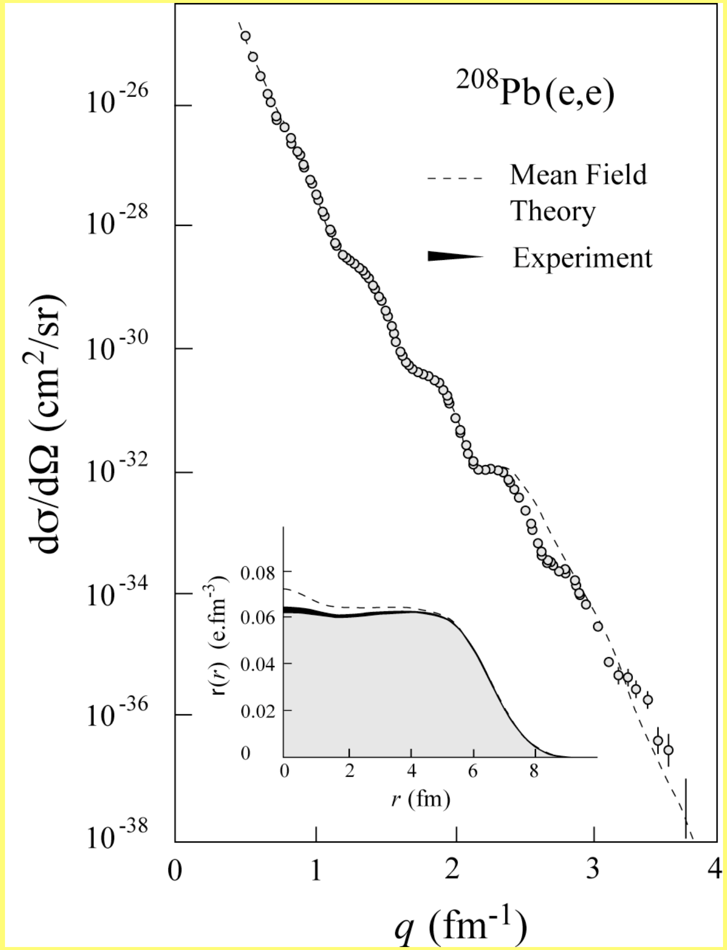
$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} |F(\mathbf{q})|^2$$

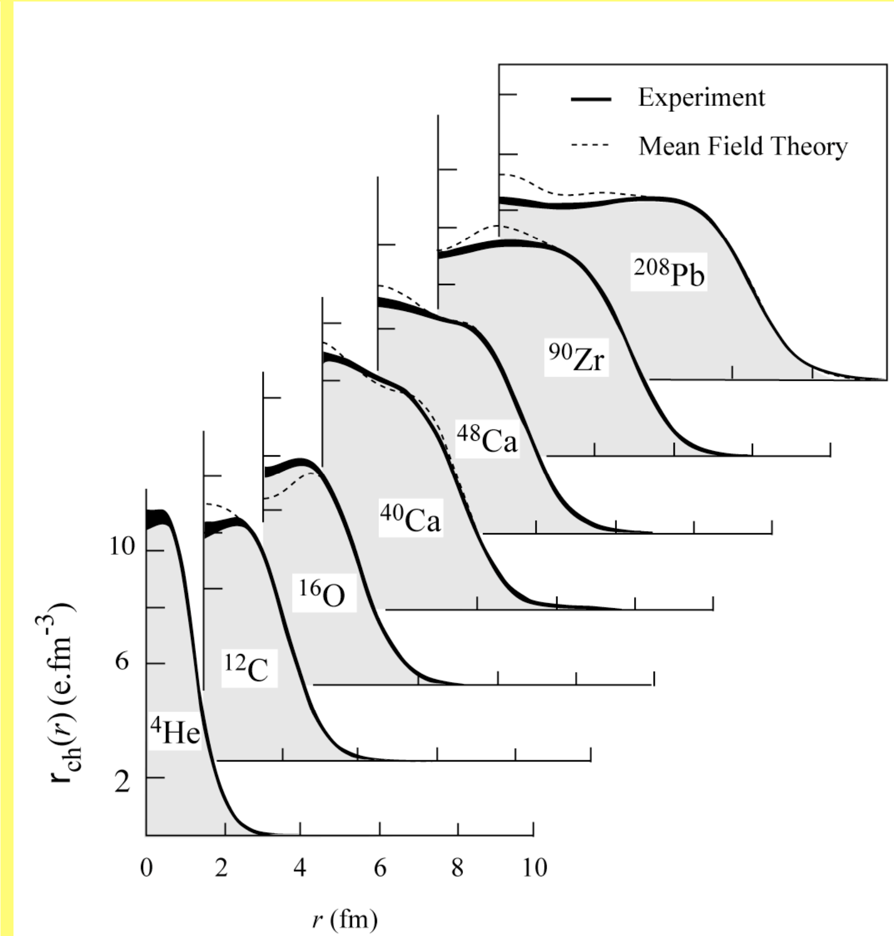
"Nuclear Diffraction"

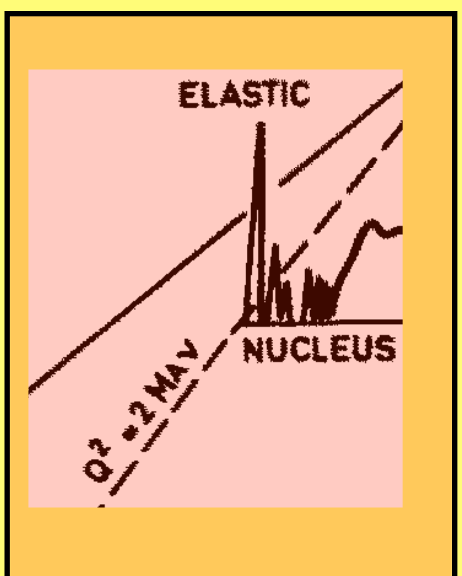



$$\rho(r) = \frac{1}{2\pi}\int F(q)\frac{\sin(qr/\hbar)}{(qr/\hbar)} 4\pi r^2 dr$$

# I. Elastic (e,e′) Scattering ⇒ charge distributions



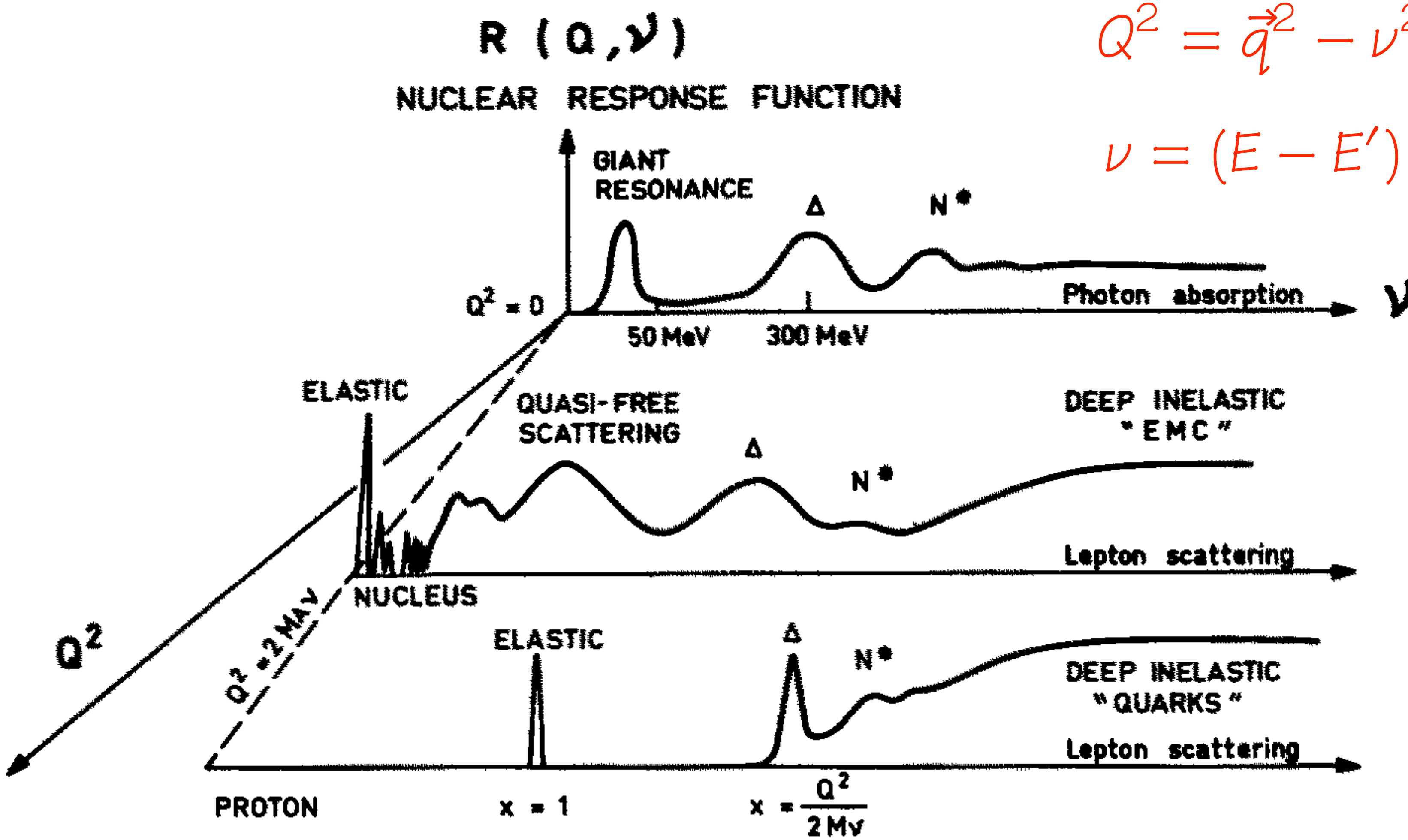




In '70s large data set was acquired on elastic electron scattering (mainly at Saclay) over large $Q^2$-range and for variety of nuclei

"Model-independent" analysis of these data provided accurate results on charge distribution for comparison with the best available theory: Mean-Field Density-Dependent Hartree-Fock

# Nuclear Response Function




$Q^2 = \vec{q}^2 - \nu^2$

$\nu = (E - E')$ $\quad (\nu \equiv \omega)$