The Neutron Electric Form Factor The Neutron is not Neutral

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Discovery of the Neutron

J. Chadwick

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Letters to the Editor

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Possible Existence of a Neutron

Ir has been shown by Bothe and others that beryllium when bombarded by a-particles of polonium emits a radiation of great penetrating power, which has an absorption coefficient in lead of about 0.3 (cm.)⁻¹. Recently Mme. Curie-Joliot and M. Joliot found, when measuring the ionisation produced by this beryllium radiation in a vessel with a thin window, that the ionisation increased when matter containing hydrogen was placed in front of the window. The effect appeared to be due to the ejection of protons with velocities up to a maximum of nearly $3 \times 10^{\circ}$ cm. per sec. They suggested that the transference of energy to the proton was by a process similar to the Compton effect, and estimated that the beryllium radiation had a quantum energy of $50 \times 10^{\circ}$ electron volts. This again receives a simple explanation on the neutron hypothesis.

If it be supposed that the radiation consists of quanta, then the capture of the α -particle by the Be⁹ nucleus will form a C¹³ nucleus. The mass defect of C¹³ is known with sufficient accuracy to show that the energy of the quantum emitted in this process cannot be greater than about 14×10^6 volts. It is difficult to make such a quantum responsible for the effects observed.

It is to be expected that many of the effects of a neutron in passing through matter should resemble those of a quantum of high energy, and it is not easy to reach the final decision between the two hypotheses. Up to the present, all the evidence is in favour of the neutron, while the quantum hypothesis can only be upheld if the conservation of energy and momentum be relinquished at some point.

J. CHADWICK.

Cavendish Laboratory, Cambridge, Feb. 17.

Neutrons account for $\approx \frac{1}{2}$ mass of ordinary matter No net electric charge Proton - Neutron mass difference: $M_n - M_p = 1.293$ MeV Free neutrons are unstable: $\tau = 888.6s$ $n \rightarrow p + e + \bar{\nu_e}$ Protons and Neutrons have Structure!

Early Indications

- * Anomalous magnetic moments of p and n
 - O. Stern, Nature 132 (1933) 169

	μ Dirac	μ observed
Proton	1 n.m.	+2.79 n.m.
Neutron	0	-1.91 n.m.

 * Non-zero neutron charge radius from scattering of thermal neutrons on atoms

$$\langle r_{ne}^2 \rangle = \frac{3m_e a_0}{m_n} b_{ne} = -0.113 \pm 0.003 \pm 0.004 \,\mathrm{fm}^2.$$

 * Experiments on Nucleon Structure go back to the mid 1950's at Stanford, see Nuclear and Nucleon Structure, R. Hofstader, W.A. Benjamin (1963).

Quarks, Gluons and QCD



$$proton = uud + gluons + q\bar{q}$$

neutron =
$$udd$$
 + gluons + $q\bar{q}$

Quantum Chromodynamics

- * 6 flavors of quarks, come in 3 colors, interact through the exchange of colored gluons
- Confinement (no free quarks or gluons)



- * No analytic solution
- * Asymptotic Freedom at high energies (pQCD)
- Responsible for residual interaction between protons and neutrons in nucleus



Probing the Ground State Substructure

- * Elastic electron scattering is ideal
- * Well understood (QED) electromagnetic interaction dominates
- * Interaction is "weak" : $\alpha = 1/137$
 - Perturbation theory works
- * But No Free Neutron Targets !!
 - Quasi elastic or elastic scattering from a nucleus
 - Deuterium (or ³He) preferred
 - * Amenable to "exact" calculations of nuclear structure

Elastic Electron Scattering - The Basics



Multi-Gev electrons allow $\Delta r \approx 10^{-13}$ m

Elastic Scattering Experiments

→ Rutherford discovered atomic nucleus through scattering alpha particles (He⁺⁺)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E\sin^4(\theta/2)}$$

→ Mott worked on consequences of the scattering electrons (spin $\frac{1}{2}$)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \cos^2(\theta/2)}{4E \sin^4(\theta/2)}$$

→ Dirac (point-like) nucleon with finite mass and recoil

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \cos^2(\theta/2)}{4E \sin^4(\theta/2)} \frac{E'}{E} \left[1 + \frac{Q^2}{2M^2} \tan^2(\theta/2) \right]$$

$$E' = \frac{E}{1 + \frac{2E}{M}\sin^2(\theta/2)}$$

4-momentum transfer, $Q^2 = 4EE'\sin^2(\theta/2)$

Form Factors

→ First introduced to describe the scattering on an extended charge distribution, $\rho(r)$, such that $\int \rho(r) d^3r = 1$



We define the form factor as the Fourier transform of the spatial distribution function,

 $F(q) = \int e^{iqr} \rho(r) d^3r$

Charge distribution

Form Factor

point	$\rho(r) = \delta(r - r_o)$	$F(q^2) = 1$	unity
exponential	$\rho(r) = \frac{a^3}{8\pi} e^{-ar}$	$F(q^2) = \left[\frac{1}{1+q^2/a^2}\right]^2$	dipole
Yukawa	$\rho(r) = \frac{a^2}{4\pi r} e^{-ar}$	$F(q^2) = \frac{1}{1+q^2/a^2}$	pole
Gaussian	$\rho(r) = \left(\frac{a^2}{2\pi}\right)^{3/2} e^{-(a^2 r^2/2)}$	$F(q^2) = e^{-(q^2/2a^2)}$	Gaussian

Form factor modifies the cross section formula in a simple way:

$$\frac{d\sigma}{d\Omega} \Rightarrow \frac{d\sigma}{d\Omega} \mid F(q^2) \mid^2$$

Form factors and Rosenbluth Formula

Proper accounting for the anomalous magnetic moments and form factors we get the Rosenbluth formula. F_1 and F_2 are the Dirac and Pauli form factors and have the normalization: $F_1^p = 1$ $F_1^n = 0$ $F_2^p = 1.79$ $F_2^n = -1.91$

$$\frac{d\sigma}{d\Omega} = \sigma_{\text{Mott}} \frac{E'}{E_0} \left\{ (F_1)^2 + \tau \left[2 \left(F_1 + F_2 \right)^2 \tan^2 \left(\theta_e \right) + \left(F_2 \right)^2 \right] \right\}; F_{1,2} = F_{1,2}(Q^2)$$



(a) Mott curve for spinless point-like proton
(b) Rosenbluth curve for a point-like proton with the Dirac magnetic moment (without anomalous magnetic moment) (F₁(q²) = 1, F₂(q²) = 0)
(c) Rosenbluth curve with contribution from anomalous magnetic moment for point-like proton (F₁(q²) = 1, F₂(q²) = κ = 1.79)
✓ The deviation of experimental data from curve (c) was interpreted as an effect from proton form factors - finite size proton. Later data was fitted with a dipole form for the form factors which implied an exponential charge distribution and an rms radius of

$$\big\langle r_E^2 \big\rangle_{(\rm proton)}^{1/2} = \big\langle r_M^2 \big\rangle_{(\rm proton)}^{1/2} = 0.86 \ {\rm fm}. \label{eq:rescaled}$$

Formalism

Sachs Form Factors:
$$G_E(Q^2) = F_1(Q^2) - \tau F_2(Q^2) \qquad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_{Mott}}{(1+\tau)} \frac{E'}{E_0} \left[G_E^2 + \underbrace{\tau(1+(1+\tau)2\tan^2(\theta/2))G_M^2}_{-1} \right]$$



$$Q^2 = 4EE'\sin^2(\theta/2) \qquad \tau = \frac{Q^2}{4M^2}$$

- * $G_{E,M}$ contain all the structure information
- * Separate G_E and G_M by angular dependence via a Rosenbluth separation (see underbrace)
- * For a point like probe G_E and G_M are the FT of the charge and magnetizations distributions in the nucleon, with the following normalizations

$$Q^2 = 0$$
 limit: $G_E^p = 1 \ G_E^n = 0 \ G_M^p = 2.79 \ G_M^n = -1.91$

G_E^n Interpretation

In the NR limit (Breit Frame), G_E is FT of the charge distribution $\rho(r)$:

$$G_E^n\left(\mathbf{q}^2\right) = \frac{1}{\left(2\pi\right)^3} \int d^3 r \rho(\mathbf{r}) e^{\left(i\mathbf{q}\cdot\mathbf{r}\right)} = \int d^3 \mathbf{r} \rho\left(\mathbf{r}\right) - \frac{\mathbf{q}^2}{6} \int d^3 \mathbf{r} \rho\left(\mathbf{r}\right) \mathbf{r}^2 + \dots = 0 - \frac{\mathbf{q}^2}{6} \left\langle r_{ne}^2 \right\rangle + \dots$$



Why measure G_E^n ?

- * FF are fundamental quantities
- * Test of QCD description of the nucleon Symmetric quark model, with all valence quarks with same wf: $G_E^n \equiv 0$ $G_E^n \neq 0 \rightarrow$ details of the wavefunctions



- More sensitive than other for factors to sea quark contributions
- * Soliton model: $\rho(r)$ at large r due to sea quarks

Dong, Liu, Williams, PRD 58 074504

Necessary for study of nuclear structure.

- ★ Few body structure functions
- * Explains $\langle \mathbf{r}_{\mathrm{ch}}^{\mathbf{2}} \rangle$ of ⁴⁸Ca as compared to ⁴⁰Ca

Proton Form Factor Data (pre-1998)

Rosenbluth formula, Rosenbluth separation:



G_E^n Measurements

- * No free neutron target \longrightarrow Use deuteron
- * proton dominates neutron \longrightarrow detect neutron
- * G_M^n dominates $G_E^n \longrightarrow$ use G_M^n to advantage
- * Inclusive cross section measurements on deuteron:
 - Elastic e D scattering at small angles:
 - \rightarrow dependence on nucleon nucleon potential
 - \rightarrow subtraction of dominant proton contribution
 - Quasielastic e D scattering
 - $\rightarrow Rosenbluth \ separation$
 - \rightarrow Sensitive to deuteron structure
- * Double Polarization measurements

asymmetry measurement detection of neutron in coincidence

- \rightarrow less sensitive to deuteron structure
- \rightarrow avoid Rosenbluth separation
- \rightarrow avoid subtraction of proton contribution

- $D(\overrightarrow{e}, e'\overrightarrow{n})p, \overrightarrow{D}(\overrightarrow{e}, e'n)p, \overrightarrow{^{3}He}(\overrightarrow{e}, e'n)pp$

${\cal G}_{\cal M}^n$ unpolarized and polarized



G_E^n via e - D elastic scattering

No free neutron – extract from e - D elastic scattering:

small θ_e approximation



Galster Parametrization: $G_E^n = -\frac{\tau \mu_n}{1+5.6\tau}G_D$

 G_E^n at large Q^2 through ${}^2\mathrm{H}(e,e')X$

PWIA model σ is incoherent sum of p and n cross section folded with deuteron structure.

- $\sigma = (\sigma_p + \sigma_n) I(u, w)$ $= \varepsilon R_L + R_T$
- * Extraction of G_E^n : Rosenbluth Separation $\Rightarrow R_L$ Subtraction of proton contribution
- * Problems:

Unfavorable error propagation Sensitivity to deuteron structure

SLAC: *A. Lung et al, PRL.* 70, 718 (1993) \rightarrow No indication of non-zero G_E^n



If G_E^n is small at large Q^2 then F_1^n must cancel τF_2^n , begging the question, how does F_1^n evolve from 0 at $Q^2 = 0$ to cancel τF_2^n at large Q^2 ?

Theory

- * Ground state QCD structure is a strong coupling problem: currently unsolvable
 - Lattice calculations of form factors just beginning
- * Models: try to capture aspects of QCD solution
 - Bag and Quark models
 - Vector Meson Dominance (VMD)
- * pQCD predicts large Q^2 behavior- Q^4 scaling

Models of Nucleon Form Factors				
$F(Q^2) = \sum_i \frac{C_{\gamma V_i}}{Q^2 + M_{V_i}^2} F_{V_i N}(Q^2)$				
breaks down at large Q^2				
Lu, Thomas, Williams (1998)				
$F_2 \propto F_1\left(rac{M}{Q^2} ight)$ helicity conservation				
Counting rules: $F_1 \propto \frac{\alpha_s^2(Q^2)}{Q^4}$				
$Q^2 F_2/F_1 \rightarrow \text{constant}$				
JLAB proton data: $QF_2/F_1 \rightarrow \text{constant}$				
GK, Lomon				
Dong (1998)				
point form (Wagenbrunn)				
light front (Cardarelli)				
Holzwarth				
Miller				
pQCD (Ralston) LF (Miller)				



How to measure small quantities like G_E^n

Use spin observables since they often result from interference between amplitudes

Very Schematically

some operator $\mathcal{O} = \mathcal{O}_{Big} + \mathcal{O}_{Small}$

unpolarized crossection: $d\sigma \propto |\langle f | \mathcal{O}_{Big} | i \rangle|^2 + |\langle f | \mathcal{O}_{Small} | i \rangle|^2$ while spin observables contain terms like: $\langle f | \mathcal{O}_{Big} | i \rangle^* \langle f | \mathcal{O}_{Small} | i \rangle$ which is linear in small quantity but with a large coefficient. For the form factors : $\mathcal{O} \propto G_E G_M$ instead of $\mathcal{O} \propto G_E^2 + G_M^2$

Two techniques

- * Recoil Polarization
- * Beam-Target Asymmetry

CEBAF and Hall C

G_E^n from spin observables

No free neutron targets – scattering from 2 H or 3 He– can not avoid engaging the details of the nuclear physics.

Minimize sensitivity to the how the reaction is treated and maximize the sensitivity to the neutron form factors by working in quasifree kinematics. Detect neutron.

- * Indirect measurements: The experimental asymmetries $(\xi_{s'}, A_V^{ed}, A_{exp}^{qe})$ are compared to theoretical calculations.
- * Theoretical calculations are generated for scaled values of the form factor.
- * Form factor is extracted by comparison of the experimental asymmetry to acceptance averaged theory. Monte Carlo
- * Polarized targets

The deuteron and ³He only approximate a polarized neutron Scattering from other unpolarized materials, f dilution factor



Recoil polarization



Elastic scattering of polarised nucleons on unpolarised protons has analysing power $\epsilon(\theta_n)$ due to spin-orbit term V_{LS} in NN interaction.

Left-right asymmetry is observed if the proton is polarized vertically.



Recoil Polarization – Principle and Practice

- * Interested in transferred polarization, P_l and P_t , at the target
- * Polarimeters are sensitive to the perpendicular components only, P_n^{pol} and P_t^{pol}

Measuring the ratio P_t/P_l requires the precession of P_l by angle χ before the polarimeter.

* If polarization precesses χ (e.g. in a dipole with \vec{B} normal to scattering plane):

$$P_t^{\text{por}} = \sin \chi \cdot P_l + \cos \chi \cdot P_t$$

For $\chi = 90^{\circ}$, $P_t^{\text{pol}} = P_l$ and is related to G_M^2

For $\chi = 0^{\circ}$, $P_t^{\text{pol}} = P_t$ and is related to $G_E G_M$

* G_E^n/G_M^n via ²H($\vec{e}, e'\vec{n}$)p in JLAB's Hall C - Charybdis and N-Pol

 G_E^n in Hall C, E93-038

Recoil polarization, ${}^{2}\mathrm{H}(\vec{e}, e'\vec{n})p$

- * In quasifree kinematics, $P_{s'}$ is sensitive to G_E^n and insensitive to nuclear physics
- * Up–down asymmetry $\xi \Rightarrow$ transverse (sideways) polarization $P_{s'} = \xi_{s'}/P_e A_{\text{pol}}$. Requires knowledge of P_e and A_{pol}
- * Rotate the polarization vector in the scattering plane (with Charybdis) and measure the longitudinal polarization, $P_{l'} = \xi_{l'}/P_e A_{pol}$
- * Take ratio, $\frac{P_{s'}}{P_{l'}}$. P_e and A_{pol} cancel
- * Three momentum transfers, $Q^2 = 0.45, 1.13$, and $1.45 (GeV/c)^2$.
- * Data taking 2000/2001.

G_E^n in Hall C via ${}^2\mathrm{H}(\vec{e},e'\vec{n})p$



G_E^n in Hall C via ${}^2\mathrm{H}(\vec{e},e'\vec{n})p$



Taking the ratio eliminates the dependence on the analyzing power and the beam polarization \rightarrow greatly reduced systematics

$$\frac{G_E^n}{G_M^n} = K \tan \delta \quad \text{where} \quad \tan \delta = \frac{P_{s'}}{P_{l'}} = \frac{\xi_{s'}}{\xi_{l'}}$$



Left: Coincidence TOF for neutrons. Difference between measured TOF and calculated TOF assuming quasi-elastic neutron. Right: ΔTOF for neutron in front array and neutron in rear array.

 ΔTOF is kept as the four combinations of (-,+) helicity, and (Upper,Lower) detector and cross ratios formed. False asymmetries cancel.

$$r = \left(\frac{N_U^+ N_D^-}{N_U^- N_D^+}\right)^{1/2} \qquad \xi = (r-1)/(r+1)$$

G_E^n in Hall C via $^2{\rm H}(\vec{e},e'\vec{n})p$



Results through ${}^{2}\mathrm{H}(\vec{e},e'\vec{n})p$





Experimental Asymmetry

Quasi-Elastic Scattering off Polarized Deuteron

$$\epsilon = P_e \frac{(1-\beta)A_e + (1+\alpha\beta)P_t^V A_{ed}^V + (1-\beta\gamma)P_t^T A_{ed}^T}{(1+\beta) + (1-\alpha\beta)P_t^V A_d^V + (1+\beta\gamma)P_t^T A_d^T}$$

 P_t^V , P_t^T = vector, tensor polarization α , β , γ = normalization ratios

- * Deuteron supports a tensor polarization, P_t^T , in addition to the usual vector polarization, P_t^V
 - This can lead to both helicity dependent and helicity independent contributions

After (symmetric) acceptance averaging and ignoring small P_t^T

$$\epsilon = \frac{1+\alpha\beta}{1+\beta} P_e P_t^V A_{ed}^V$$
 or
$$A_{ed}^V = \frac{1+\beta}{(1+\alpha\beta) P_e P_t^V} \epsilon$$

 G_E^n extracted via A_{ed}^V from data and MC simulation

Beam–Target Asymmetry in E93-026

 $^{2}\overrightarrow{\mathrm{H}}(\overrightarrow{e},e'n)p$

 $\sigma(h,P) \approx \sigma_0 \left(1 + hPA_{ed}^V\right)$

h: Beam Helicity

P: Vector Target Polarization

T: Tensor Target Polarization $T = 2 - \sqrt{4 - 3P^2}$

 A_d^T is suppressed by $T\approx 3\%$

Theoretical Calculations of electrodisintegration of the deuteron by H. Arenhövel and co-workers

E93-026 $\overrightarrow{\mathbf{D}}(\overrightarrow{e}, e'n)p \quad \left(\overline{\sigma}(h, P) = \sigma_0 \left(1 + hPA_{ed}^V\right)\right)$

A_{ed}^V is sensitive to G_E^n

has low sensitivity to potential models

has low sensitivity to subnuclear degrees of freedom (MEC, IC) in quasielastic kinematics

Sensitivity to G_E^n – Insensitivity to Reaction







Target Stick



Neutron Detector

- * Highly segmented scintillator
- * Rates: 50 200 kHz per detector
- * Pb shielding in front to reduce background
- * 2 thin planes for particle ID (VETO)
- * 6 thick conversion planes
- * 142 elements total, >280 channels

- Extended front section for symmetric proton coverage
- * PMTs on both ends of scintillator
- * Spatial resolution $\simeq 10$ cm
- * Time resolution $\simeq 400 \text{ ps}$
- Provides 3 space coordinates, time and energy



Neutron Detector





Experimental Technique for $\overrightarrow{\mathrm{D}}(\overrightarrow{e},e'n)p$

For different orientations of *h* and *P*: $N^{hP} \propto \sigma(h, P)$

Beam-target Asymmetry:

$$\epsilon = \frac{N^{\uparrow\uparrow} - N^{\downarrow\uparrow} + N^{\downarrow\downarrow} - N^{\uparrow\downarrow}}{N^{\uparrow\uparrow} + N^{\downarrow\uparrow} + N^{\downarrow\downarrow} + N^{\uparrow\uparrow}} = hPfA_{ed}^V$$



Data and MC Comparison



Extracting G_E^n



E93026 Results





Laboratory	Collaboration	$\mathbf{Q^2}(\mathbf{GeV/c})^{2}$	Reaction	Reported
MIT-Bates	E85-05	0.255	$^{2}\mathrm{H}(\tilde{\mathrm{e}},\mathrm{e}^{\prime}\tilde{\mathrm{n}})$	1994
		<0.8	$^{2}\tilde{H}(\tilde{e},e'n)$	Planned
		<0.8	${}^{3}\tilde{\mathrm{He}}(\tilde{\mathrm{e}},\mathrm{e'n})$	Planned
Mainz-MAMI	A3	0.31	${}^{3}\tilde{\mathrm{He}}(\tilde{\mathrm{e}},\mathrm{e'n})$	1994
	A3	0.15, 0.34	$^{2}\mathrm{H}(\mathrm{\tilde{e}},\mathrm{e}^{\prime}\mathrm{\tilde{n}})$	1999
	A3	0.385	${}^{3}\tilde{\mathrm{He}}(\tilde{\mathrm{e}},\mathrm{e'n})$	1999
	A1	0.67	${}^{3}\tilde{\mathrm{He}}(\tilde{\mathrm{e}},\mathrm{e'n})$	1999/2003
	A1	0.3, 0.6, 0.8	$^{2}\mathrm{H}(\tilde{\mathrm{e}},\mathrm{e}^{\prime}\tilde{\mathrm{n}})$	Analysis
NIKHEF		0.21	$^{2}\tilde{H}(\tilde{e},e'n)$	1999
Jefferson Lab	E93026	0.5, 1.0	$^{2}\overline{\tilde{H}}(\tilde{e},e'n)$	2001/2004
	E93038	0.45, 1.15, 1.47	$^{2}\mathrm{H}(\tilde{\mathrm{e}},\mathrm{e}^{\prime}\tilde{\mathrm{n}})$	2003
	E02013	1.3, 2.4, 3.4	${}^{3} ext{He}(ilde{ ext{e}}, ext{e'n})$	Approved

Conclusions

- * G_E^n remains the poorest known of the four nucleon form factors.
- * G_E^n is a fundamental quantity of continued interest.
- * Significant progress has been made at several laboratories by exploiting spin correlations
- * Data under analysis is of sufficient quality to test QCD inspired models.
- * Future progress likely with new experiments and better theory.